# PHILOSOPHY AND MODERN SCIENCE

HAROLD T. DAVIS

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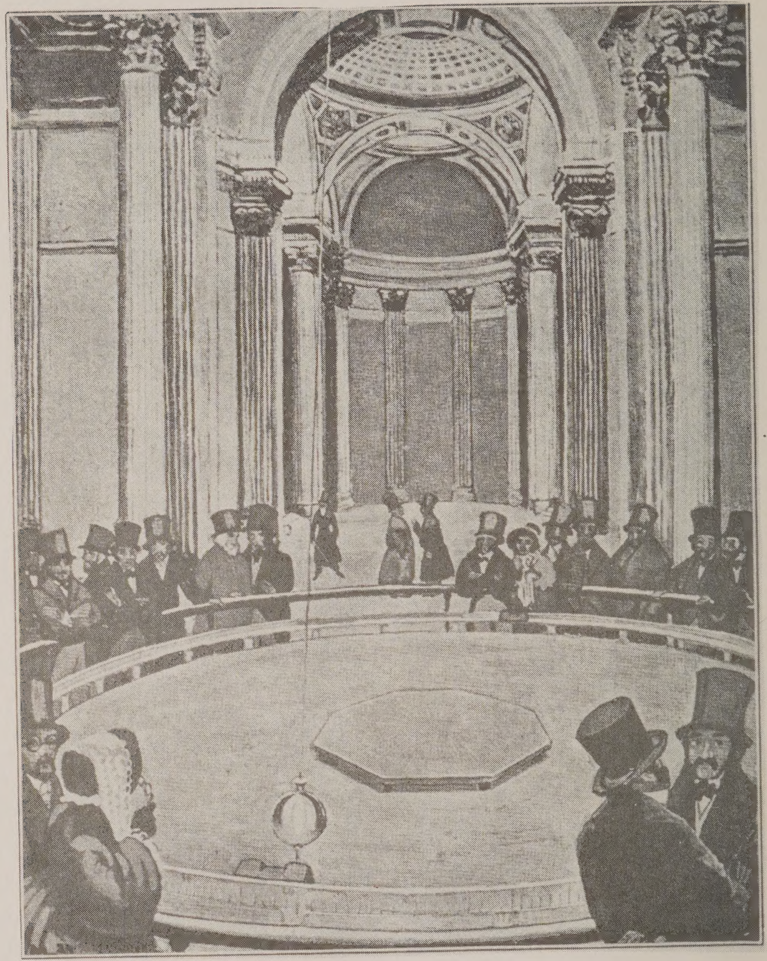

PLATE I

The Foucault pendulum experiment was performed in the Pantheon at Paris in 1851. One explanation of its mystery is connected with modern limitations imposed upon space and time. (From Hanotaux's *Histoire de la Nation Française*, vol. xiv).

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# PHILOSOPHY
# and
# MODERN SCIENCE

By

HAROLD T. DAVIS
Northwestern University

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TO MY MOTHER

This Book is Affectionately Dedicated.

The poet Horace to his friend Iccius:

"You, in the very midst of the contagious itch of gain, still have a taste far from mean, still set your thoughts on lofty themes: what causes hold the sea in check, what rules the year, whether stars roam at large of their own will or by law, what hides the moon's disk in darkness, what brings it into light, what is the meaning and what the effects of Nature's jarring harmony."

Epistle XII, Book 1, lines 14-19.

# PREFACE TO THE SECOND EDITION.

Twenty years have come and gone since the first edition of this book was written under the stimulating influence of the astonishing discoveries of modern physical science. But in that two decades and more, a mere moment in the long history of mankind, there have transpired events of vast importance, which in some respects over-shadow any that have been recorded in the previous annals of the race. These events are the discoveries of science; they are the logical fruition of the trend of speculation which has been set forth in the first edition of this book.

The story which had been unfolded in the pages written before 1931 used as its connecting thread the nature of energy. This was traced from the first vague gropings of the scientists of classical antiquity, through the intellectual renaissance of the sixteenth and seventeenth centuries, down to the mature speculations of modern scientists. What is the character of energy? How is it created in the sun and the other stars? How is it related to matter and what is its nature in the space between material objects? An essential clue to this relationship was provided by the famous equation of Einstein, $E = mc^2$, which up to 1931 and even later had more the status of a brilliant and appealing speculation than a fact of experimental physics. The astronomers seized upon it avidly as the basis of an explanation of stellar energies; the physicists viewed it with vague uneasiness for it promised them more energy than perhaps they wished for in their exploration of the mysteries of the atomic nucleus.

In the first edition of the book the quest of the alchemists for the transmutation of metals, for the philosopher's stone of the ancients, was described in the pioneer investigations of Rutherford and his associates. Intriguing as these discoveries were, however, they fell far short of the alchemist's goal. The tools had not yet been forged by means of which the exploration into the depths of the atom could be carried on. These tools were high-speed particles which could penetrate the potential barrier of the planetary electrons with their negative charges of electricity, and that of the more deep-seated protons with their positive charges. The discovery in 1932 of the neutron, a particle without charge, provided the very tool which was needed for the continuation of the quest. But high velocities of all particles involved were necessary also and the invention of such powerful instruments as the cyclotron, the high voltage generator, and the betatron have provided the great energies which were required in the search.

To indicate the intensity of the investigations of the past twenty years one needs only to enumerate a few of the momentous discoveries. In addition to the neutron, there has been added to the family of atomic particles the ephemeral positron with a life cycle of a minute fraction of a second. From the study of cosmic radiation there has emerged "the intermediate particle," the elusive meson, whose potentialities are as yet but dimly understood. Artificial radioactivity has been discovered and a number of these synthetic compounds have been created in the laboratory with half lives which vary from less than a second to many years. A large number of isotopes have been discovered, from the spectacular deuterium of hydrogen, better known to the lay public as the constituent of heavy water, to the portentous uranium 235. Several new transuranic elements have been created, among them the ephemeral neptunium with its very short half life, and the more stable plutonium and several others.

And finally, to climax this orgie of discovery the great secret of transmutation has achieved a dramatic and terrible realization in the vast energies released by atomic fission, that is to say, the splitting of atoms into large fragments. This event has shaken the world of the common man to its foundation. Hitherto content to allow the scientist the solitude of his laboratory where he probes the mysteries of nature, the world has now awakened to the fact that his secrets have a terrifying application to its own destiny. The man of science has entered the arena of international politics armed with a weapon of unparalleled power. The equation of Einstein has finally achieved a terrible reality. The philosophy of science, which appeared to have little relationship to the ordinary affairs of man, has now emerged as one of its most vital problems. The genie has come forth from its bottle enveloped in a pillar of fire and no power is there to restore it once more to its prison.

In the first edition of this work the philosophical problems were concerned very largely with those questions which enveloped the subject of energy. Thus we discussed the meaning of external principles in nature, the proposition of least action, the maximizing of entropy and the fatal heat-death of the universe which was its consequence. We were concerned with the problem of continuity as it appears in the phenomena of waves; and the related problem of the discrete, as it applies to the minute particles of physics. From these were derived the concepts of zero and infinity, and the antinomies of pure reason which have so greatly concerned the classical philosophers. Determinism, the laws of probability, freedom of the will and other questions with roots in the writings of the ancients appeared as concomitants of some of the most exciting measurements from the laboratory. In these was found rich pastorage for philosophical discussion.

But in the new edition the reader will find that other and more personal matters have intruded in the text. The natural philosopher has expanded beyond the walls of his laboratory. In the terror of the atomic bomb the problems have changed from those of abstract philosophy to those which involve the nature of man himself. Problems of economics, standards of living, political philosophies, and the nature of society itself are now thrust into the picture of the new physics. What was at one time suspected by a few is now apprehended by all men that the problems of the natural world are also problems of the social world as well. It is to the explanation and the interpretation of this new trend of natural philosophy that some of the ensuing pages are devoted.

1953.

H. T. Davis.

## PREFACE TO THE FIRST EDITION.

The present book tells the romance of modern physical science. It attempts to humanize certain data and speculations which have emerged from a body of recent astonishing experiments. It begins with some problems first proposed by the Greeks and traces the story of their development during subsequent centuries. Particularly are the strange concepts of these recent days set forth as the logical culmination of past experience.

The author has long cherished the belief that we are witnessing in the present development of science the uniting of two great streams of speculative thought which have their sources in the high plateaus of Greek philosophy. They diverged during the course of the centuries, one stream flowing through the philosophies of Descartes, Berkeley, Kant, Poincaré, and Mach, the other through the experiences of Galileo, Newton, Fresnel, von Helmholtz, Faraday, Lord Kelvin, Maxwell, and Lorentz. In this modern era we have witnessed the merging of the two streams. It is difficut to say whether Einstein and his great colleagues in this golden age of science are subjective philosophers or investigators of physical nature. Their work is revolutionary, but it has produced not so much a factual as a conceptual renaissance. Relativity, the wave mechanics, cosmic radiation, and the mysteries of the new concepts of energy and the atom have caused us to turn a speculative eye upon the most cherished principles of the older physics. It is difficult to say, for example, whether the theory of gravitation is to be regarded as a development of geometry, or whether it is to be thought of as an explanation of objective experience.

The present volume attempts to set forth as simply as possible the basic postulates of physics and to trace their implications. Technical language has been avoided as far as the subject matter will permit. The philosophical aspect of the problems has been constantly emphasized. It has been shown that the concept of the ether is found in the "extension" of Descartes. The history of man's attempt to endow this "necessity of reason" with material properties has been carefully developed down to the recent experiments at the basis of the wave mechanics, where it disappears in the mystical postulates of Schrödinger, Dirac, Heisenberg, and de Broglie.

The book was originally undertaken to prove to a friend that there is much that is poetical in the story of modern science. His contention was that "east is east; and west is west; and never the twain shall meet." But to one who will look past the long rows of figures in Leverrier's tables of the planets, or beyond the dreary pages of formulas in Maxwell's treatise on electricity and magnetism, there is a land of mystery and magic.

1931.

# PHILOSOPHY AND MODERN SCIENCE

# CHAPTER 1.

## THE PROBLEM OF ENERGY

"Indeed the scientific activity is the only one which is obviously and undoubtedly cumulative and progressive. . . . When one reads the history of science one has the exhilarating feeling of climbing a big mountain."

George Sarton in *The History of Science and the New Humanism.*

### 1. *Preliminary Concepts.*

The quest for energy is as old as the race itself. Observing the sinews of the lion, the strength of the elephant, the force of the wind, the heat of the sun, and the power of the cataract, early man devoted certain of his efforts to the utilization of the mysterious entity which appeared to flow through these natural objects. For the most part, however, his interest was utilitarian; the urge was for practical devices by means of which natural sources of energy could be converted into useful work. In this application, however, early man never attained any singular achievement, and the reason for this, perhaps, was because he failed to turn a speculative eye upon the nature of the energy which he desired to use.

The object of the present work is to tell the story of the dramatic discoveries which have led step by step to our present knowledge of the nature of energy and of its relationship to the material universe with which it is so fundamentally connected. These investigations have progressed by many paths, some of which have seemed far removed from the ultimate objectives of the search. They have introduced numerous *concepts* upon which modern science has sought to found its theories; they have produced such terms as "the luminiferous ether, universal gravitation, action, entropy, wave electrons, photons, etc." They have led from such abstract concepts as cosmic space to the terrifying reality of atomic bombs. It is to the description of the various theories and of their ultimate coördination in a grand concept of the fundamental unity of matter and energy that these pages will be devoted.

Philosophical speculation must be woven inevitably into the fabric of the story, since those natural scientists who have contributed most to the understanding of energy, almost without exception, have been natural philosophers as well. But their statements about the new concepts which they introduce will never be the frequently vague and unsatisfactory statements of traditional metaphysics. For, however strange these concepts may appear in words, they can always be reduced to numerical calculation and the formulas of mathematics. There is no achievement in science which is higher than this.

To some there is a natural abhorrence in mathematizing the objects of experience. Although there is an unquestionable gain in the accuracy of description to give the wave lengths of the light which forms the golden glow of a sunset, the soul of the poet is rudely shocked by this intrusion of the scientific spirit into his subjective domain. The philosopher for the most part also seems to prefer more latitude in his epistemology than is provided by the restraints of mathematical and numerical boundaries. The scientist, however, feels vague uneasiness unless his theory is properly buttressed by mathematical formula and numerical calculation. The most widely cited statement of this conflict is one made by Lord Kelvin in a popular lecture in 1889 which puts the matter thus: "I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind."[1]

But in the beginning, as man grappled with the mysterious forces observed by him, gravitation which needed no material medium to convey its influence from one object to another, heat and light which flowed across empty space, he was forced necessarily into the position of making assumptions that offended what might be called his rational intuitions. He avoided the pitfalls of metaphysical speculation, however, by formulating his concepts in the form of *postulates.*

One of the most conspicuous differences between a scientific discipline and one that does not employ the scientific method is found in the scientist's use of postulates, that is to say, affirmations which are to be used as temporary propositions that account for existing facts, or which may be admitted as the basis for deductive reasoning. Perhaps it is as much for its introduction into science of the postulational method as for any other reason that the *Elements* of Euclid, written about 300 B. C., should have attained such a conspicuous and enduring eminence. There is no doubt that the appearance of this work so early in the history of science was a remarkably fortunate event, since the development of geometry as a postulational discipline provided a perfect model for the guide of those who investigated the phenomena of more obscure domains of knowledge.

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[1] *Popular Lectures and Addresses* by Lord Kelvin, London, Vol. 1, 1889, p. 73; also, see *Life of Lord Kelvin,* by S. P. Thompson, Vol. 2, 1910, p. 792.

Today many examples can be found which illustrate the development of science through the modification of fundamental postulates. Propositions which were believed to be completely valid in one period of scientific history have sometimes been replaced by others, or have been modified in essential particulars by the discovery of new facts. Newton's law of gravitation is an example of a postulate which has played a long and important part in the history of physics and astronomy. But the proposition that it is after all a postulate and not an absolute truth, or a complete law of nature, has been strikingly revealed in recent years by the modifications both in fact and theory which have been imposed upon the law by Einstein's theory of gravitation. The nature of these changes will be discussed in more detail later in the book. The point at issue is the recognition of the postulational character of the original statement of the law.

### 2. *The Anthropomorphic Character of Scientific Concepts.*

It seems a curious thing when one reflects about it that most of our discoveries in physical science should fit so neatly into purely rational concepts. In other words we are not constantly surprised by many of the things we find out about nature. In a certain sense there seems to be a sort of anthropomorphic character to the behavior of energy and matter. By this we mean that we can make equations to describe their properties and the marvel is that the equations which are derived purely from logical concepts are in such remarkable accord with the facts. It is little wonder that in building concepts about the Deity who according to our postulates has organized and ordered nature, we should have given him anthropomorphic character. Some satirist has remarked that if animals could have formulated conceptions of the world they would have pictured God in their images. This idea, ancient as man's thought, appears as follows in a fragment from Xenophanes of Colophon (570-480 B. C.)[2]:

"But if cattle or lions had hands, so as to paint with their hands, and produce works of art as men do, they would paint their gods and give them bodies in form like their own — horses like horses, cattle like cattle."

With our characteristically human methods of arriving at abstract concepts through analogy it is not surprising that we have approached the problems of nature as if they were all subject to what we are pleased to call rational belief.

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[2] See A. Fairbanks: *The First Philosophers of Greece*, 1898, p. 67. Also E. Hiller and W. T. Bergk: *Anthologia Lyrica sive Lyricorum Graecorum Veterum praeter Pindar*, 1901, p. 54 (17).

The age of science has been founded upon the postulate that nature is rational. By this word we shall mean that nature obeys well-ordered rules which are understandable by human intelligence and conform to models that man constructs upon the basis of his intuition. It is a curious fact, however, that the early intuitions of man led in the direction of the mystical and the supernatural. Man fled in terror before the thunder storm in the belief that it was evidence of an avenging Deity, instead of trying to give it a rational explanation. Mythology clings about the motions of the sun, the moon, and the stars. There did not seem to be the slightest disposition among the vast majority of mankind and most of the poets to see in nature anything but caprice. The black plague was but an act of God before which man stood in stark impotence. Although the most casual survey will show that a great many things were lying within the reach of easy discovery which would have hastened materially the age of science, there seemed to be a peculiar antipathy to the idea that nature was rational. The annals of mediaeval history are full of miracles and the credulity of that period would pass belief if it were not for similar material evidence in the present stage of civilization.

It is a proper question to ask why man has always clung so tenaciously to the mystical view of nature. It was not primarily due to the fear felt by some that authority in religion was menaced, although this had a powerful influence during the middle ages. For the same venomous criticism which overwhelmed Copernicus (1473-1543) when he avocated the heliocentric theory of the universe appeared also against Aristarchus (c. 280 B. C.) for advancing the same theory some twenty centuries earlier.[3] It seems incredible that so beautiful a discovery as Galileo's that Jupiter was encircled by moons should have been met with a black and menacing prejudice. Why were not the minds of his day quickened by the discovery that nature can be investigated by instruments of precision? Why should the attempt of Copernicus to bring simplicity and order to the motion of the planets have met with sinister intolerance? We are able to study the same phenomenon in our own day in the antagonism that the proponents of evolution have encountered in proposing a rational rather than a mystical explanation of creation.

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[3] This is on the authority of Plutarch in his essay: *On the Face Appearing in the Face of the Moon*, 6: See Chapter 5, Section 1.

It is not our purpose, however, to discuss this curious problem of the reluctance of man to accept the discoveries of science, especially when they were in conflict with cherished mysticism. It is far more instructive to start with the scientific reformation and to show how the postulate of a rational and intelligible nature has led to the magnificent discoveries of modern science and then has gone beyond to a new realm where rational intuition is no longer sufficient. In other words science which started out so auspiciously with the telescope of Galileo and with Newton's theory of the moon has returned through recent discoveries to a remarkable new metaphysics. Rational concepts again merge into a kind of mysticism, but with this difference, that the new mysticism is founded upon experiments carried out by instruments of the highest precision.

As we progress in our discoveries we are beginning to find a limit to the anthropomorphic analogy. The rational belief with which science regards nature and the actual behavior of nature seem to be at least in partial accord. The attitude of science that all things can be explained in terms that do not conflict with the processes of our minds is certainly not in conflict with the major part of natural phenomena. However, there seems to be a limitation to this postulate. It is much the same as trying to fit a plane surface to a sphere. At every individual point the plane will fit, but if we progress to other points the divergence appears. For what are called first-order phenomena, nature is in harmony with rational belief. Newton's laws of motion account for all the major movements of the planets, but they fail to account for the tiny discrepancy in the motion of the perihelion of the planet Mercury, as we shall later describe.

One of the most fundamental of our rational beliefs is the existence of an objective nature. Philosophers have long debated this point and the famous skepticism of George Berkeley and David Hume culminated in a denial of the possibility of demonstrating the independent existence of the material world. The pendulum has swung since then and will probably continue to swing between the purely mechanistic view that all things belong to an external reality which is gradually discovered by our minds and the subjective view that all things are entirely created by us. But for the purpose of science it seems necessary to postulate an objective world. Otherwise, nature would correspond to the character of its creator. Existing wholly within the mind it would be necessarily subjective; laboratory experiments could not reveal anything that was not in accord with mental processes and science would be merely an exploration of the behavior of the mind. On such a hypothesis it would seem impossible to reach conclusions that would surprise us, such, for example, as those which led to the theory of wave mechanics.

Assuming that nature exists objectively, the scientist explores it from the point of view of what we have called rational belief. It is true that this expression is difficult to define and postulates by different people as to what constitutes rational belief would certainly show marked differences; for most, however, it is probable that these differences would be inconsequential. Let us consider, for example, that significant portion of modern physics which rests upon the propositions which underlie the theory of probability. The foundation of this theory is certainly a rational belief. To put it in its simplest terms, the theory of probability rests upon the belief that if a coin is tossed sufficiently often the ratio of the number of times it falls heads to the number of tosses will approach the fraction ½ as a limiting value. This proposition cannot be proved, but it is so convincing nevertheless that our minds are willing to admit it without statistical or other evidence.

Nothing more radical than the model of penny-tossing was used as the fundamental postulate in studies of the thermal behavior of gases. It is no wonder, therefore, that the theory of thermodynamics appears to rest upon a foundation more secure than that of other domains of physical science. And yet it was the failure of the law of the equipartition of energy (See Chapter 10) in this fundamental theory that led to the perplexing difficulties of quantum mechanics. The anthropomorphic model failed as soon as investigations had been carried far enough. Rational concepts appeared to break down and a very radical recasting of the entire problem was required.

Although the difficulties of which we speak have now been absorbed into the general pattern of modern physics and the consternation which they created originally is now remembered rather as an astonishing incident in the history of science than as a matter of current importance, the truth is that the paradoxes and the beliefs in mutually contradictory propositions have never been resolved. Is it any wonder, therefore, that we should find in contemporaneous opinion the expression of doubt about the fundamental rationality of nature? "It is *possible*," said Bertrand Russell, "that the desire for rational explanation may be carried too far," although he avoids a categorical expression of opinion.[4]

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[4] *The A, B, C of Atoms*, New York, 1923, p. 157.

The most lucid statement of this point of view is that of A. S. Eddington, who wrote as follows:[5]

"It is one thing for the human mind to extract from the phenomena of nature the exact laws over which it has no control. It is even possible that laws which have not their origin in the mind may be irrational, and we can never succeed in formulating them. This is, however, only a remote possibility; probably if they were really irrational it would not have been possible to make the limited progress that has been achieved. But if the laws of quanta do indeed differentiate the actual world from other worlds possible to the mind, we may expect the task of formulating them to be harder than anything yet accomplished by physics.

"The theory of relativity has passed in review the whole subject-matter of physics. It has unified the great laws, which by the precision of their formulation and the exactness of their application have won the proud place in human knowledge which physical science holds today. And yet, in regard to the nature of things, this knowledge is only an empty shell—a form of symbols. It is knowledge of structural form, and not knowledge of content. All through the physical world runs that unknown content, which must surely be the stuff of our consciousness. Here is a hint of aspects deep within the world of physics, and yet unattainable by the methods of physics. And, moreover, we have found that where science has progressed the farthest, the mind has but regained from nature that which the mind has put into nature.

"We have found a strange foot-print in the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last, we have succeeded in reconstructing the creature that made the foot-print. And Lo! it is our own."

Shall we explore the lead suggested by these eminent philosophers? We can do no better than to begin with the story of the scientific reformation.

### 3. *The Historical Origins of Physical Science.*

Following a sterile period of more than ten centuries, genuine scientific discovery resumed its progress about the fifteenth century and began to flourish with Johannes Kepler (1571-1630) and Galileo Galilei (1564-1642) in the sixteenth. In a very real sense this was a revitalization of a trend of thought which, initiated by Greek genius more than three centuries before the beginning of the Christian era, had developed in a remarkable manner even into the period of the decline of the Roman empire. To attain a proper background for the understanding of the scientific development of our times it is necessary to appreciate the scope of this Greek movement.

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[5] *Space, Time, and Gravitation,* Cambridge, 1920, pp. 200-201.

It was more especially in the fields of astronomy and mathematics that Greek science excelled, a dual development that is readily understood from the dependence of one discipline upon the other. The relative simplicity of the experimental tools, an elementary astrolabe for the one, a stylus and a plate of wax for the other, undoubtedly contributed much to this advance over other forms of scientific knowledge.

Among the earliest of the great astronomer-mathematicians we find Eudoxus (c. 408-355 B. C.), whose fragmentary writings indicate that a substantial advance had been made prior to the founding of the Alexandrian Museum (c. 300 B. C.), which is the focal point of most of ancient science. There is abundant evidence to show that Eudoxus was the intellectual father of Euclid and that a book of such perfection as the *Elements* could scarcely have been produced without a rich inheritance. But Greek science before the Alexandrian epoch was sporadic at best. Such names as Thales (640-546 B. C.) of Miletus, Pythagoras (580?-500? B. C.), the mystic, Zeno of Elea (5th century B. C. ), the dialectician, Democritus (c. 460-370 B. C.), the atomist, and a few others emerge from the haze of these early centuries.

To understand the heritage of the modern renaissance one must appreciate the significance of the successors of the members of the earlier Greek schools. The story should properly begin with Aristotle (384-322 B. C.), more perhaps for his influence upon Alexander the Great than for his own contributions to physical science. Extensive as these were both in volume and scope, they provided often more errors to the understanding of physical phenomena than cogent facts. Aristotle's proposition that the velocity of falling bodies is proportional to their weights is but one example of the many defects which, because of the enormous influence of the philosopher in Medieval times. created difficult barriers to the later progress of physical science. Aristotle, however was the teacher of both Alexander the Great and his friend and companion Ptolemy, who later became the first king of Hellenistic Egypt. There is little doubt that the influence of Aristotle's teaching of natural philosophy was the primary incentive which inspired Ptolemy to found in Alexandria the great Museum. This was the first institution to have among its primary objectives the study of science. Astronomy, mathematics, and medicine were fundamental curricular subjects and these departments were originally headed by the most distinguished scholars that the Greek world could provide. Euclid, for example, was the leader of the mathematical group. The riches acquired by Alexander in his Persian conquests were used to establish adequate buildings and a library which surpassed all others in the Mediterranian world. More than 400,000 volumes are estimated to have been among its stores.

It is beyond the scope of this work to enumerate all the eminent scholars who found leisure and opportunity to advance knowledge within the walls of the Museum. For almost every important thinker of the Hellenistic and the Roman periods was in one way or another associated with the activities and history of this notable institution. Preëminent among these in the early years of the Museum was Archimedes (287? - 212 B. C.), who, although a native of Syracuse, had intimate associations with the Alexandrian mathematicians and astronomers. In the works of this broad scholar science reached one of its highest points. He was a pioneer in establishing the principles of both mechanics and hydrostatics. He invented the water screw to assist in irrigation, extended the knowledge of the lever, and stated the laws of fluid pressure. His greatest achievements were in mathematics as may be inferred from the fact that he grasped the fundamental idea of the calculus and was able to evaluate certain integrals.

Among the notable achievements of the Alexandrian school was the measurement of the size of the earth, a feat ascribed to Eratosthenes (c. 273-c.192 B. C.) who was also a mathematician of considerable ability. In studies of the earth and its relationship with the moon, not only were measurements of considerable accuracy made on the size and distance of the moon, but its influence in creating the tides was also discovered. Thus we find the following passage attributed to Posidonius (c. 130-50 B. C.): "For when the moon rises above the horizon to the extent of a zodiacal sign (that is to say, 30 degrees), the sea begins to swell, and perceptibly invades the land until the moon is in the meridian; but when the heavenly body has begun to decline, the sea retreats again, little by little until the moon rises a zodiacal sign above her setting." From this we may infer that only by the narrowest margin did this early astronomer fail to discover one of the most profound laws of modern science, a discovery thus left to the greater genius of Newton.

In its study of the mechanism of the solar system Greek science reached perhaps its highest attainment. Although the heliocentric theory originally proposed by Aristarchus had been rejected since stellar displacements which are an inevitable consequence of this hypothesis had not been observed, notable progress was made in the actual description of planetary movements. Hipparchus (c. 130 B. C.), who is often called the father of astronomy, discovered the precession of the equinoxes, that is to say, the motion of the two points in the sky where the plane of the ecliptic in which the earth moves around the sun intersects the plane of the equator. This was, indeed, a remarkable discovery and reveals more perhaps than any other thing the high attainment of ancient science. For the precession itself is a very small motion (50.4 seconds of arc per year), as is shown from the fact that approximately 258 centuries will be required for the equinoxes to make a complete circuit of the sky. It is thus astonishing that this small motion should have been detected even by the acute observations of Hipparchus; it is also a triumph for the accuracy of the observations of his predecessors, Aristyllus and Timocharis, astronomers attached to the Museum, who about a century and a half earlier had made a catalogue of stars. The precession of the equinoxes as estimated by Hipparchus (45 seconds of arc per year) was only five seconds less than that finally established by modern astronomical observations.

The intellectual successor of Hipparchus was Claudius Ptolemy (c. 139 A. D.), an Alexandrian astronomer and geographer, whose famous treatise the *Syntaxis Mathematica*, or as it is more commonly called the *Almagest*, or *the Great Composition* from the Arabic, represented the culmination of classical astronomical knowledge. This notable work contained a detailed account of the motions of the planets on the assumption of a geocentric system. The representation of planetary motions by means of cycles and epicycles which it promulgated will be described in more detail later in this work. Among Ptolemy's most interesting contributions, derived at least in part from Hipparchus, was the formulation of a remarkably perfect trigonometry, both plane and spherical, which included a table of chords (essentially a table of sines) accurate to six decimal places. Scarcely less important than the *Almagest* was Ptolemy's work on *Geography*, which described the regions of the known world, established a system of geographical coördinates including a prime meridian, and provided a scheme of projection by means of which he could represent scientifically upon a plane map the position of places actually located upon the spherical surface of the earth. This map of Ptolemy was neglected during the Middle Ages, but it became a standard for the instruction of cartographers when it was rediscovered in the fifteenth century.

Although ancient genius was strangely deficient in its ability to apply the results of scientific discovery to the work of the world, there was not lacking a knowledge of some of the basic principles. Most notable among the names associated with early engineering is that of Hero of Alexandria, who probably flourished about 150 B. C., although some scholars attribute a date to him as late as 150 A. D. His work was undoubtedly influenced by a certain Ctesibius, who emerges through the dim curtain of the past as an ingenious engineer of the early Ptolemaic period. To him is ascribed the invention of the water-clock, the catapult, and the hydraulic organ.

Hero has been accorded the invention of numerous devices most of which apply in one way or another the principle of the vacuum. Among these are found two forms of the steam engine, one of the turbine type and the other of the reciprocating type, The latter was used to open and close the doors of a temple by means of a sacrificial fire built upon an adjoining altar. Hero also invented a fire engine, perfected the water organ, applied mathematics to surveying, and constructed an ingenious instrument called the *dioptra*, which served the same purpose that the theodolite does for modern civil engineering. A number of his pneumatic toys revealed an unusual insight into the properties of air. The following quotation from Hero's *Treatise on Pneumatics* will be recognized as the prototype of certain passages in modern treatises on the kinetic theory of gases:

"The particles of air are in contact with each other, yet they do not fit closely in every part, but void spaces are left between them, as in the sand on the seashore; the grains of sand must be imagined to correspond to the particles of air, and the air between the grains of sand to the void spaces between the particles of air. Hence, when any force is applied to it, the air is compressed, and, contrary to its nature, falls into the vacant spaces from the pressure exerted on its particles; but when the force is withdrawn, the air returns to its former position from the elasticity of its particles, as is the case with horn shavings and the sponge, which, when compressed and set free again, return to the same position and exhibit the same bulk."

Although enough has probably been said to indicate the high attainments of ancient science, the story would not be complete without further mention of the galaxy of scholars who advanced the knowledge of mathematics. From the time of Euclid to the beginning of the fifth century of our era, when the forces had set in which were leading to the dissolution of the Roman empire, there had been associated with the Museum a succession of very competent mathematicians. These are generally divided into two groups, one of which comprised what is called the first Alexandrian school and the other the second Alexandrian school. The former began with the founding of the Museum and embraced the period of the Ptolemaic dynasty, which may be said to have ended with the death of Cleopatra in 30 B. C. The second Alexandrian school began with the Christian era and ended with the death of the mathematician-philosopher Hypatia in 415 A. D.

To the first school belong the names of Euclid, Archimedes, and Eratosthenes, some of whose achievements we have already mentioned. Included also is Apollonius of Perga, who flourished during the reigns of Ptolemy Euergetes I (247-221 B. C.) and Ptolemy Philopater (221-203 B. C.). His famous treatise in eight books on the *Conics* gained for him the name of the Great Geometer. Lesser names in this period of the Museum are those of Diocles and Nicomedes. The former invented the curve called the *cissoid*, an ivy-like figure by means of which he could solve the ancient problem of the duplication of the cube; the latter invented the *conchoid*, a shell-like curve by means of which an angle can be trisected.

Claudius Ptolemy, whose contributions have already been mentioned, belonged to the second Alexandrian school. Second only to him was Diophantus (c. 250 B. C.), who produced a great work entitled *Arithmetica*, which gained for him the title of the father of algebra. Its influence on the theory of integers has extended down to our own times. Of lesser note was Menelaus (c. 98 A. D.), who contributed a theorem known by his name which plays a conspicuous part in the modern theory of projective geometry. Pappus, who flourished around 380 B. C., was one of the last mathematicians of the Museum. His principal contribution consisted of two theorems by means of which the centers of gravity of plane areas and plane curves are related to the volumes and areas of surfaces generated by revolving the plane areas and curves about an external axis. Modern students of mechanics are usually astonished to find the name of an ancient Greek attached to theorems which are proved most easily by the machinery of integral calculus.

The final names in the history of the mathematics of the second school are those of Theon (c. 350 B. C.) and his daughter Hypatia. Father and daughter were the editors and commentators of the work of earlier mathematicians. Theon produced an edition of Euclid's *Elements* and a commentary on the *Almagest;* Hypatia devoted her attention to the works of Diophantus and Apollonius. The death of Hypatia, which was the culmination of the conflict beween the Museum and the authority of the early church, marks the end of scientific advance in the classical period.

This brief summary of the achievements of the early physical scientists is a necessary concomitance to any real understanding of the modern stream of discovery. Only one unaware of the importance of inheritance could fail to realize and appreciate the debt which the modern world owes to the ancient one. We turn, therefore, with more confidence to a description of the scientific reformation with which began our modern adventures in the understanding of nature.

### 4. *The Scientific Reformation.*

For a thousand years after the decline of ancient scholarship there was little scientific activity in the world. The works of the Graeco-Roman culture were transported into Arabia by the victorious armies of the Saracens. There they were preserved and in this rather infertile soil produced a few fruits mainly in the fields of mathematics and astronomy. The name of Al-Khowarizmi, who wrote a monumental work on *Algebra* about 825, is preserved in the mathematical term "algorithm"; but the name of the Persian, Omar Khayyam (c. 1043-1131) is better known as that of the author of the *Rubaiyat* than as that of a mathematician and scientist. But the light of progress in the western world was dim and even the work of such an enlightened scholar as Roger Bacon (1214-1294?) did little to dispel the darkness of the medieval centuries.

A revival of the Greek trend, as we have indicated earlier, began with the work of Johannes Kepler (1571-1630) and Galileo Galilei (1564-1642) in the sixteenth century. Kepler had access to the great astronomical tables which had been constructed by Tycho Brahe (1546-1601) in his magnificent observatory on the island of Hveen in the Sound between Denmark and Sweden. Adopting the heliocentric theory of Copernicus (1473-1543) as the simplest hypothesis for the description of planetary motions, Kepler soon made the discovery that the orbits of the planets were essentially ellipses with the sun at one of the foci. Two other laws, which we shall discuss in detail later, were also distilled from the available data; and it became evident for the first time that genuine advance had been made at last beyond the discoveries of Hipparchus and Ptolemy. Galileo, also, overcoming the difficulties of ignorance and superstition with which he was beset, had succeeded in founding the science of mechanics by comparing theory with experiment and had given to mankind his telescope. His discovery, by means of this new instrument,of the four major satellites of Jupiter, the rings of Saturn, the phases of Venus, the mountains of the moon, and spots upon the surface of the sun had stimulated the imagination of his contemporaries. The power of this new force is also attested by the black hostility which it aroused, for the novelty of scientific ideas often seems to be proportional to the superstitious fears which they invoke. The age of objective science was at hand.

As David Brewster has cogently observed: "It is a remarkable circumstance in the history of science, that astronomy should have been cultivated at the same time by three such distinguished men as Tycho, Kepler, and Galileo. While Tycho, in the 54th year of his age, was observing the heavens at Prague, Kepler, only 30 years old, was applying his wild genius to the determination of the orbit of Mars, and Galileo, at the age of 36, was about to direct the telescope to the unexplored regions of space."[6] But it is even a more remarkable circumstance that these three astronomers should have been succeeded in the same century by a man whose genius transcended all of them. Sir Isaac Newton (1642-1727), reflecting upon the nature of material objects and the forces which disturb them, was able to make a profound synthesis of the empirical discoveries of his predecessors. This synthesis was published in 1687 in Newton's principal work, the *Philosophiae Naturalis Principia Mathematica*, which has been called by David Brewster "the brightest page in the records of human reason." In this volume the law of gravitation is given for the first time and the discoveries of Kepler are shown to be mathematical consequences of this law.

Sir Isaac Newton was born on Christmas day, 1642 in the manor house of Woolthorpe about six miles south of Grantham in the county of Lincoln. Just as it was the good luck of Christopher Columbus to have been born at a time when the old world was on the verge of discovering the new, so it was the fortune of Newton to have been born in a period when Kepler's magnificent discoveries were thrusting the idea of gravitation upon men's minds. But it is, perhaps, most fortunate of all for science that so great a genius should have appeared at this beginning of the scientific movement, for Newton by his powers united in a single work what might have been scattered in the writings of many less important books. This noble figure became the rallying post for those who saw the vision of the scientific revolution. If there is any value in selecting a group of heroes whose lives may be a guide and inspiration, Newton is preëminently the hero of the scientific revolution. Alas, even in this age, immensely enriched by the labors of those who followed the ideals of Newton small thought is ever given by the average man to the achievements of this great leader! Carlyle has written entertainingly of the hero as a priest, the hero as a god, the hero as a king, the hero as a man of letters, but where do we find evidence that the man of science is regarded as a hero?

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[6] *Martyrs of Science*, London, 1841, p. 189.

Apart from its revolutionary contributions to mathematics, which will not concern us here, the work of Newton falls into two parts, the first having to do with gravitation, the second with the phenomenon of light. Countless generations of men have experienced with Matthew Arnold that sensation felt when one looks upon the restless action of the tides:[7]

> "Listen! you hear the grating roar
> Of pebbles which the waves draw back, and fling,
> At their return, upon the high strand.
> Begin, and cease, and then again begin,
> With tremulous cadence slow, and bring
> The eternal note of sadness in. . .
> But now I only hear
> Its melancholy, long, withdrawing roar,
> Retreating, to the breath
> Of the night wind, down the vast edges drear
> And naked shingles of the world."

But to Newton there was in the ceaseless action of the tides a cosmic challenge. In them he saw more than a restless expanse of water deriving its activity from some hidden principle of nature. By a superb leap of the imagination he sensed a connection between the distant moon and the puzzling activity of the tides. To him there had occurred the principle of *action at a distance* which at a single stroke united all the particles of matter of the universe by a force which varied inversely as the squares of their distances apart and directly as their masses. The genius of the discovery lay in the fact that Newton saw the possibilities latent in the postulate that two bodies, though separated by empty space, could still exert measurable influence upon one another. The ancient astrologers, it must be remarked, had maintained a similar postulate, but the difference between them and Newton was that he could foretell infallibly the effect of the gravitational influence, while their system failed in all crucial tests.

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[7] *"Dover Beach."*

The second contribution made by Newton was his set of experiments on light and his subsequent formulation of a theory to account for what he had found. Here, as in the case of gravitation, he was faced with the necessity of making a postulate. In the light of modern knowledge we know that he might have offered either of the following hypotheses:

1. Light is a manifestation of energy transmitted from the bright object to the eye by means of tiny corpuscles.
2. Light is the manifestation of energy transmitted from the bright object to the eye by means of undulations in a tenuous medium which pervades all space.

Between these postulates one has at first on rational grounds little hesitation in choosing with Newton the first, yet its complete discomfiture during the nineteenth century shows that the second more metaphysical postulate was in better accord with the empirical tests upon which a physical theory must stand or fall.

### 5. *Action at a Distance.*

The striking thing that we are to observe here is this. Newton introduced the age of reason into science. He stands preëminently as one who showed how the motions of the heavenly bodies can be reduced to exact calculation. He pointed the way of the scientific method by showing that one might have on the one hand a subjective theory and upon the other a set of experimental data to check against it. And yet was a rational theory ever built upon a more metaphysical foundation than his theory of gravitation? Was an idea ever introduced into rational thinking that is more repellent to reason than the concept of action at a distance?

During the winter of 1692-93 Sir Isaac Newton was called upon to answer several letters written to him by Rev. Richard Bentley, an eminent clergyman of that day, and in one of his replies he made this famous statement:[8]

". . . It is inconceivable, that inanimate brute matter should, without the mediation of something else, which is not material, operate upon and affect other matter without mutual contact, as it must be, if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desire you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a *vacuum*, without the mediation of any thing else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers."

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[8] From the works of Richard Bentley, in three volumes, London, 1838, edited by Rev. Alexander Dyce. See Vol. 3, p. 212.

There seems to be a common illusion among those who so calmly enjoy the fruits of this modern world of discovery, that science progresses entirely by exact calculation and the slow drudgery of laboratory technique. Nothing is further from the truth. *Science progresses rather by sudden guesses about an almost mystical behaviour in nature, although, to be sure, these inspirations are derived from the slowly accumulated facts of the laboratory.*

If there exists a law of evolution why did it take so many years to formulate it? Surely for the reason that first there had to arise in the mind of a man like Darwin the concept of some selective power in environmental conditions which would choose from the hurly-burly of life processes those factors which would lead to survival. There seems to be but little difference between the magnitude of one postulate which assumes the existence of a selective principle in nature and another which assumes that each particle of matter has an attraction for every particle across the empty space that separates them.

After Newton, the study of science was vigorously pursued by those who, forgetful of the metaphysical postulate upon which the *Principia* was built, saw only in the calulations an evidence of the rationality of nature.

### 6. *The Principle of Least Action.*

It is difficult to isolate from the many principles discovered by those who labored under this assumption, a few which were the most important to development, but there is one that must not be neglected. This postulate goes under the name of the *principle of least action* and has the strange distinction of being perhaps the most fundamental and yet the most imperfectly understood law of physics. Its basic character is seen in the fact that all the laws of mechanics are formulated in it, and its intriguing mystery resides in the numerous discomfitures of those who have tried to give it a rational existence.

The kernel of this principle may be summarized as follows: *When an event that requires energy is taking place in the physical universe, it must take place so that the mean value of the difference between the kinetic energy and potential energy involved, in the interval of time required for the event to take place, must be as small as possible.*

The reader to whom these ideas are novel can gain an appreciation of the meaning of the law by considering the path taken by a ball thrown into the air. What course will the ball take? There is no reason *a priori* to see why the ball should follow the course of that geometrical figure, called the parabola, the study of which is to be found in the works of antiquity. There are many arcs which the ball might travel. Why, then, does it pick out this particular one? In terms of our new principle the answer is that by pursuing this course the value of the difference between the gravitational pull of the earth and the kinetic energy possessed by the flying ball, averaged over the time when the ball left the hand of the thrower until it returned to earth again, is smaller than it would be for any other path.

Is there any sense to such an answer? Would it not be as rational to say that the ball takes the path of a parabola because that is the law of nature? The reply is that the law of least action which we have invoked appears to be valid not only for the path taken by the ball but for all the acts of the physical universe. It has become so fundamental a part of our system of mechanics that Einstein in creating his theory of relativity preferred to leave it alone and alter the nature of space and time instead.

The law of least action was first announced, though imperfectly, by Pierre Louis Moreau de Maupertuis (1698-1759), who unfortunately is known more for the biting sarcasm of Voltaire's "A Dissertation by Doctor Akakia" than for his scientific achievements. Voltaire gives as a "judgement of the professors of the college of wisdom" the following appraisement of the least action principle:

"The assertion that 'the product of the space multiplied by the velocity is always a minimum,' seems to be false; for this product is sometimes a maximum according to the opinion of Leibniz and as may be easily proved. It would appear that the young author took only one-half of M. Leibniz's ideas; and we, therefore, acquit him of the guilt of having ever comprehended one whole idea of M. Leibniz."

Little did Voltaire realize that the law he so lightly lampooned along with Doctor Akakia's suggestions that a city should be founded in which Latin alone was to be spoken, that a hole should be dug to the center of the earth and that the brains of giants six feet high and of hairy men with tails should be dissected the better to discover the nature of the human mind, was destined to become, as Jacobi expresses it, "the mother of our analytical mechanics."

Maupertuis first stated his principle in a memoir entitled "Accord de differentes loix de la nature qui avoient jusqu' ici paru incompatibles," which he read to the French Academy on April 15th, 1744.[9]

The laws in question had to do with the reflection and refraction of light. As is well known, a ray of light when traveling in a homogeneous medium will pass from one point to another either directly or by reflection in the shortest time and by the shortest path. P. Fermat (1650-1665), the French contender for the honor of having invented the infinitesimal calculus, generalizing this observation, had assumed that the path of a refracted beam passing into a medium of greater or less density would also be determined by this minimum principle. But in his demonstration, Fermat had been obliged to differ with Descartes who assumed that the velocity of light increased with the density of the medium in which it was propagated. Maupertuis who belonged to the corpuscular faith and regarded light as a stream of particles, wanted to keep the beautiful minimum principle of Fermat, but also desired to reconcile it with the ideas of Descartes. Hence Maupertuis' problem was to find out what minimum principle, if any, was involved and he came to the conclusion that light would travel by that path in which the *quantity of action* was least. The historic passage follows:[10]

"I must now explain what I mean by the quantity of action. A certain action is necessary for the carrying of a body from one point to another: this action depends on the velocity which the body has and the space which it describes; but it is neither the velocity nor the space taken separately. The quantity of action varies directly as the velocity and the length of path described; it is proportional to the sum of the spaces, each being multiplied by the velocity with which the body describes it. It is this quantity of action which is here the true expenditure of nature, and which she economizes as much as possible in the motion of light."

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[9] *Historie de l'Académie de Paris*, (Published 1748), pp. 417-426. *Oeuvres de Maupertius* 1756; Vol. 4, pp. 3-28.

[10] *Historie de l'Académie de Paris*, 1748, p. 423, *Oeuvres*, Vol. 4, p. 17. For an admirable account of the history of the principle of least action see an article by Phillip E. B. Jordain: "Maupertuis and the Principle of Least Action," The *Monist*, Vol. 22 1912, pp. 414-459.

Having arrived at this conclusion he ends with the following speculations:[11]

"We cannot doubt that all things are regulated by a supreme Being, who, while he has imprinted on matter forces which show his power, has destined it to execute effects which mark his wisdom; . . . Let us calculate the motion of bodies, but let us also consult the designs of the Intelligence which makes them move."

The same metaphysical cast appears in the thinking of Euler who gave form to the principle of Maupertuis and helped to place it in its present elevated position in mechanics." He said:[12]

"As the construction of the universe is the most perfect possible, being the handiwork of an all-wise Maker, nothing can be met with in the world in which some maximal or minimal property is not displayed. There is, consequently, no doubt but that all the effects of the world can be derived by the method of maxima and minima from their final causes as well as from their efficient ones."

From this we see that the idea which inspired Euler was what might be called the *extremum principle* of nature, or the assumption that all the activities of the material world could be described in terms of the minimization of some quantity over a period of time. The magnificence of such a hypothesis has had a universal appeal, not only because of its elegance and simplicity, but also for its power in unifying a large number of apparently unrelated phenomena. That the quest for the quantity which nature minimized was not an easy one is clearly revealed by the subsequent history of the idea.

An extensive correspondence was exchanged between Euler and J. L. Lagrange (1736-1813), which attempted to clarify the principle of least action. Lagrange, who devoted numerous memoirs to the calculus of variations, which had such minimizing problems as its special province, succeeded in deriving the fundamental equations of mechanics for conservative fields of force by means of the principle of least action as stated by Maupertuis and Euler. He failed, however, to find the general law by means of which non-conservative exchanges of energy could be included. This was left to the genius of Sir William R. Hamilton (1805-1865), who first stated the principle in its modern form. In his expression for the law the difference between the kinetic and potential energies is minimized over the interval of time; that the older formulation is included in the new is apparent from the fact that the sum of the two energies is a constant for conservative systems. His statement of the law is generally called *Hamilton's principle* to distinguish it from the original principle of least action as stated by Maupertuis and Euler.

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[11] *Historie,* p. 423; *Oeuvres,* Vol. 4, p. 17.

[12] Methodus inveniendi linear curvas maximi minimive proprietate gaudentes," Lausanne, 1744, p. 245. See also E. Mach: *The Science of Mechanics,* English translation by T. J. McCormack, Chicago, 2nd ed. 1902, p. 455.

It is inaccurate to say that the exchanges of energy which occur in all material systems minimize action, since examples have been constructed where this is not actually the case. In its more careful statement the principle merely affirms that the first variation of the integral of action, in the sense of the calculus of variations, is zero. This is a necessary, but not a sufficient condition, to assure that action is a minimum. Functions which satisfy this condition are called *extremals* and hence it would be more proper to speak of the *extremal principle of action*, rather than the principle of least action.

Although an unvarying success accompanied the application of Hamilton's principle to the solution of special problems during the years immediately subsequent to its publication, there remained the uneasy feeling that a law so universal, which rested upon a foundation no better than that afforded by metaphysics, might ultimately fail to solve the deeper problems of modern physics. So far these fears have proved unfounded. H. Bateman (1882-1946) has appraised the situation as follows:[13] "A firm belief in the generality of Hamilton's principle has been a guiding idea in many of the great developments of mathematical physics in recent years. Variational principles of the Hamiltonian type were used by Mie in his theory of matter, by Hilbert, Einstein, Lorentz, Tresling, Fokker, Weyl, Palatini, Whittaker and others in the development of the general theory of relativity. De Broglie and Schrödinger used it in the development of the new quantum theory and when Dirac began to develop his modification of quantum theory and ideas about matter Darwin showed that by combining atom and radiation fields into a single system the whole scheme of equations could be derived from a variational principle."

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[13] H. Bateman: "Hamilton's Work in Dynamics and its Influence on Modern Thought," *Scripta Mathematica*, Vol. 10, 1944, pp. 51-63.

But for all of this success the fact still remains that the science of mechanics, regarded as a consequence of the principle of least action, rests upon a metaphysical foundation. Action itself, the quantity which is to be minimized, is a concept that fails to make the same appeal to our *a priori* intuitions as the concept of energy. Even Newton's force of gravitation or the analogous forces of electricity and magnetism which Michael Faraday (1791-1867) so picturesquely visualizes as tubes in the ephemeral ether, convey a more realistic pattern to our senses than the concept of action. This is seen from the fact that Maxwell, and after him Hertz, actually thought of these tubes as being joined together into hoops and projected through space.[14] But where does a material analogue exist for the action of Maupertuis and Euler? For all of this, however, in the epistemology of Lord Kelvin which we have cited earlier, action belongs to the realm of what we know since even in the most complicated systems "it can be measured and expressed in numbers." It is the metaphysical origin of the principle itself which, while it challenges our imaginaton, yet repels our reason.

### 7. *The Ether Postulate.*

Returning from these speculations to another aspect of science we find that the next great concept of the materialist is that denied by Newton in favor of an atomic theory of light, namely the theory that the propagation of light consists of something analogous to vibration. But here again we are faced by a strange mystery. Newton, led to his theory of gravitation as the only possible logical explanation of the motions of the moon and the laws of Kepler, was under the categorical necessity of postulating the existence of a force which acted between two bodies through the medium of a vacuum. In the case of light we have been driven to a similar necessity equally repugnant to the senses and yet just as categorical. We must postulate the existence of a vibration which will operate in a vacuum.

James Clerk Maxwell, (1831-1879), that profound speculator to whose genius we owe the electro-magnetic theory of radiation, says at the end of his second volume on electricity and magnetism in commenting upon the physical ideas of some of his contemporaries:[15]

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[14] See H. Hertz: *Electric Waves*, English translation by D. E. Jones, London, 1893, pp. 143-147.

"There appears to be, in the minds of these eminent men, some prejudice, or *a priori* objection, against the hypothesis of a medium in which the phenomena of radiation of light and heat and the electric actions at a distance take place. It is true that at one time those who speculated as to the causes of physical phenomena were in the habit of accounting for each kind of action at a distance by means of a special aethereal fluid, whose function and property it was to produce these actions. They filled all space three and four times over with aethers of different kinds, the properties of which were invented to 'save appearances,' so that more rational enquirers were willing rather to accept not only Newton's definite law of attraction at a distance, but even the dogma of Cotes (Preface to Newton's *Principia*, 2nd edition), that action at a distance is one of the primary properties of matter, and that no explanation can be more intelligible than this fact. Hence the undulatory theory of light has met with much opposition, directed not against its failure to explain the phenomena, but against its assumption of the existence of a medium in which light is propagated."

The history of the ether fills a volume of considerable size in the longer story of the development of physical science. Some believe that its history is now at an end, since references to this "necessity of reason" have disappeared rather completely out of modern technical writings. Others feel, however, that while unsolved mysteries envelop the subject of energy, convenience demands that we have at hand an agency to which they can be consigned. Hence it is that the concept of the ether preserves a tenuous existence in the form of a kind of latent entity which is ready to be invoked when needed. Some aspects of the history of this concept and the reasons which have led to its present precarious state will be considered in detail in later chapters.

Our present concern is to indicate certain reasons why the invention of an ether was considered inevitable to the physicists of the nineteenth century. The words of Maxwell which have just been cited provide the most cogent reason for the creation of a medium that has been assumed to exist coextensive with space but different from it. That is to say, the existence of the ether has been assumed to be a necessary postulate to account for the transfer of energy, especially radiant energy, from one place to another.

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[15] *A Treatise on Electricity and Magnetism*, 3rd edition, 1904, Vol. 2, p. 492.

The light of speculation is thus focused upon the concept of energy itself, an entity which enters ubiquitously into all the activities of the universe. But in spite of the fact that energy is one of our most universal properties, for all of this the concept of energy possessed for the early physicists, and still possesses for us, certain very mysterious aspects. Let us review some of the problems presented by this strange entity.

A material object is said to possess the energy of position, or *potential energy*, if it occupies a position in a field of force such, for example, as the gravitational field of the earth. It possesses *kinetic energy* if it is in motion, this energy being measured in Newtonian physics by the well know expression, $\frac{1}{2}MV^2$, where $V$ is the velocity and $M$ is the mass of the object. Potential and kinetic energies are interchangeable in many dynamical situations, the simplest example for illustrative purposes being the oscillating pendulum. At the top of its swing the energy of the bob of the pendulum is all potential, at the bottom all kinetic. *Force* itself enters fundamentally into the problems because potential energy can be expressed as the product of force by the distance through which the material object has been moved against the force. Since *work* is required to move the object against a force, energy is frequently defined as "the capacity for doing work."

All these ideas about energy and its relationship to force are to be found inherently expressed in the properties of Newton's *Principia.* No attempt was made by Newton to explain the nature of energy, but the observed physical facts were clearly stated in a series of propositions and scholia. Thus by more than a century the concepts of mechanical energies preceded the more deeply imbedded concepts of other energies.

A revolution in our ideas about the nature of energy was initiated through a closer examinaton of the character of heat. From ancient times the flow of heat from one object to another had been observed, and hence it came to be regarded by the most thoughtful scholars as a subtle, imponderable fluid to which the name "caloric" was attached. Many attempts were made to weigh this fluid, but without success. The puzzling nature of the phenomenon is shown by the currency among the leading scientists of the eighteenth cenutry of the *phlogiston theory*, which attributed to matter a "principle of combustibility." In the burning of chemical compounds the violence of the combustion was assumed by these early scientists to be proportional to the amount of phlogiston present in the substances.

The first real light was thrown upon the nature of heat by Benjamin Thompson, Count Rumford (1753-1814), who published in 1798 a paper entitled "Enquiry Concerning the Source of Heat which is Excited by Friction." In this monograph he presented the results of his observations about the origin of the heat generated in the boring of brass cannons. Rumford concluded that the heat was not inherent in the metal itself, but was produced by the friction of the boring devices. From this it was inferred that there existed somewhere an inexhaustible supply of heat, and hence Rumford reached the following celebrated conclusion:[16] "It is hardly necessary to add that anything which any insulated body or system of bodies can continue to furnish without *limitation* cannot possibly be a *material substance;* and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner that heat was excited and communicated in these experiments, except it be *motion.*" A contemporary investigation by Sir Humphrey Davy (1778-1829) of the melting of ice by friction confirmed the correctness of this deduction.

The discovery of Rumford and Davy, however, only deepened the mystery of energy. For it failed to describe the nature of the motion with which heat was thus identified. Moreover, although these experiments proved that kinetic energy could be translated into heat energy and that this relationship was fixed by what is called the *mechanical equivalent of heat*,[17] the reversibility of the transfer involved certain difficulties. In practical engines a certain loss was always observed in the conversion of heat into equivalent work.

The perplexities of the situation were not lessened with the development of chemistry in the nineteenth century, which presented problems associated with the transformation of chemical energy into other forms. Even more perplexing were the discoveries of electric and magnetic energies, which, as we now know from the work of Maxwell and his successors, are merely aspects of the energy of radiation.

Especially in radiation was it difficult to generalize the proposition of Count Rumford that energy was motion, for the concept of motion seemed inevitably to associate itself with matter. Only by making the assumption that radiant energy was a vibration in the metaphysical ether could a rational generalization of the earlier concepts be preserved. And from this developed the scientific struggle to endow this "necessity of reason" with material properties.

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[16] *Philosophical Transactions,* Vol. 88, 1798, p. 80.

[17] The mechanical equivalent of heat asserts, in terms of foot-pounds and calories, that 778 foot-pounds of energy = 252 calories of heat.

However, one great principle was evolved from these reflections, namely the proposition that *energy is conserved.* All forms of energy are interchangeable with one another and the disappearance of heat, for example, can be exactly accounted for in the balance sheet of energy by an exact equivalent of mechanical, radiant, chemical, or any other form of the mysterious entity. Although the *first law of thermodynamics* in its original form was limited to a statement of the mechanical equivalence of the energy of heat, its broader scope affirms the equivalence of all forms of energy. The history of this astonishing generalization will be discussed at length in another chapter, for the conservative principle in recent years has been replaced by an even more remarkable principle, namely, the equivalence of matter and energy. Thus the conservation of matter on the one hand and the conservation of energy on the other have been merged into a grand generalization in which matter-energy is the entity that is unchangeable and indestructible.

The object of this section has been achieved. We have shown how the early investigators of the nature of energy first rejected the existence of a material caloric, and then were forced to the invention of a metaphysical ether for the storage and propagation of the "motion" into which the mysterious substance had been diverted. This we may call properly the postulate of the ether.

### 8. *The Discovery of Entropy.*

Since our object at this time is to survey the concepts that have guided laboratory experiment during the scientific revolution, we cannot now penetrate further into these speculations on the nature of the ether but must proceed to another doctrine that has had profound influence upon scientific progress. We refer to what is commonly called the *second law of thermodynamics*, or the law of *entropy*.

In the earliest histories of thought the physical universe was classified under three heads, namely, solids, liquids, and gases. A second method of classification consisted in dividing things into cold bodies and hot bodies, where a certain arbitrary temperature not far from the center of the annual temperature range of the surface of the earth was used as the dividing line of the classification. The question might now be asked: Is there any connection between these two classifications?

The study of thermodynamics started logically when data were accumulated on the thermal properties of gases, and it was found approximately at the same time by J. A. C. Charles (1746-1823), and J. L. Gay-Lussac (1778-1850) in France that the pressure exerted by a gas in a container of constant volume varies directly as the temperature, provided this temperature be calculated so that its zero is 460 degrees below the zero of our ordinary Fahrenheit thermometer, or 273 degrees below zero on the centigrade scale. It also became apparent that this temperature range was essential to the classification of things into solids, liquids, and gases, since the state of any particular kind of matter depends upon the value of its temperature, measured from the absolute zero of the gas thermometer. Iron, for example, is commonly regarded as a solid and air a gas because most of us do not work either in a Bessemer steel plant or in a factory devoted to the manufacture of liquid gases.

Proceeding from these elementary ideas now familiar to all of us, however obscure they may have been to the early workers in science, we now ask the question: Why should there exist a lower bound to temperature? Is it the result merely of our definition of temperature and therefore a limit put subjectively into nature or should we really be surprised at it as at a fact discovered about objective nature?

Most people will agree that the existence of a lower bound to temperature is a startling fact. The human mind dislikes to think in terms of bounds. We regard the concept of boundless space as an inalienable right of the human imagination, and it is disturbing to be told that the velocity of light is the ultimate velocity of material things since it is so easy to imagine a velocity double that of light. However finite the material world in which we live, we feel resentful at restriction and hold with Byron,[18]

"Eternal Spirit of the chainless Mind,
Brightest in dungeons, Liberty."

Hence, any fact wrested from the world of matter that imposes a bound to that which might conceivably be boundless is at first sight an important and somewhat disconcerting affair. Why should temperature be bounded and bounded at a level not so remotely distant from the very range of temperature within which we live?

To put the answer somewhat crudely, we may say that matter acts as a storehouse or container of thermal energy. Temperature is merely a gauge of the intrinsic energy content of matter, and when all energy has been pumped out of matter then the gauge sinks to zero. The mystery of the absolute zero is, in this sense, no more a mystery than the fact that one's car stops when the gasoline tank is empty.

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[18] "The Prisoner of Chillon."

But in another way absolute temperature enters into one of the most mysterious concepts in physics, that of entropy. In order to gain some appreciation of this idea let us enquire into the nature of heat energy from this point of view. Suppose for example, that we have a piece of iron at the temperature of ice and apply a source of heat to it. Its molecules will gradually absorb the energy of the hot body; it will begin to radiate heat; it will turn dark red, bright red, and finally attain the brightness of incandescence. At this stage let us remove the source of heat and apply a piece of ice until the iron has once more attained its first temperature. During the course of this experience, what has the iron gained and what has it lost again? According to R. J. E. Clausius (1822-1888), the father of thermodynamics, the iron has gained and lost a quantity of *entropy* which in the first case is by definition equal to the sum of each little increment of heat gained divided by the absolute temperature at which it was added, and in the second equal to the sum of each increment of heat lost divided by the absolute temperature at which the exchange took place. To put it otherwise, a mysterious entity has flowed through the iron, being first acquired at the expense of the hot body and then returned again to the ice. The iron has merely acted as a conveyor between the hot body and the cold, similar, for example, to the bed of a river along which the water of the mountain snows is conveyed to the water of the ocean. But the mystery remains. What is entropy and does it have objective existence? Is it something created in the mind of Clausius or is it a thing as real as Newton's force or Maxwell's ether or Maupertuis' action?

Mysterious though it may be and remote from all of our *a priori* feelings of reality as it is, a careful weighing of the evidence seems to force the idea of an objective entropy upon us as unescapably as the law of gravitation has led us to the concept of action at a distance.

The next question that we should like to ask is this: What has become of entropy once it has been absorbed into the substance of the ice? Is this not analogous to the question, where does all the water go that flows so ceaselessly toward the sea?

In the literature of social philosophy the most melancholy dictum that the writer knows is to be found in a passage of Emerson's essay on "Self-reliance" where he says:

"Society never advances. It recedes as fast on one side as it gains on the other. Its progress is only apparent, like the workers of a treadmill. It undergoes continual change; it is barbarous, it is civilized, it is Christianized, it is rich, it is scientific; but the change is not amelioration. For everything that is given, something is taken."

In the literature of science there is no more melancholy thought than that of Clausius who said: "Heat cannot of itself pass from a colder to a hotter body," "the entropy of the universe tends to a maximum."

Water cannot flow up hill; neither can entropy return to its first value unless it be put there by the expenditure of work. In the mind of Nicolas Leonard Sadi Carnot (1796-1832), a reflection of this kind led to the concept of an ideal system called *the Carnot cycle.* This ideal system was to act so that the heat that flowed down through the substance of the iron from the hot body to the cold was to be stored in the form of work and used again to pump the heat back from the cold body to the hot, or in terms of the richer concept, was to restore entropy to its former value. Unfortunately for those who seek in this a method for manufacturing a perpetual motion machine, the ideal engine of Carnot cannot be attained in reality. There is always a certain amount of heat that can never be pumped back again. Entropy never quite attains its former level, but is always a little greater, since entropy may be regarded as *a measure of the difference between energy and the free energy that one can use in driving the machines of the world.*

There is an elemental quality in this concept. Maupertuis had said that there was something which nature always strives to make a minimum. The dictum of Clausius sees in nature a quantity that always strives toward a maximum. The machines of commerce work for a day and the net result is that the time is brought a little nearer when all the heat will have flowed down hill from the higher levels and free energy will have disappeared forever. Society never advances but recedes as rapidly on one side as it gains on the other. The energy upon which mankind depends for his existence is forever flowing down the temperature gradients that occur in all the activities of nature. Entropy forever increases to the fatal maximum. Was there ever pictured a more melancholy destiny for the universe and for the society of man?

Very naturally there has existed a speculative appeal in this concept of entropy, as a property of nature which, when it changes, always changes in one direction. A passage illustrating this point and bearing

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[19] *Vorlesungen über Thermodynamik,* English trans., by A. Ogg, London, 1903, p. 103.

also upon the question of the objective validity of natural law is taken from the *Treatise on Thermodynamics* by Max Planck (1858-1947):[19]

"The gist of the second law [of thermodynamics] has nothing to do with experiment; the law asserts briefly that *there exists in nature a quantity which changes always in the same sense in all natural processes.* The proposition stated in this general form may be correct or incorrect; but whichever it may be, it will remain so, irrespective of whether thinking and measuring beings exist on the earth or not, and whether or not, assuming they do exist, they are able to measure the details of physical or chemical processes more accurately by one, two, or a hundred decimal places than we can. The limitations of the law, if any, must lie in the same province as its essential idea, in the observed Nature, and not in the Observer. That man's experience is called upon in the deduction of the law is of no consequence; for that is, in fact, our only way of arriving at a knowledge of natural law. But the law once discovered must receive recognition of its independence, at least in so far as Natural Law can be said to exist independent of Mind. Should any one deny this, he would have to deny the possibility of natural science."

The object of this discussion, introduced so early into our study of concepts that underlie modern physics, is not to give a full explanation of entropy, but to prepare the way for the later discussion of energy. It is well for us merely to see in as full view as possible this metaphysical character of the theory of heat which takes as one of its most fundamental concepts a thing which, though mathematically defined and so thoroughly measurable that its values are recorded in elaborate tables calculated to several places of decimals, is, nevertheless, quite beyond rational intuitions.

The savage as he sat at evening time on the banks of the Mississippi river may occasionally have reflected upon the source of the father of waters. Why did it flow forever toward the sea without finally emptying the source? If the dictum of Clausius be true why does not all the energy reach its final level? Why in the lapse of geologic time have not the stars all burned out, the motion of the planets ceased and all matter come to rest? Why does entropy which always seeks its maximum never arrive?

This thought has been expressed before by others and the answer seems to be that in the macrocosmic universe the second law of thermodynamics may have an exception although none has ever been found in the microcosmic system such as we have within the limits of laboratory experiment.

The recent speculations of the physicists as to the origin of cosmic radiation, which will be surveyed in more detail in a later chapter, appear to indicate that the second law of thermodynamics may not be true in the cosmic universe and that entropy is not necessarily approaching a maximum value. Heat may run uphill just as water does under the sucking action of the sun. These matters, however, are still dark and the truth is uncertain.

From another point of view the theory of heat has been fully developed upon the hypothesis that a hot body consists essentially of a large number of minute particles (molecules) whose mean kinetic energy, that is to say the mean energy of their velocities, determines the temperature. These molecules are distributed according to the laws of probability, some being rapidly moving ones and some being slow, according to the energies which they have acquired. In other words, the uniform temperature of a body is due to a statistical distribution of moving molecules and would be very different if it were determined by the energy of its most slowly moving molecule or by the energy of its most rapidly moving one. That part of the second law of thermodynamics which says that heat energy always flows from a hot body to a cold body can thus be picturesquely expressed by the statement that one can never separate the swift molecules from the slow, once they have been thoroughly mixed, just as one can never unscramble an egg.

This concept immediately suggests a question. What do we mean by statistical law? In another chapter the matter will be further explored. For our present need it will be sufficient to affirm that statistical law arises out of a complex of causes too intermingled for their separate influences to be apprehended and such that no single one dominates the situation. But does statistical law rule the activity of the universe or in isolated systems of activity do we find dominating influences guiding the destiny of events? The answer is obvious. Wherever intelligence enters the situation statistical laws are submerged in obedience to a more powerful factor. This, perhaps, is why satisfactory results are not always obtained when the statistical model is applied to biological or psychological problems where the mysterious attribute of intelligence may enter the material under investigation. Let us consider, for example, the statistical problem presented by the dealing of four hands of cards. To make our experiment let us hand the deck to a child who knows nothing about cards and suggest that he distribute them into four piles. Let us regard his arrangement as a fortuitous event; let him, if you please, occupy the position of a card-dealing mechanism. The miracle is soon revealed. The small intelligence of the child is sufficient to bring us into the presence of the impossible. He will almost immediately separate the cards into the four suits. In a few moments he has accomplished what probably has never happened in the history of the world by chance.

We return to the problem which suggested this digression. How can we unscramble an egg or separate the molecules according to their velocities from the intricate mixture of a gas? The answer is that this miracle may be accomplished in theory by the proper introduction of intelligence. It is possible that in the distribution of molecules the swift ones might be separated from the slow provided a *sorting demon* of molecular size and with proper equipment were set to the task. This interesting idea is due to Clerk Maxwell who puts the matter in these words:[20]

"One of the best established facts in thermodynamics is that it is impossible in a system enclosed in an envelope which permits neither change of volume nor passage of heat, and in which both the temperature and the pressure are everywhere the same, to produce any inequality of temperature or of pressure without the expenditure of work. This is the second law of thermodynamics, and it is undoubtedly true so long as we deal with bodies only in mass and have no power of perceiving or handling the separate molecules of which they are made up. But if we conceive a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are still as essentially finite as our own, would be able to do what is at present impossible to us. For we have seen that the molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower ones to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics."[21]

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[20] *Theory of Heat*, p. 328.

[21] Even this seemily impregnable argument of Maxwell has been assailed in this agnostic age by attributing a kind of entropy to the demon himself. See an article by L. Szilard in the *Zeitschrift für Physik*, Vol. 53, 1929, p. 840. P. Clausing later, however, controverts the position of Szilard. *Ibid.*, Vol. 56, 1929, p. 671.

### 9. *Science—The Perplexing Mysteries.*

With this remark we have reached the goal set by the present chapter. We have surveyed some of the great concepts upon which the founders of modern physics have built their structure. They are concepts with which the giants of the nineteenth century struggled and turned over as unsolved puzzles to their successors of the twentieth. The vast amounts of laboratory data that have been accumulated in attempting to explore their consequences, the volumes of formulas to which they have led, and the machines that have been created from them, form one of the richest heritages ever passed on by one century to another.

But the object of this volume is to show the foundation upon which rests physical science in general and the nature of energy in particular. As we have seen the problems become more perplexing and mysterious as we proceed. Science is not and cannot be wholly material and rational so long as action at a distance, the ether of radiation, the action of Maupertuis, and the entropy of Clausius remain as the foundations of its structure. Great progress in investigation has been made during the first part of the present century until, in the enthusiastic estimate of some critics, the total scientific knowledge of the world has more than doubled since 1900. But progress toward the mystery of reality has merely led to newer mysteries. The perplexing puzzle today presents a greater challenge to those who would pick up pebbles with Newton on the beach where "the immense ocean of truth extends itself unexplored."

## CHAPTER 2.

# THE PHILOSOPHICAL BACKGROUND OF MODERN PHYSICS.

"But the discoveries of great men never leave us; they are immortal, they contain those eternal truths which survive the shock of empires, outlive the struggles of rival creeds, and witness the decay of successive religions. All these have their different measures and their different standards; one set of opinions for one age, another set for another. They pass away like a dream; they are as the fabric of a vision, which leaves not a rack behind. The discoveries of genius alone remain; it is to them we owe all that we now have; they are for all ages and all times; never young and never old, they bear the seeds of their own life; they flow on in a perennial and undying stream; they are essentially cumulative, and, giving birth to the additions which they subsequently receive, they thus influence the most distant posterity, and after the lapse of centuries produce more effect than they were able to do even at the moment of their promulgation."

Henry Thomas Buckle in his *Introduction to the History of Civilization in England.*

### 1. *The Value of Abstract Speculation.*

Some philosopher has aptly said: "Every science begins as philosophy and ends as art; it arises in speculation and flows into achievement." In the beginning there are vague gropings after the basic forms of nature; speculation takes the place of exact knowledge; hypotheses are advanced long before the discovery of essential data; the unfettered imagination runs far ahead of the toilsome work of the laboratory. The theory of biological evolution, for example, was inherent in the writings of ancient philosophers twenty centuries or more before the extended studies of Lamarck, Wallace, and Darwin gave it standing as a scientific principle. The heliocentric theory of the solar system appeared in Greek and medieval astronomy centuries before Bessel reported the discovery of the first stellar parallax. For this reason philosophy has sometimes been called by its adherents the mother of the sciences.

But science has not always valued this philosophical inheritance. After it has attained its strength and the vague terms of its early concepts have been crystalized into the language of laboratory instruments and mathematical symbols, science looks with a measure of distrust upon its origin. Science advances while philosophy recedes.

This attitude, however, is an unfortunate one for the progress of knowledge. Nearly every great advance has been attended by some speculative principle which is more philosophy than science. Could any one have foreseen in the abstract postulates of the theory of relativity the emergence of the mighty force of atomic energy? And yet the relationship which this theory established between matter and the space surrounding it proved to be in the end the most important key to the whole problem.

Abstract speculation has too often been neglected or made subordinate to the more spectacular gains in pragmatic values that have come in such profusion from science. The Greeks, hampered by their lack of instruments and almost completely destitute of tables of data acquired from laboratory studies, turned their meditations inward. Such problems as the trisection of an angle and the duplication of the cube by means of straight edge and compasses intrigued their fancy. The science of mathematics made essential gains, but, except in a few scattered instances, scant thought was expended upon its application to natural phenomena.

And yet the value to modern science of the abstract speculations of the Greeks and their modern successors cannot be over estimated. They are to be found at the very core of many scientific theories, especially where they relate to the mysteries of energy. Those who scorn these philosophical origins of modern science might well reflect upon the following passage from the work of C. S. Peirce (1839-1914), physicist, mathematician, and logician, who, although the founder of pragmatism, still saw clearly the practical values in pure speculation:[1]

"Kepler's discovery rendered Newton possible, and Newton rendered modern physics possible, with the steam engine, electricity, and all other sources of the stupendous fortunes of our age. But Kepler's discovery would not have been possible without the doctrines of conics. Now contemporaries of Kepler. . . were abandoning the study of geometry . . . because they said it was so *utterly useless*. There was the future of the human race almost trembling in the balance; for had not the geometry of conic sections already been worked out in large measure, and had their opinion prevailed, that only sciences apparently useful ought to be pursued, the nineteenth century would have none of those characters which distinguished it from the *ancien regime.*"

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[1] From his *Collected Papers*, Vol. 1, Cambridge, Mass., 1931, p. 32.

The problems with which we shall be concerned principally in this chapter relate to the structure of material objects and to the time and space in which they are embedded. The struggle of the philosophers with these deep concepts will invoke such problems as the nature of continuity in contrast to the nature of the discrete. The present puzzle in physics concerning the difference between photons, or particles of light, and electrons, or particles of matter, presents problems analogous to those found in the more abstract speculations of the mathematical philosophers.

The geometry of space and the nature of time have been favorite themes with those who have studied nature by speculation. The one-wayness of time and the mystery of the three-dimensional character of space have long intrigued the imagination of the philosophers. The value of exploring such abstract entities is shown in one of the most startling discoveries of the nineteenth century, which revealed that geometry is not a unique discipline and that many equally valid geometries can be constructed according to the fancy of the mathematician. In one problem he may use the geometry of Euclid, but in another he may invoke a geometry the basic postulates of which are in direct contradiction with those of Euclid. The difference comes in the way one wishes to regard the nature of space and the concept of infinity.

Enough has now been said to indicate the importance of abstract speculation in the history of science. This importance is especially apparent in the history of energy, which depends so fundamentally upon the relationships between space, time, and matter. We shall proceed, therefore, to an examination of a few of the pertinent philosophical systems of the past, which form a necessary background to the proper understanding of physical phenomena.

## 2. *Zeno's Paradoxes.*

What were some of the striking features of Greek thought where it touched upon matters of metaphysics? We find, for example, Anaximenes of Miletus (6th century B. C.) asserting that air is the first principle of nature, from which we get by condensation and rarefaction, fire, wind, clouds, water, and earth.

Heraclitus (c. 540-475 B. C.), an early Greek philosopher who lived at Ephesus, said that the fundamental form of existence was fire and that all things were in eternal flux. Matter continuously transforms into fire and fire back again into matter. The law of the world is change; nothing is quiet and that which seems at peace is still subject to unseen movements. His concept of the universe is one of ceaseless energy.

In direct contrast to the philosophy of Heraclitus is that of Zeno of Elea (5th century B. C.) who has left us some of the most perplexing paradoxes of philosophy. As late as 1915 Florian Cajori (1859-1930) published a long series of articles on the Zeno problem, and explanations of the difficulty still not infrequently appear in the public press.[2] We shall have occasion in a later chapter to discuss the theory of point sets and the problem of atoms versus the continuum. Hence it is worth recording here that these puzzles are direct descendants of the Zeno problem. Perhaps it was his reflections concerning the flux and movement of the world of Heraclitus which led Zeno to the pregnant idea that the supposition of the real existence of things manifold and changing leads to contradictions. His philosophy was thus made to assume that nothing moves, and he defended his position with the following arguments:

1. Motion cannot begin, because a body in motion cannot arrive at another place until it has passed through an unlimited number of intermediate places.

2. Achilles cannot overtake the tortoise, because as often as he reaches the place occupied by the tortoise at a previous moment, the latter has already left it.

3. The flying arrow is at rest; for it is at every moment only in one place.

These arguments can be put a little more vividly, perhaps, as follows. Drop an elastic ball upon a smooth table and watch it bounce. Let us suppose that at each impact with the table top it rebounds through half of the distance through which it falls. The question is: Will the ball ever come to rest? At first thought we might be inclined to say no, because, on the assumption of a rebound every time, the ball must bounce an infinite number of times. To be sure these bounces will soon approach molecular size and then submolecular size, but nevertheless there will be an infinite number of them. As a matter of fact we know either from experience or from calculation based on the law of falling bodies, that the ball will come to rest in a very few seconds. Indeed, it is not a difficult mathematical exercise to show that if the fall is through a distance of sixteen feet, the ball will come to rest in less than six seconds. This problem will seem more real if one will verify this statement by making the calculation oneself.

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[2] "History of Zeno's Arguments on Motion," *American Mathematical Monthly*, Vol. 22, 1915.

The point of Zeno's paradox that we wish especially to stress is the connection which it effects between space and time through the concept of velocity. Let us think of the space through which the ball bounces as being infinitely divisible. We must similarly regard an interval of time as being capable of unlimited divisibility. It is in the subtle connection between the space-concept, the time-concept and the limiting values of the two sequences involved in the motion of the ball that the paradox resides. What is velocity? How is the progress of matter through space connected with the partitioning of time?

Have the paradoxes of Zeno been answered? Cajori at the end of his notable papers, to which we have already alluded, makes the following interesting observation:[3]

"As now we pause and look backward, we see that a full and logically correct explanation of Zeno's arguments on motion has been given by the philosophers of mathematics. Looking about us, we see that the question is still regarded as being in an unsettled condition. Philosophers whose intellectual interests are remote from mathematics are taking little interest in the linear continuum as created by the school of Georg Cantor. Nor do they offer a satisfactory substitute. The main difficulty is not primarily one of logic; it is one of postulates or assumptions. What assumptions are reasonable and useful? On this point there is disagreement. Cantor and his followers are willing to assume a continuum which transcends sensuous intuition. Others are not willing to do so. Hence the divergence. In the Koran there is a story that, after the creation of Adam, the angels were commanded to make due reverence. But the chief of the angels refused, saying: 'Far be it from me a pure spirit to worship a creature of clay.' For this refusal he was shut out from Paradise. The doom of that chief, so far as the mathematical paradise is concerned, awaits those who refuse to examine with proper care the massive creation by our great mathematicians without which the tiniest quiver of a leaf on a tree remains incomprehensible."

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[3] Pp. 296-297.

### 3. *The Atomists.*

Closely associated with the Zeno problem, which focuses our attention upon the infinitesimal, is the concept of the atomic nature of the universe. Leucippus (5th century B. C.) and Democritus (c. 460-370 B. C.) among the Greeks and Lucretius (c. 98-55 B. C.) among the Romans were exponents of the atomistic philosophy. According to them nature was formed from the "full" and the "void." The full consisted of indivisible, primary particles of matter, or atoms, which are distinguishable from one another only in form, position and arrangements. For example, the atoms of fire and of the soul are round; pleasant tastes are caused by smooth, and bitter tastes by jagged atoms.

The philosophy of the atomists reaches full flower in the poem of Lucretius, *De Rerum Natura*, written about 54 B. C., where the poet lets his bold imagination range over most of the problems of man's relation to the universe. This poem is remarkable not only for the fact that it anticipates many of the concepts important to modern scientific thought, such, for example, as the atomic structure of matter, Darwin's theory of evolution, conservation of matter and energy, etc., but also in its exploration of the concept of infinity.

C. J. Keyser has pointed out the very modern point[4] of view which Lucretius seems to have had with regard to the concept that lies at the basis of mathematical philosophy. Atoms in the view of Lucretius must lie between an upper and a lower bound and finite sums of them are always finite. But since the matter of the universe is infinite the number of atoms of which matter is composed is also infinite. The following striking passage is characteristic of the arguments of the poet:[5]

"Again unless there shall be a least, the very smallest bodies will consist of infinite parts, inasmuch as the half of the half will always have a half and nothing will set bounds to the division. Therefore between the sum of things and the least of things what difference will there be? There will be no distinction at all; for how absolutely infinite soever the whole sum is, yet the things which are smallest will equally consist of infinite parts. Now since on this head true reason protests and denies that the mind can believe it, you must yield and admit that there exist such things as are possessed of no parts and are of a least nature. And since these exist, those first bodies also you must admit to be solid and everlasting."

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[4] "The Role of the Concept of Infinity in the Work of Lucretius," *Bulletin of the American Mathematical Society*, Vol. 24, 1917-18.

[5] Book I, lines 615-627. From the translation by H. A. J. Munro, 1910, p. 15.

We see in this passage a mind struggling with the great concept of infinity. Does matter consist of ultimate particles which cannot be further sub-divided and a finite sum of which makes up the structure of any given piece of matter, or can matter be infinitely divided and is the object in your hand merely the materialization of the limit of an infinite sum of the infinitesimal quantities with which the science of mathematics plays? A careful study of the words of Lucretius which we have quoted will yield the conclusion that the poet, in that gray age of science, was quite unaware of the existence of the latter possibility and that his gem of thought is clouded. We shall return to these questions in a later chapter.

Turning from a study of the ultimate structure of matter Lucretius next gives thought to the extension of space and the possibility of conceiving that the universe has a bound. This passage has more than historical interest for us because it focuses the common concept of the boundlessness of space into a passage of rare beauty. Since the discoveries of modern astronomy, particularly as they apply to the great recessive velocities of the spiral nebulae, and modern speculations as to the cause of the inertia of matter, will lead us to a consideration of this problem of metaphysics, it seems worth while to set forth the arguments by which Lucretius decided that space was without a bound:

"Well then the existing universe is bounded in none of its dimensions; for then it must have had an outside. Again it is seen that there cannot be an outside of nothing, unless there be something beyond to bound it . . . . And it matters not in which of its regions you take your stand; so invariably, whatever position any one has taken up, he leaves the universe just as infinite as before in all directions. Again if for the moment all existing space be held to be bounded, supposing a man runs forward to its outside borders and stands on the utmost verge and then throws a winged javelin, do you choose that when hurled with vigorous force it shall advance to the point to which it has been sent and fly to a distance, or do you decide that something can get in its way and stop it? For you must admit and adopt one of the two suppositions; either of which shuts you out from all escape and compels you to grant that the universe stretches without end. For whether there is something to get in its way and prevent its coming whither it was sent and placing itself in the point intended, or whether it is carried forward, in either case it has not started from the end. In this way I will go on and, wherever you have placed the outside borders, I will ask what then becomes of the javelin. The result will be that an end can no where be fixed, and that the room given for flight will still prolong the power of flight . . . Lastly, one thing is seen by the eyes to end another thing; air bounds off hills, and mountains air, earth limits sea and sea again all lands; the universe, however, there is nothing outside to end."[6]

It is instructive to observe that Lucretius, while willing to admit the infinite nature of space, denied this possibility to the microcosm by assuming the existence of indivisible atoms. Although we have already indicated the nature of these primordial entities, it is a matter of interest to observe how the poet avoided his dilemma of an infinite space and a finite atom. This he achieved by assuming that space can contain two things only, a vacuum or a plenum. About the latter Lucretius says:[7] "There exist, therefore, certain bodies which can completely fill the places which they occupy, and distinguish empty space from full. These bodies can neither be broken in pieces by being struck with blows externally, nor again, can be decomposed by being penetrated internally, nor can they be made to yield if attempted by any other method." A few lines later the poet completes his description as follows:[8] "Primordial atoms are therefore of pure solidity, which, composed of the smallest points, closely cohere; not combined of a union of any other things, but rather endowed with an eternal, simple, and indissoluble existence, from which nature allows nothing to be broken off, or even diminished, reserving these primordial atoms as seeds for her production."

That these ideas of Lucretius exerted considerable influence upon more modern thought is indicated by the following passage from the pen of Sir Isaac Newton. An estimate of the difficulty of these concepts and an explanation of the slow progress made by science in developing the atomic theory of matter is afforded by the fact that a span of nearly eighteen centuries separated the words of the poet from the words of Newton. The latter wrote as follows:[9]

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[6] Book I, lines 958-1001. Munro's translation, p. 23.

[7] Book I, lines 526-530. [8] Book I, lines 609-614.

"It seems probable to me that God, in the beginning, formed matter in solid, massy, hard, impenetrable, movable particles, of such sizes and figures and with such other properties, and in such proportion to space, as most conduced to the end for which He formed them; and that these primitive particles, being solid, are incomparably harder than any porous body compounded of them; even so very hard as never to wear or break in pieces: no ordinary power being able to divide what God himself made one in the first creation."

Our estimate of *De Rerum Natura* must be that of others who have been struck by the power and imagination of the poet. Alas, Lucretius lived in an age when speculation could not be checked by the cold facts of laboratory experiment; when the tool invented by Newton and Leibniz and sharpened by their followers, when the telescope of Galileo and instruments of precision were not available to turn dreams into theories and speculations into formulas. The poem contains a statement of most of the problems of natural philosophy and proceeds to their solution only in so far as the unaided mind can go.

### 4. *The Philosophy of Descartes.*

In passing over the philosophical system of Plato, Aristotle and the other Greeks we do not mean to slight this noble heritage, but a philosophy of natural phenomena developed without competent objective data must necessarily have had small influence upon the real progress of scientific thought. We pass, therefore, to the system of René Descartes (1596-1650) whose theory of vortices played a conspicuous part in the pre-Newtonian period of science.

In this ambitious theory Descartes assumed that there was no such thing as empty space and that matter was spread not as atoms, but continuously throughout the whole universe. Naturally in the motion of this all-pervading matter, whirlpools or vortices would be set up as whirlpools are set up in flowing water. As the particles rub against one another some are worn round like stones in a stream and the part that is worn off becomes a sort of cosmic dust. This dust tends to accumulate at the center of the vortices where it forms suns and stars. The globular particles are thrown like cream in a separator to the outside and form the atmosphere of the sun. By exercising his imagination upon these hypotheses Descartes was able to construct a theory which accounted for many of the phenomena of nature. He, however, like Lucretius, lacked in considerable measure, though not completely, the experimental observations with which Newton was later able to convince the intelligence of his contemporaries and win belief for his theory of light and gravitation.

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[9] *Opticks*, 3rd ed., 1721. pp. 375-376.

It will be illuminating to record the precise thoughts of Descartes on the questions with which we shall later be most concerned, namely those which have to do with definitions of space, time, and the fluid that fills the "void."

Thus we find in the eleventh principle of the second book of his *Principles of Philosophy* (1644) a discussion of the meaning to be attached to the statement "that space is not different from corporeal substance":

"And it will be easy for us to recognize that the same extension which constitutes the nature of body likewise constitutes the nature of space, nor do the two mutually differ, excepting as the nature of the genus or species differs from the nature of the individual, provided that, in order better to see what is the true idea that we have of body, we take a stone and reject from it all that does not belong to the nature of body. Let us then first reject hardness, because, if one should reduce this stone to powder, it would no longer have hardness, and yet would remain a body; let us reject color, because we have seen occasionally some stones so transparent that they have not had color; let us reject weight, because we have seen fire, which, although it is very light, still possesses body; let us reject cold, heat, and all of the other qualities of this kind, because we do not think that they are in the stone, or rather that the stone does not seem to change its nature because it is either hot or cold. After we have thus examined this stone we find that the real idea which forces us to think that it is a body, consists in this alone, that we perceive distinctly that it is a substance extended in length, breadth, and depth; and this is comprised in our idea of space, not only of that which is full of body, but that which is called empty."

Having thus arrived at the concept of extension Descartes proceeds to analyze the meaning of empty space. Is it possible to conceive of a perfect vacuum or must all extension be endowed with corporeal properties? The keynote of the philosophy of the French thinker is to be found in his answer to this question. With him there is no vacuum. Space is filled with an ethereal fluid whose mutual reaction with matter comprises the totality of phenomena. We who reflect upon the revelation of modern science are not yet sufficiently satisfied about the nature of radiation and the method by which heat reaches us from the surface of the sun to be wholly scornful of the following passages which are the sixteenth and seventeenth principles in the second part of Descartes' philosophy:

"With regard to a vacuum, in the philosophical sense of the word, where we mean a place in which there is no substance, it is evident that there is no such place in the universe, because the extension of space or of internal place is no different from the extension of body. And since from the fact that a body is extended with regard to length, breadth and depth, we have reason to conclude that it is a substance, because it is quite inconceivable that that which is nothing has extension, we must therefore conclude the same thing of space which one supposes to be empty, that is to say, since it has extension it must necessarily also have substance.

"But when we take this word according to ordinary usage, and we say that a place is empty, it is an established fact that we do not wish to say that there is nothing at all in this place or in this space, but merely that there is nothing of that which we presumed ought to be there. Thus because a pitcher is made to hold water, we say that it is empty when it contains only air; and if there is no fish in a fish-pond we say that it is empty, although it be full of water; thus we say that a vessel is empty, when in place of the merchandise which it is designed to carry, it is loaded only with sand, in order that it might be able to resist the impetuous violence of the wind: it is in this same sense that we say that space is empty when it contains nothing that affects our senses, even though it contains created matter and an extended substance. For we consider ordinarily only those bodies which are near to us in so far as they create in the organs of our senses, impressions so strong that we are able to perceive them. And if, in place of our knowing what we ought to understand by the words vacuum or nothing, we conceive hereafter of a space where our senses perceive nothing as containing nothing created, we shall fall into an error as great as if, because one ordinarily says that a pitcher is empty in which there is only air, we therefore judge that the air which it contains is not a substantive thing."

Those who have followed the warfare between the corpuscular theory of light and the theory of propagation of waves in a continuous medium, and the struggle of the concepts of the continuum in mathematics, will see here that Descartes arrays himself upon the side of the continuists. In another passage he very definitely indicates his break with the atomists, when he shows the difference between the postulate of an ethereal fluid throughout empty space and the postulate of Democritus that the material world was constructed out of atoms which were separated from one another by complete emptiness (Principle 202, Part 4). In order to finish the story that we are tracing we must examine the attitude of Descartes toward the extent of his primary substance. This, he says, is infinite for the reasons quoted below. Would one say that he has made an advance over the arguments of Lucretius?

"We also know that this world, or the material extent which composes the universe, has no limit," says Descartes in the twenty-first principle of part two, "because whatever limit we wish to imagine, we are still able not only to imagine some indefinitely extended spaces beyond, but such spaces as we have conceived them to be; namely, that they contain a body of indefinite extent. For the idea of extent which we conceive in any space whatever is the same idea which we ought to have of body."

It is not our purpose to pass judgment upon the character of these concepts of Descartes. We are merely interested in tracing the philosophical background from which modern science arose; but we can easily see how the minds of scientists that followed were prepared by such doctrine for the concept of an ether filling the vacuum of empty space and extending without limit in all directions.

At the basis of Descartes' idea of the primary nature of "extent," which is the basic property of matter and hence of space, there is merely a postulate as to the nature of reality. Let us, for example, consider the concept represented by the modern word "ether." We first proceed to endow it with a set of properties; we conceive of it as space-filling, tenuous beyond the limits of measurement of our material instruments, perfectly elastic, fluid, etc. We bring to bear upon the concept the machinery of mathematics and translate such of these properties as we can into the language of symbols. We deduce certain consequences of our definitions and then seek to verify them by instruments of precision. Should the concept break down because experimental data fail to confirm the mathematical conclusion, it is discarded. One by one the properties are thus tested and rejected until only the primary one of extension remains. Is the ether thus denied existence? Without doubt its sensory existence is severely jeopardized, but is this sufficient to deny it objective reality? In other words, how many and what properties are necessary to objective existence?

We turn finally to the brief consideration which Descartes gave of time. Time to him was subjective and resided in the mind rather than in the object.

Thus he said in the 57th principle of Part I the following:

"Of these qualities or attributes, there are some which are in the things themselves, and others which are only in our thought; thus, for example, time which we distinguish from duration considered in general, and which we call the measure of movement, is only a certain way of regarding duration for we do not think that the duration of things which are changing is different from that of things which are not: for it is evident that if two bodies move during an hour, the one rapidly and the other slowly, we do not consider more time to exist for one than for the other, although we suppose more movement to have taken place in one than in the other. But finally in order to comprehend the duration of all things under the same measure, we usually avail ourselves of the duration of certain regular movements which make days and years, and this we call time. Hence this adds nothing to the notion of duration, generally considered but a mode of thinking."

### 5. *Are there Postulates of Normal Intuition?*

We must now pause for a slight digression in the historical account of the philosophical speculations which preceded the age of science and ask this question. Is it possible to formulate a set of postulates of normal intuition; that is to say, are there certain primary statements of belief which can be set down without fear of reasonable differences arising among rational people? We have seen how Lucretius and Descartes held as a fundamental tenet that space was limitless, although they differed completely in their opinion as to the ultimate divisibility of matter. Is this limitlessness of space, admitting that it is beyond the possibility of experimental proof, a fundamental postulate of all reasonable people? Is it more rational to believe that space is boundless rather than that it is bounded or is the question wholly without meaning? It is of considerable interest to note that in a debate on the theory of relativity so able a mathematician and astronomer as the late W. D. MacMillan attempted to formulate a creed of rational belief. It will be illuminating to the reader to survey these postulates of normal intuitions thoughtfully and to analyze his own reactions toward them. MacMillan advanced the following propositions:[10]

1. The physical universe is continuous in time.
2. The physical universe is not bounded in space.
3. There exist physical units, which for finite intervals of time preserve their identities and exhibit characteristic properties. These are, for example, electrons, atoms, molecules, stars, galaxies, etc.
4. The sequence of physical units is infinite both ways. That is to say, there is no largest physical unit and there is no smallest one.
5. The phenomena of nature occur always in such a way that certain relations remain invariant.
6. The energy within a region of space does not increase or decrease unless there is a corresponding decrease or increase in some other region of space.
7. The universe does not change always in any one direction.
8. The geometry of the physical universe is Euclidean.
9. The time of the physical universe is Newtonian.

One of the objects contemplated by this volume is to exhibit the struggle of modern physics with these tenets. Guided by the researches of Einstein, Millikan, Michelson, Shapley and many others science has now accumulated experimental evidence from the laboratory and the sky which bears directly upon these philosophical problems. That the evidence is conclusive is doubtful; that it is stimulating to the imagination and encouraging to bold speculation is to be preëminently admitted.

### 6. *Newton's Absolute Space and Time.*

We proceed now to consider the philosophy of Newton which finds its place in the postulates of rational intuition cited above. *Velocity*, according to Newtonian mechanics, may be described as the limit of an increment of space divided by an increment of time; *acceleration* as the limit of an increment of velocity divided by an increment of time; *force* as mass times acceleration. Construed in less technical language these definitions are the abstract formulation of the common observation that a stone falling under the influence of gravitation moves with a constantly increasing velocity. These ideas seem simple enough until we reflect that in them there is found a basic relationship between space and time as it seems to apply to the objects of the material universe. In them time appears as an implicit variable, a quantity which cannot be dissevered from the velocity or acceleration to which it pertains.

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[10] *A Debate on Relativity*, Open Court, 1927, pp. 42-43.

It is important for subsequent chapters to have a clear concept of what Newton thought regarding time and space. In the first part of the *Principia*, he makes these celebrated remarks:[11]

"I do not define time, space, place, or motion, as being well known to all. Only I must observe that the vulgar conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which, it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.

"1. Absolute, true, and mathematical time, of itself, and from its own nature flows equally without regard to anything external, and by another name is called duration; relative, apparent, and common time is some sensible and external (whether accurate or unequable) measure of duration by means of motion . . . .

"2. Absolute space, in its own nature, without regard to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute space.

"3. Place is a part of space which a body takes up, and is according to the space, either absolute or relative.

"4. Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another."

It will be clear from a study of these definitions that Newton regarded space and time as being real entities, things of an absolute physical character which existed exterior to our mental perception of them. To him there was an absolute space and an absolute time and if all material objects were removed it is conceivable, although the point is not explicitly made, that the framework of absolute time and space would still exist.

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[11] This is a resume of a scholium appended to his initial definitions, pp. 6-7 of the Glasgow edition (1871).

P. W. Bridgman some years ago brought an interesting point of view to bear upon these concepts of philosophy, by means of which he would eliminate many questions as meaningless.[12] His idea is that every concept of physics or of mathematics must be measured in terms of the actual operations necessary to define it. Are there such things as absolute space and absolute time or is the question without meaning? To explore these possibilities one must investigate the operations by which time and space are measured and he will see that these are all relative. Hence, employing the criterion that "*the concept is synonymous with the corresponding set of operations*," we see that it is meaningless to speak of absolute time and space. Other questions to which Bridgman suggests that this criterion of reality should be applied are: "May time have a beginning or an end?," "Why does time flow?," "May space be bounded?," "May space or time be discontinuous?," "Why does nature obey laws?," etc. Perhaps this is the solution of the problem of philosophy, but while man sees or thinks he sees the possible synthesis of natural law in such abstract concepts as entropy and action he will not wish to dismiss thus lightly from his speculations any abstractions which might lead to deeper insight. It is but a step from the vacuum of Descartes to the ether of Maxwell.

### 7. *The Contributions of Locke and Leibniz.*

It is difficult to estimate the relation of *An Essay Concerning Human Understanding*, the principle work of John Locke (1632-1704), to the scientific revolution, but there is no doubt that it exerted a powerful influence upon the speculative thought of that formative period. Locke was much influenced by the arguments of Descartes with regard to extension and duration, but he was unwilling to believe that a vacuum necessarily possessed attributes of a body. "Our own clear and distinct ideas plainly satisfy us, that there is no necessary connection between space and solidity, since we can conceive the one without the other."[13] The following naïve argument shows that his belief in space was similar to that of Newton as something having an objective validity:[14]

"For if they [plain men] had not the idea of space without body, they could not make a question about its existence; and if their idea of body did not include in it something more than the bare idea of space, they could have no idea about the plenitude of the world; and it would be as absurd to demand whether there were space without body, as whether there were space without space, or body without body, since these were but different names of the same idea."

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[12] *The Logic of Modern Physics*, Macmillan, 1927, pp. 28-32.

[13] Book 2, Chap. 13, 22. [14] Book 2, Chap. 13, 24.

Locke's concept of duration adds nothing to that of Descartes and is concerned mainly with the distinction between duration "considered as going in one constant, equal, uniform course"[15] and the means taken to measure it. He devotes a chapter to the mutual relationship which exists between time and space, and reaches the conclusion that "expansion and duration do mutually embrace and comprehend each other; every part of space being in every part of duration, and every part of duration in every part of expansion."[16] That this mutual embrace could be reduced to formulas and applied in tracing the movements of the planets, the motion of the tides and other phenomena of the material world was certainly very far from his immediate thoughts.

The philosophy of Gottfried Wilhelm Leibniz (1646-1716), the contemporary of Newton who shares with him the honor of discovering the principle of the calculus, centers about a mystical theory of *monads*. Monads are to be thought of as primary entities like atoms, but atoms in which there is a spark of intelligence. They differ from the atoms of Democritus and Lucretius in the assumption that they have no measurable dimensions, but are points and are endowed with active forces which consist of ideas. Had Leibniz been able to show that collections of his monads violated some of the laws of abstract probability by reason of their intelligence, such as might be expected from a group of Maxwell's demons mixed up in a collection of material atoms, then his idea would have had far more objective reality. As it is, monadology is but an interesting historical incident in the struggle of man toward the light.

The philosophy of Descartes exerted a strong influence upon Leibniz and he has recorded in various places his opinions concerning the principles of the French thinker. That one which has for us perhaps the most significance is his question whether the essence of a body consists merely in extension.

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[15] Book 2, Chap. 14, 21. [16] Book 2, Chap. 15, 12.

"If the essence of body consists in extension, this extension alone ought to be sufficient to account for all the properties of body. But this is not so. We notice in matter a quality, called by some *natural inertia*, by which body resists in some way motion; so that it is necessary to employ some force to set it in motion (even making abstraction of the weight) and a large body is moved with more difficulty than a small body."[17]

We shall see later how this idea of Leibniz persisted through speculation, gained an entire new meaning in the behavior of the Foucault pendulum and the motion of the gyrocompass, and led to the attempt of Mach to find its explanation in the other material of the universe. "All this shows," says Leibniz, "that there is in matter something other than what is purely geometrical; that is, than extension and its changes pure and simple."[18]

### 8. *Immanuel Kant and the Subjective View.*[19]

A new point of view, new in the sense that ideas expressed in the writings of his predecessors here appear systematically stated and logically developed, was injected into philosophy with the work of Immanuel Kant (1724-1804), who in 1781 published his celebrated *Critique of Pure Reason.* Those who hesitate to include this name in a history of the development of physical ideas would do well to recall that Kant's early thought was dominated by the work of Newton and he shares with Laplace the credit for having first stated with systematic arguments the nebular hypothesis of the origin of the solar system. That his speculations finally led him to advance the possibility of the subjective existence of nature comes more impressively from him than from one who had struggled in vain with the ideas of Newton.

These pages are tracing the growth of speculative thought and one must not imagine that the ideas of Kant are without ancestry. Great theories which we encounter for the first time in our study often seem like isolated bursts of imagination; we marvel at their complexity, their applications, the far reaching vistas that they open. They seem like the great buttes, isolated on the high plateaus of western landscapes, gigantic masses towering in striking relief against the sky, the "monuments more enduring than bronze" of which the poet Horace wrote. But isolation is rarely the case in speculative thought, which resembles, rather, the growth of mountain ranges rising from foot hills to mesas and from mesas to the great peaks. The work of Kant was but the logical culmination of the idealistic view of George Berkeley (1685-1753), Bishop of Cloyne, and David Hume (1711-1776), who sought to undermine the materialism of their age by denying the reality of the world. Matter to them was but a complex of ideas. This was the inheritance of Kant.

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[17] "Letters on the Question, Whether the Essence of Body Consists in Extension" 1691. Translated by G. M. Duncan: *The Philosophical Works of Leibniz*, 2nd ed. New Haven, 1908, p. 42.

[18] *Ibid.*, p. 44.

[19] To those whom an apology is due for the use of this term, we refer to the transcendental idealistic point of view.

In his book on the critique of reason, Kant turns away from the objective space and time of Newton and argues that space and time do not exist of and by themselves or as properties of matter, but they are what he calls *a priori* concepts of the human mind. Without the human mind there would be neither space nor time just as there would not be sound without a human ear to hear it. In other words he wishes to make the whole question a subjective one and would deny that we could ever arrive at any concept of either that could not be derived from pure reason alone. Time and space in his philosophy lose their objective significance; they are not entities which we can study by means of the instruments of science. The geometry of space resides in the mind and one can make any set of postulates with regard to it that he may desire, provided they satisfy the canons of logic and are consistent. There can never be any question as to whether the geometry of Euclid, or that of Riemann or of Lobachevski is the true geometry of space, because the mind through pure reason can not differentiate between them.

One is reminded when he reflects upon the consequences of this full subjective philosophy of the conversation between Alice and the twin brothers Tweedledee and Tweedledum. Tweedledee exclaimed:

". . . if he [the Red King] left off dreaming about you [Alice] where do you suppose you'd be?"

"Where I am now, of course," said Alice.

"Not you!" Tweedledee retorted contemptuously. "You'd be nowhere. Why, you're only a sort of thing in his dream!"

"If that there King was to wake," added Tweedledum, "you'd go out—bang!—just like a candle!"

"I shouldn't!" Alice exclaimed indignantly. "Besides, if I'm only a sort of thing in his dream, what are *you*, I should like to know?"

"Ditto," said Tweedledum . . . ; "you know very well that you're not real."

"I *am* real!" said Alice, and began to cry.

Kant's metaphysics with regard to space and time are summarized below in his own words and can be compared with the views previously stated by Newton:[20]

"1. Space does not represent any property of objects as things in themselves, nor does it represent them in their relations to each other; in other words, space does not represent to us any determination of objects such as attaches to the objects themselves and would remain, even though all subjective conditions of the intuition were abstracted.

"2. Space is nothing else than the form of all phenomena of the external sense, that is, the subjective condition of the sensibility, under which alone external intuition is possible.

"3. Time is not an empirical conception, that is, one derived from experience.

"4. Time is necessary representation, lying at the foundation of all our intuitions.

"5. Time has only one dimension; different times are not coexistent but successive.

"6. Time is not a discursive or, as it is called, general conception, but a pure form of the sensuous intuition.

"7. The infinity of time signifies nothing more than that every determined quantity of time is possible only through limitations of one time lying at the foundation."

To put this in simpler language, the mind of man is so constructed that he is born with an inherent knowledge or feeling for three dimensional space and a consciousness of the sequence of events. He is thus able without confusion to locate objects about him. If he is to have a cup of tea at half past three at the house of a friend he will not find, as sometimes happens in dreams, that the house and the hour keep changing as he tries to find them.

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[20] Sections 1 and 2 of Part I.

### 9. *The Antinomies of Reason.*

Before we can hope to appreciate the real revolution which modern discoveries in physics have effected and to appraise their philosophical importance it is necessary for us to understand the meaning of the word *antinomy* as it appears in Kant's philosophy.

After he had defined the catagories of pure reason, Kant turned to the interesting problem of why certain concepts always implied their opposites and why attempts to ascertain *the truth* of one or the other always failed. He was led to the conclusion that there exist fundamental antinomies, or contradictions, in reason, which appear whenever we try to pass from human experience to the absolute. Of these antinomies the following four occupy a central place in the Kantian dialectic:[21]

I. *Thesis*: The World has a beginning in time, and is also limited as regards space.

*Antithesis*: The World has no beginning and no limits in space; it is infinite regards both time and space.

II. *Thesis*: Every composite substance in the world is made up of simpler parts and nothing anywhere exists save the simple or what is composed of the simple.

*Antithesis*: No composite thing in the world is made up of simple parts, and there nowhere exists in the world anything simple.

III. *Thesis*: Causality in accordance with laws of nature is not the only causality from which the appearances of the world can one and all be derived. To explain these appearances it is necessary to assume that there is also another causality, that of freedom.

*Antithesis*: There is no freedom; everything in the world takes place solely in accordance with laws of nature.

IV. *Thesis*: There belongs to the world, either as its part or as its cause, a being that is absolutely necessary.

*Antithesis*: An absolutely necessary being nowhere exists in the world, nor does it exist outside the world as its cause.

It is rather an amazing contemplation to reflect that there exist propositions about the absolute in nature which, though mutually contradictory, may both be assumed as postulates from the applications of which we may expect no contradictions. The present situation in physics, as will be revealed in the ensuing pages, presents other antinomies from laboratory experiments which have the same contradictory nature. The tenets of the modern theory of wave mechanics, for example, assume that light is for one purpose a corpuscular phenomenon and for another a continuous undulation.

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[21] *Critique of Pure Reason.*. Transcendental Dialectic, book 2, chap. 2, sec. 2.

The antinomies of Kant struck fire in the thought of Georg W. F. Hegel (1770-1831) and led him to his difficult philosophy of the Absolute Idea. This system may be characterized by the term *dialectic* by which Hegel meant the process of exhibiting the incomplete character of any concept except the all inclusive concept of the Absolute Idea. Because of the fundamental antinomies in reason any concept is incomplete, but through its implications we may establish a kind of successive approximation through an infinite sequence of stages to the forever unattainable absolute.

### 10. *Psychological Duration.*

This subjective concept of time is sometimes confused with *duration*, which Henri Bergson (1859-1941) picturesequely describes as "the continuous progress of the past that gnaws into the future and which swells as it advances".[22] The philosophical picture which we have tried so briefly to trace would scarcely be complete without pushing the subjective view to the limit. It is true that here we must leave philosophy and pass into psychology, where the subjective and the objective are so strangely mingled that one cannot trace the threads; but the peculiar mystery of matter immersed in a three dimensional continuum of space and a one dimensional continuum of time which is perceived and studied by a living scientist most certainly has a subjective as well as an objective side. One can scarcely be accused of mysticism in any approach that he might make to this puzzling problem with which the great speculative genius of the past has so often struggled. Therefore, without further apology, let us consider what we might term the psychological aspect of time and space.

The matter has been vividly put by Thomas De Quincey (1785-1859) in his *Confessions of an English Opium Eater* in which he describes some of the bizarre dreams that came to him after prolonged use of the terrifying drug. These words are poetry rather than science.

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[22] *L'evolution creatrice*, 6th. ed. Paris, 1910, p. 5; English trans. by A. Mitchell, New York, 1911, p. 4.

"The sense of space, and in the end the sense of time, were both powerfully affected. Building, landscapes, etc. were exhibited in proportions so vast as the bodily eye is not fitted to receive. Space swelled, and was amplified to an extent of unutterable infinity. This, however, did not disturb me so much as the vast expansion of time. I sometimes seemed to have lived for seventy or one hundred years in one night; nay, sometimes had feelings representative of a millennium, passed in that time, or, however, of a duration far beyond the limits of any human experience."

Again in the sequel: *Suspiria de Profundis* we have a vivid picture of subjective time:

"In the Opium Confessions I touched a little upon the extraordinary power connected with opium (after long use) of amplifying the dimensions of time. Space, also, it amplifies by degrees that are sometimes terrific. But time it is upon which the exalting and multiplying power of opium chiefly spends its operation. Time becomes infinitely elastic, stretching out to such immeasurable and vanishing termini, that it seems ridiculous to compute the sense of it, on waking, by expressions commensurate to human life. As in starry fields one computes by diameters of the earth's orbit or of Jupiter's, so, in valuing the virtual time lived during some dreams, the measurement by generations is ridiculous—by millennia is ridiculous; by aeons, I should say, if aeons were more determinate, would be also ridiculous. On this single occasion, however, in my life, the very inverse phenomenon occurred. . . . Instead of a short interval expanding into a vast one, upon this occasion a long one had contracted into a minute."

Are these but dreams and have they nothing to do with reality? This subjective character of existence as it relates to man in his environment has, of course, been the subject of extensive study. It is at the basis of the Bergsonian philosophy. William James (1842-1910), whose vision laid the foundation for a rational insight into what was personal and what was nature, considers psychological time in various places in his *Principles of Psychology*, from which the following statement is quoted:[23]

"Thus it appears indubitable that all space relations except those of magnitude are nothing more or less than pure sensational objects.

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[23] Vol. 2, 1890, pp. 151 and 154.

But magnitude appears to outstep this narrow sphere. . . . Suppose no feeling but that of a single point ever to be awakened. Could there possibly be the feeling of any special whereness or thereness? Certainly not. Only when a second point is felt to arise can the first one acquire a determination of up, down, right or left, and these determinations are all relative to that second point."

In another place he calls attention to the subjective character of time by citing the phenomenon that time *appears* to flow so much more rapidly in old age than in youth.[24] In fact this may be roughly stated by the formula that a year at age $x$ appears to represent approximately $1/x$ of one's life. For example, in childhood a year seems like one tenth of one's life; at 50 a year in passing appears but as one fiftieth of the whole span.

Lamartine, the mystic French poet, has expressed this well in "Le Lac," a poem which catches the spirit of what we have called psychological duration:

"O time suspend your flight, and you, glad hours,
    Suspend your rapid course,
Let us delight in swiftly passing joys;
    These days without remorse.

Enough of wretched ones implore thy flight,
    Speed on the passing suns,
Take with their days their cares and weary pain,
    Forget the happy ones.

But I demand in vain some moments more,
    The time slips by and flees;
I say to night: Yet tarry here! and dawn
    Is glowing through the trees.

Love then, love then, and through the rushing hours
    Enjoy the present day;
Man has no port; time has no shore or bound;
    It goes; we pass away."

### 11. *Poincaré and the Foundations of Geometry.*

We are now upon the threshold of the modern world and we might expect the philosophy of those who are the heirs of Newton and Kant and the immediate predecessors of Einstein to indicate real progress in exact thinking about the concepts of time and space. In order to exhibit the best of pre-Einsteinian thought we can profitably examine the ideas of two, a mathematician and a physicist, whose thought has played a conspicuous part in the new science. The men to whom we refer are Henri Poincaré (1854-1912) and Ernst Mach (1838-1916). Henri Poincaré, was mathematician, physicist, astronomer, and philosopher. During the course of his thirty-three years of active scientific work he produced more than 1,500 memoirs in addition to numerous books. For the fifteen years before his death he was by common consent the dominant figure in those scientific groups with which he was associated. His mind ranged over most of the speculative problems of these four branches of science and his philosophical ideas have been condensed in a series of essays first published under three heads: *Science and Hypothesis*, *The Value of Science*, and *Science and Method*, and later collected in one volume called *The Foundations of Science.* These essays are particularly noted for their brilliant style, and many quotations from them have found their way into scientific literature.

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[24] *Principles*, vol. 1, 1890, p. 625.

Because of the scope of the philosophy of Poincaré it is difficult, if not impossible, to condense it into a single system. His aim is to give careful scrutiny to the foundations of the physical sciences, particularly where they are mathematical in character. He focuses his attention upon the concepts of space and time as they appear in geometry and in the equations of dynamics; he keeps forever before him the mystery of probability "so strongly does this vague instinct which lets us discern probability defy analysis."[25] What, he asks, are the basic concepts of mathematics and wherein is the secret of their application to the world of matter?

It would be impossible in the scope of a few paragraphs to systematize Poincaré's philosophy, since it is so interwoven with the specific phenomena of physics; this is the main reason, perhaps, why his ideas have made little progress among laymen, who would find many of his observations without meaning because of their ignorance of the physical situations which occasion them. Hence his speculations will be introduced from time to time in this book at those places where the special phenomenon is being discussed. His essays form a volume to be read slowly at intervals of time. It is a book to which one may turn for moments of meditation.

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[25] *Foundations of Science*, p. 30.

Perhaps the keynote of Poincaré's attitude toward science is contained in the following statement:[26]

"The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living. Of course I do not here speak of that beauty which strikes the senses, the beauty of qualities and of appearance; not that I undervalue such beauty, far from it, but it has nothing to do with science; I mean that profounder beauty which comes from the harmonious order of the parts and which a pure intelligence can grasp. This it is which gives body, a structure so to speak, to the iridescent appearances which flatter our senses and without this support the beauty of these fugitive dreams would be only imperfect, because it would be vague and always fleeting. On the contrary, intellectual beauty is sufficient unto itself, and it is for its sake, more perhaps than for the future good of humanity, that the scientist devotes himself to long and difficult labors."

His approach to the problem of space is through the mathematical concept of geometry. Geometrical space to Poincaré has five fundamental properties: "(1) It is continuous; (2) it is infinite; (3) it has three dimensions; (4) it is homogeneous, that is to say, all its points are identical one with another; (5) it is isotropic; that is to say, all the straight lines which pass through the same point are identical one with another."[27] Geometrical space is to be compared with *perceptual space*, "the frame of our representations and our sensations."

In this geometrical space we have set up three geometries by purely logical processes. These geometries differ from one another on the basis of the postulate with regard to the number of straight lines that can be drawn through a single point parallel to a given straight line.

Since this idea, in spite of its long history, is still novel to many people, it is perhaps well to make a brief digression for the purpose of explaining it. To Euclid, "parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction." One is immediately struck by the fact that this definition is essentially negative since parallel lines are characterized by what they do not do.

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[26] *Foundations of Science*, pp. 366-367.

[27] *Foundations of Science*, pp. 66-67.

Euclid in order to proceed with his deductions soon found it necessary to introduce into his geometry a postulate regarding the nature of parallel lines. This postulate can be stated in various ways, but for our purpose it will be convenient to express it in the affirmation that through a given point there is one and only one parallel to a given straight line.

The real nature of this postulate did not become apparent until a long succession of geometers had tried in vain to prove it. Then the idea seemed to occur simultaneously in various parts of the mathematical world, to Carl Friedrich Gauss (1777-1855) in Germany, to Nicholaus Lobachevski (1793-1856) in Russia, and to Wolfgang Bolyai (1775-1856) in Transylvania, that the postulate of Euclid could not be proved, in other words, that a systematic geometry could be constructed without it.

The new Non-Euclidean geometry which resulted, began with the definition: "In relation to a line, all the lines of a plane can be divided into *intersecting and non-intersecting* lines. The latter will be called parallel, if in the pencil of lines proceeding from a point they form the limit between the two classes; or in other words, the boundary between the one and the other." Hence from this positive, in contrast to Euclid's negative, definition of parallel lines we derive the postulate that more than one line can be drawn through a point so as to be parallel to another line. This definition also introduced the idea of an *angle of parallelism*, that is to say, the angle between the parallel and the perpendicular from the given line to a point, *P*, on the parallel. This angle of parallelism changes with the distance of the point *P* from the given line.

Later the great German mathematician Georg F. B. Riemann (1826-1866) pointed out that a third possibility existed and that another geometry, now commonly referred to as Riemannian, could be constructed on the postulate that no line could be drawn through the given point parallel to a given line.

One striking feature of these three geometries is exhibited in the theorem which concerns the sum of the angles in a triangle. According to Euclid, the sum of the angles in any triangle equals two right angles; with Lobachevski this is no longer true and the sum of the angles of a triangle is always less than two right angles; Riemann on the contrary shows that the sum is always greater than two right angles.

Bewildered by this strange situation, mathematicians asked which was *the true* geometry. An answer was sought for by the astronomers. If Lobachevski is correct then the parallax of every star,[28] no matter how far away it may be, is always positive; if Riemann's geometry is the geometry of space then the parallaxes tend to become negative with distance; if Euclid has defined the true behaviour of parallels, then the parallaxes are all positive but have the limit zero as the distances of the measured stars increase. But parallax is a very difficult thing to measure because of its extreme smallness and the numerous errors of observation that tend to mask it. Some negative parallaxes have been found; the largest positive parallax is 0.75"; most of them are zero. No answer seems possible from this source until instruments of greater precision are invented.

But has the question any meaning? In a celebrated passage Poincaré makes the following statement.[29]

"The *axioms of geometry therefore are neither synthetic a priori judgments nor experimental facts.*

"They are *conventions*; our choice among all possible conventions is *guided* by experimental facts; but it remains *free* and is limited only by the necessity of avoiding all contradiction. Thus it is that the postulates can remain *rigorously* true even though the experimental laws which have determined their adoption are only approximative.

"In other words, *the axioms of geometry* (I do not speak of those of arithmetic) *are merely disguised definitions.*

"Then what are we to think of that question: Is the Euclidean geometry true?

"It has no meaning.

"As well ask whether the metric system is true and the old measures false; whether Cartesian coordinates are true and polar coordinates false. One geometry cannot be more true than another; it can only be *more convenient.*"

In considering the concept of time Poincaré fixes his attention upon the idea of simultaneity. "When we say that two conscious facts are simultaneous, we mean that they profoundly interpenetrate, so that analysis cannot separate them without mutilating them."[30] He considers time in connection with the individual who is studying phenomena and asks the questions: "Can we transform psychological time, which is qualitative, into a quantitative time? Can we reduce to one and the same measure facts which transpire in different worlds?"

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[28] For the definition of parallax see Section 5, Chapter 4.

[29] *Foundations of Science*, p. 65.

His answer to the first question is put in the following words: [31]

" . . . *We have not a direct intuition of the equality of two intervals of time.* The persons who believe they possess this intuition are dupes of an illusion. When I say, from noon to one the same time passes as from two to three, what meaning has this affirmation?

"The last reflection shows that by itself it has none at all. It will only have that which I choose to give it, by a definition which will certainly possess a certain degree of arbitrariness. Psychologists could have done without this definition; physicists and astronomers could not. . . ."

As to the definition of simultaneity of distant events, Poincaré affirms that this is wholly a matter of convenience.[32]

"This simultaneity of two events, or the order of their succession, the equality of two durations, are to be so defined that the enunciation of the natural laws may be as simple as possible. In other words, all these rules, all these definitions are only the fruit of an unconscious opportunism."

He cites, for example, the appearance of the new star in 1572, a description of which is vividly given in the works of Tycho Brahe. That tremendous cosmic upheaval took place in a very distant part of the sky, perhaps two hundred light years from the earth. The conflagration of which Tycho had just become aware was thus, in point of fact, an event that took place before the discovery of America; but is there any meaning to be attached to the statement that the phenomenon occurred after the "formation of the visual image of the isle of Espanola in the consciousness of Christopher Columbus"? The answer made by Poincaré is that this statement has meaning only as the result of a convention in accordance with the dictum stated above.

The laws of probability also played an important part in the thoughts of Poincaré and he has devoted many pages to his reflections upon this subject. "Has probability been defined? Can it even be defined? And if it cannot, how dare we reason about it?" Since these questions will be the subject of a subsequent chapter we shall be satisfied at this point merely to state them and shall postpone our consideration of their answers until a more systematic development of basic ideas is possible.

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[30] *Foundations of Science*, p. 223. [31] *Foundations of Science*, p. 224.

[32] *Foundations of Science*, p. 224.

Can we estimate the importance of the philosophy of Poincaré? It is too early, perhaps, to reach a just appraisement, and to trace the force of his arguments through so many phases of speculation. The least that can be said is that he focused the attention of his contemporaries upon the speculative relationship between abstract ideas and the specialized collections which comprise the various branches of modern science.

### 12. *Ernst Mach—Relativity and Inertia.*

When Poincaré died in Paris on July 17th, 1912, the whole scientific world felt the shock of this great loss. His passing left the feeling that a light of truth had gone out. Eulogies were written in most of the languages of the world, and there was universal expression among astronomers, mathematicians, and physicists that the race would wait long before the appearance of another such universal genius.

Contrast this with the passing of Ernst Mach who died on February 19, 1916, near Munich. At that time there was little expression throughout scientific journals of the value of his contributions to knowledge. He died neglected as he had feared. But whose duty was it to appraise his works? Was he psychologist, philosopher, or physicist? To the psychologist he seemed an enigma; to the philosophers his points of view upon the problems of mechanics appeared to lack significance because the genius of an Einstein was required to appreciate their consequences and formulate them in such language that they could be tested by objective experimentation; with the physicist Mach himself indicated his complete schism in a communication directed to Max Planck, wherein he said:[33]

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[33] See Paul Carus: "Mach and his Work," *Monist*, Vol. 21 (1911), pp. 19-42, in particular p. 33. The present tendency seems to be to agree with Mach. We thus find A. E. Ruark and H. C. Urey beginning their treatise on *Atoms, Molecules and Quanta* (1930), with the remark: "Once, when the famous Boltzmann had concluded a lecture on atoms and molecules, the equally famous Mach arose and said, in effect, You do not know that molecules exist. Boltzmann replied, I *know* that there are molecules. Mach answered, You *do not*, and so the debate ended. Today, the most ardent lover of modern atomic theory would side with Mach. . . "

"We can see that the physicists are on the surest road to becoming a church, and are already appropriating all the customary means to this end. To this I simply answer: If belief in the reality of atoms is so essential to you I hereby abandon the physicist manner of thought, I will be no regular physicist. I will renounce all scientific recognition; in short the communion of the faithful I will decline with best thanks. For dearer to me is freedom of thought."

Ernst Mach was born in 1838 at Turas, Moravia, and was educated at Vienna. He was professor of mathematics at Graz in 1864 and in 1867 became professor of physics at Prague where he was rector magnificus in 1879-80. He held the chair of philosophy at the University of Vienna (1895-1901) when he retired. His early youth and first teaching experience were spent in comparative poverty, against which he struggled with rare patience and resolution.

The essence of the philosophy of Mach is to be found in his acceptance of sensations as the essential realities. These he calls the elements of the world and the quotation already cited shows how strongly he differed from those who believed that atoms are the realities. The book upon which his greatest reputation rests is *The Science of Mechanics*, published in 1883, which is a critical and historical account of the development of this fundamental subject.

The greatness of Mach's philosophy now appears to be his anticipation of the theory of relativity. He strongly differed with Newton's concept of an absolute time and an absolute space, which he characterized as mediaeval. In an attempt to replace it with a better point of view he found himself called upon to consider the mutual relationship of things, not in a small isolated system, but in the actual complicated and vast system of universal nature. The following statements are now recognized as legitimate anticipations of the theory of relativity:[34]

"No one is competent to predicate the things about absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be produced in experience. All our principles of mechanics are, as we have shown in detail, experimental knowledge concerning the relative positions and motions of bodies. Even in the provinces in which they are now recognized as valid, they could not, and were not, admitted without previously being subjected to experimental tests. No one is warranted in extending these principles beyond the boundaries of experience. In fact, such an extension is meaningless, as no one possesses the requisite knowledge to make use of it.

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[34] *The Science of Mechanics.* Trans. by T. J. McCormack, Chicago, 2nd. ed., 1902, pp. 229-230; 5th ed., 1942, pp. 280-282.

"Let us look at the matter in detail. When we say that a body *K* alters its direction and velocity solely through the influence of another body *K*, we have asserted a conception that it is impossible to come at unless other bodies *A*, *B*, *C*, . . . are present with reference to which the motion of the body *K* has been estimated. In reality, therefore, we are simply cognizant of a relation of the body *K* to *A*, *B*, *C*, . . . If now we suddenly neglect *A*, *B*, *C*, . . . and attempt to speak of the deportment of the body *K* in absolute space, we implicate ourselves in twofold error. In the first place we cannot know how *K* would act in the absence of *A*, *B*, *C*, . . . ; and in the second place, every means would be wanting of forming a judgment of the behavior of *K* and of putting to the test what we have predicated,—which latter therefore would be bereft of all scientific significance."

You have seen earlier in the chapter how Leibniz made the inertia of matter a property as fundamental to its definition as the property of extension. Newton in his *Principia* merely incorporated the objective fact of inertia but did not inquire into its cause. With Mach this speculation takes a leading role and he now asks the question: "Why is the inertia of matter?" Is it, as Leibniz believed, inherent in the nature of matter itself, or is there an outside cause? Can it be that inertia is a reflection of the existence of the other matter in the universe? To put the question picturesquely, suppose that we consider all the material in the universe except a single stone to be blotted out of existence. Would this stone now resist motion as it does in the actual world? Would it move in a straight line? Or is such a question meaningless without the presence of other matter to create a frame of reference? All matter, excepting the sun, the planets, their satellites, the asteroids, and a small amount of cosmic dust, is so tremendously remote from our little earth that the influence of its gravitational pull is too infinitesimal to be measured by the most refined of instruments. How, then, are the stars, the nebulae, the great dark masses of the sky connected, if at all, with the behaviour of bodies on our earth? To be sure they form a sort of statistical reference frame which we can consult by means of telescopes in order to exhibit our annual motion about the sun, our daily rotation and the drift of the solar system, but have they, perhaps, a more fundamental connection with the intrinsic properties of matter? Can we find in them the source of material inertia? As one swings a pail of water about his head, is the fact that the water clings to the pail and does not spill, a cosmic phenomenon whose explanation ultimately goes back to the total quantity of matter in the universe?

One can see that Mach was dealing with a great idea and that merely to have asked the question created a fruitful field for speculation. This point of view is a genuine advance over the rather sterile philosophy of Descartes, who merely saw in matter the one fundamental property of extension.

The following is a significant passage and is representative of the idea which Mach enlarges in a magnificent way, to his immortalization.[35] Surely Einstein had a rich inheritance.

"Let us now examine the point on which Newton, apparently with sound reasons, rests his distinction of absolute and relative motion. If the earth is affected with an *absolute* rotation about its axis, centrifugal forces are set up in the earth: it assumes an oblate form, the acceleration of gravity is diminished at the equator, the plane of Foucault's pendulum rotates, and so on. All these phenomena disappear if the earth is at rest and the other heavenly bodies are affected with absolute motion round it, such that the same *relative* rotation is produced. This is, indeed, the case, if we start *ab initio* from the idea of absolute space. But if we take our stand on the basis of facts, we shall find we have knowledge only of *relative* spaces and motions. *Relatively*, not considering the unknown and neglected medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally *correct*; only the latter is more simple and more *practical.* The universe is not *twice* given, with an earth at rest and an earth in motion; but only *once*, with its relative motions, alone determinable. It is, accordingly, not permitted us to say how things would be if the earth did not rotate. We may interpret the one case that is given us, in different ways. If, however, we so interpret it that we come into conflict with experience, our interpretation is simply wrong. The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise."

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[35] *The Science of Mechanics*, pp. 231-232.

This is a remarkable statement when we reflect that it was written prior to the historical papers of Einstein. Mach had surely gone far beyond the absolutism of Newton, the subjectivity of Kant, and had seen in the problem of the inertia of matter a connection between man and the cosmic structure. What rich speculation is awakened as we read: "When, accordingly, we say that a body preserves unchanged its direction and velocity *in space*, our assertion is nothing more or less than an abbreviated reference to *the entire universe*."[36] There are no isolated systems and no isolated experiments. In every action we have a direct connection with the remainder of the universe no matter whether its separate particles be miles or millions of light years from us. Certainly there exists in all philosophy no richer speculation than this, and the triumph of it is to be found in the pages of the special and general theories of relativity.

### 13. *The Intellectual Evolution.*

One who reads the history of science must be struck by the frequency of the coincidence of discovery. There is great emotional appeal in the following story, related in the biography of Lord Kelvin. Lord Kelvin, then William Thomson, at the beginning of his studies, had gone to Paris, fired with a devotion to the mathematical ideas of Joseph Fourier and George Green. The latter, some years before, had privately published his investigations in a work entitled "An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism," (1828) but its contents were little known in England and completely unknown in France.

One night about three weeks after the arrival of William Thomson, eager footsteps were heard before the door of his lodging and with a hasty knock Jacques Charles Sturm burst into the room. The visitor was in a state of high excitement and asked permission to see the manuscript of Green which Thomson had with him. Turning the pages rapidly, Sturm scanned the contents of the essay. As his eye fell upon the formulas with which Green anticipated his theorem on the equivalent distribution,[37] he suddenly jumped from his chair crying: "Ah, voilà mon affaire."

This is a kind of tragedy, but it is a tragedy that has been and will be enacted many times. Ideas are not the product of individuals but rather the product of movements, to which each individual has added his increment of thought. Logarithms were invented by Baron John Napier, say the text books, but they were certainly independently found by the Swiss, Joost Burgi. Newton discovered the law of gravitation, but very probably the law would soon have appeared in the writings of another. Certainly the idea of the inverse square law is to be found in the conjectures of Newton's contemporaries Robert Hooke, E. Halley, C. Wren, and C. Huygens. The unfortunate quarrel between the followers of Leibniz and those of Newton over the priority of the discovering of the calculus is too well known to need telling here. We have already commented upon the discovery of non-Euclidean geometry, and similar coincidences could be multiplied without end.[38]

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[36] *The Science of Mechanics*, p. 233.

[37] *Life of Lord Kelvin*, by Silvanus P. Thompson, 1910, Vol. 1, p. 119.

Ideas are the product of an intellectual evolution in which individuals are but incidents. In the dark tangle of modern quantum mechanics and the theory of radiation there is a bright truth to be discovered. We shall greatly reward, as we should, the individual who first has the vision, but the solution will really be the solution of the race. Struggle as he may, the individual can only see as far as the data of the instruments of his period and the intellectual vision of his predecessors permit. Newton himself admitted that he stood upon the shoulders of giants; it was his great individual glory that he was the first to interpret what he saw.

Swiftly and imperfectly we have traced the golden thread of a single thought from the magnificent Greeks to the beginning of the present age. Has the race made real progress in examining this philosophical problem of the nature of space and time and the relation of matter to its framework? Did Descartes make a contribution in his vacuum that was not empty? Did Leibniz hit upon a fundamental aspect of the problem in the attention which he gave to inertia? Has Newton helped us by systematizing the arguments which would show that space and time formed an eternal, unchanging, absolute framework of reality? What shall we say to the categorical imperative of Kant and the psychical duration of William James? Does Mach see the furthest of all when he insists that there is not a single isolated system in the universe, but that every part of matter is related to the cosmic structure by the effects of inertia and all phenomena are but relative?

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[38] For a list of similar coincidences see W. F. Ogburn; *Social Change*, New York, 1922, pp. 90-102.

# CHAPTER 3.

## IS THERE AN ETHER?

"Alice laughed. 'There's no use trying,' she said: 'one can't believe impossible things.'

"'I daresay you haven't had much practice,' said the Queen. 'When I was your age, I always did it for half-an-hour a day. Why, sometimes I've believed as many as six impossible things before breakfast . . . '."

Lewis Carroll in *Through the Looking-Glass.*

### 1. *Galileo and Inertia.*

In the last chapter it was shown that there were several ways of looking at space and time. Sir Isaac Newton advanced the theory that we were to regard these entities as absolute and unchangeable factors independent of matter for their existence. They are, perhaps, what is meant by the trite expression "eternal verities." The idea might be regarded as a sort of absolutism. Space and time in the philosophy of Newton together make up the unchanging framework of the world, a four-dimensional manifold into which the phenomena of nature fit and where they may be thought of as residing.

Immanuel Kant, perplexed by the attempts of others to give reality to space and time, and under the influence of the idealism of Berkeley and Hume, adopted the full subjective view. Time and space are the eternal verities, not objectively, but subjectively. They are *a priori* intuitions of the human mind. Destroy the mind and you have destroyed all.

Ernst Mach, reflecting upon the point of view of Leibniz that matter possesses inertia as well as extension, had a new revelation according to which time and space were aspects of a cosmic relativity. Space and time in his opinion are not entities with an independent existence, fixed and unchangeable, but are to be thought of as attributes of matter. Create but a single electron and you have started to manufacture space and time. Create two electrons and with them space and time appear, although their existence is limited to the neighborhood of the newly created matter. In other words space and time grow with the growth of matter, just as in a certain sense there is more red in two red apples than in one. With the creation of matter inertia appears and may be regarded as a phenomenon related to the totality of matter in the universe.

In this chapter we shall survey the history and the arguments relating to the creation of the meta-physical space-filling medium which we are accustomed to call the *ether,* or in more recent times, the *luminiferous ether.* To a certain extent this word has tended to disappear from modern scientific literature, but the phenomena which occasioned it remain the most fundamental ones in physics. These are the electro-magnetic radiations. Their relationship to matter, the nature of their energy, and their basic significance in the problem of the transmutation of the elements give to them a primary place in all modern speculations about the nature of the world. Hence the ether, which was created in the first place to provide a medium for the propagation of electro-magnetic waves, cannot be dismissed without a careful survey of its history.

We shall begin our discussion with a review of the meaning of the word *inertia.* This word appears not infrequently in common language and for most purposes may be defined as that property of matter which enables it to resist a change of motion. The meaning of this seems simple enough; it is in keeping with the nature of things to experience a resistance, different from friction, when one moves a marble across a table top.

But let us take our view from another vantage point and push our speculations to some entertaining conclusions. We ask you to regard a historical incident. A few years before the birth of Sir Isaac Newton, a famous Italian astronomer by the name of Galileo Galilei (1564-1642) found himself in serious difficulty with the clergy of his day. He had made the heretical statement that the earth and not the sun was in motion.

That was, alas, an age in which only a timid few held the view that one could learn anything from experiment. In fact it was not only difficult but dangerous to differ with authority provided the authority was sufficiently seasoned with years; to try to prove oneself correct by appealing to the evidence of phenomena was to flirt with death.

While Galileo sojourned at Padua in the year 1600 a public spectacle was held which was calculated to make anyone pause who too earnestly wanted to look about him and tell others what he saw. Filippo Giordano Bruno (1548-1600) was burned at the stake in order to expiate the crime of holding to the Copernican view that the earth and not the other heavenly bodies was in motion. Galileo had already experienced an unpleasant difficulty in his teaching experience at Pisa where he had dared to differ from

PLATE II

Galileo "with a sincere heart and unfeigned faith" denying his belief in the motion of the earth. From a painting by J. V. Robert Henry.

the Aristoteleans who argued, because the statement was found in the writings of the master, that a heavy weight always fell more rapidly than a light one. Galileo, not satisfied with this authority, offered to prove that Aristotle's statement was wrong and climbed the celebrated leaning tower with a heavy shot and a light one. The two were observed to fall together making one sound as they struck the ground, but this evidence was regarded with unfriendly eyes. It only succeeded in discrediting the young experimenter and making him an object of suspicion.

Galileo, however, was a man of courage and persisted in his dangerous pursuits. Among other things he invented an instrument by means of which distant objects could be brought to nearer view and with it he was able to discern the moons of Jupiter. Did the world welcome these new discoveries? Was not the imagination of his contemporaries stirred by this remarkable new information? It would be bewildering to us of this modern age to interpret the black suspicion of those days if there were not evidence today of the same distrust in spite of the many benefactions conferred by objective discovery. This attitude of man toward the discoveries of truth is a perplexing mystery.

How characteristic of the bigotry of the day is the attitude which inspired the following letter written by Galileo to Kepler:

"Oh, my dear Kepler, how I wish we could have one hearty laugh together! Here, at Padua, is the principal professor of philosophy whom I have repeatedly and urgently requested to look at the moons and planets through my glass, which he pertinaciously refuses to do. Why are you not here? What shouts of laughter we should have at this glorious folly! And to hear the professor of philosophy at Pisa laboring before the grand duke with logical arguments, as if with magical incantations, to charm the new planets out of the sky."[1]

We return to the main theme of our story which was the dangerous predicament in which Galileo found himself as the result of his heresy of statement that the earth moved. Bruno had dared to spread such malicious doctrine and had met death by public torture; Galileo, however, had gained some prestige among a few, whose imaginations had been fired by his discovery of the moons of Jupiter and the possibilities of the telescope, and some protection was to be expected from high authority. However he trod upon dangerous ground.

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[1] See Sir Oliver Lodge: *Pioneers of Science*, London, 1919, p. 106.

Unfortunately Galileo had little if any experimental evidence for his assertion. The size of the sun and the nature of the stars were practically unknown. It did not seem strange to the people of that day that the firmament with the sun, the planets, and all of the stars should revolve around the earth once in twenty-four hours. Our own pity of their ignorance is largely derived from the fact that we know the stars to be huge suns, larger than our own, which are at such immense distances from us that we have to use the distance that light travels in a year as a yard-stick to be able to talk about them. If anyone tried to tell us that the earth stood still and that the stars revolved around it, we could effectively answer him by computing the incredible velocity with which Polaris, the pole star, for example, would have to move in order to make the circuit of the earth in a single day. Since Polaris is of the order of 300 light-years from us, a simple calculation shows that it must attain a velocity nearly 700,000 times that of light itself.

Let us change the facts of history and let us imagine that Galileo performed the following experiment: that he had asked permission to suspend a heavy weight by a long cord from the dome of a cathedral and had set it in motion above a circle drawn on the floor. Imagine the picture, the dim light from the tinted windows falling upon the dark circle of the clergy and the heretic. And what would they have seen in that breathless hour? The pendulum, as though plucked by some cosmic hand, would have been seen to change gradually its plane of oscillation, and whereas it was swinging in a North and South direction at twelve o'clock, it would have been found to oscillate in a different plane at six. How could the accusers of Galileo have answered or explained away this strange phenomenon? As he arose from his knees after repenting the abjuration of his heresy would he then, as the apocryphal legend affirms he did, have had to mutter under his breath those famous words, "e pur si muove" (and yet it moves)? To his accusers Galileo might have said that the property of inertia keeps the weight swinging in its cosmic path, while the earth itself rotates under it.

It is amazing that this simple phenomenon was not observed until the middle of the nineteenth century. If weather maps had been available as we now know them it would have struck the attention of anyone who studied them that the great weather cyclones (or "lows") of the northern hemisphere always whirl in one direction, namely counterclockwise, while in the southern hemisphere the spiral motion is clockwise. The great currents of the ocean which so profoundly modify our weather are examples of the same phenomenon. One may wonder whether those who witnessed the sufferings of Bruno and those who heard poor, aged Galileo say: "with a sincere heart and unfeigned faith I abjure, curse, and detest the said errors and heresies," would have seen the implication of this tendency of natural currents.

### 2. *Foucault's Pendulum Experiment.*

And what is the explanation of the motion of the pendulum and the direction of the vortical motions of the earth? It is customary to say that these phenomena prove the rotation of the earth. The motion of the pendulum is not caused by any transcendental or mysterious force from without, but is due simply to the fact that the earth turns under the swinging weight. The latter is held in its plane by the property of inertia. And yet perhaps the explanation is not so simple since the experiment was performed for the first time in 1851 by Jean Foucault (1819-1868). Simple facts have not usually remained undiscovered for so many years.

Foucault performed his experiment in the Pantheon at Paris, from the dome of which he hung a heavy iron ball about a foot in diameter by a wire more than 200 feet in length. A circular rail about twelve feet in diameter was built on the floor of the Pantheon and a little ridge of sand constructed upon it in such a way that at each vibration the ball would record a mark to indicate its point of passage. The ball was then displaced from the vertical by a small thread and left for a few hours until it had come absolutely to rest. The thread was burned and the ball set in motion across the ridge of sand. And what did those who witnessed it behold? Slowly, hour after hour, the ball changed its plane of vibration. The floor of the Pantheon was in visible motion. The principle for which Bruno died and for which Galileo suffered was vindicated. The earth moved! Those who visit Washington may see a Foucault pendulum swinging in the building of the National Academy of Sciences.

And yet let us consider a simple question. With respect to what is the earth in motion? This is a legitimate question from the point of view of Mach, to whom all motion is relative. The point can be put more strikingly by making the assumption that the sun and all the stars and the nebulae and all the matter of the universe except the earth are completely destroyed. If this step seems a little difficult to take, let us suppose merely that the earth is far enough removed from all visible matter to be essentially in complete isolation; that is to say, let the sky be completely devoid of light from any source. Then, since we should have no outside point of reference in the solitudes of space, should we be able to say whether the earth revolved or not? Under these strange circumstances would the Foucault pendulum change its plane from hour to hour, and would the weather cyclones, if any existed, persist in revolving in a single direction? Now let but a single point of light break through our darkness from a single star on the edge of a huge, but very distant, universe. If this single point of light appeared to change its position in the sky from hour to hour could we hold this as evidence that we were in rotation?

There are several explanations that have been advanced to account for the fact, truly surprising when we review it thus speculatively, that we are able to detect the rotation of the earth without reference to outside bodies.

The first of these hypotheses is, strangely enough, a denial of the reality of the rotation of the earth. We may assume that the earth is actually non-rotating and that the stars really have the tremendous velocities which would result from such an assumption. A. S. Eddington, whose interpretation of the theory of relativity and whose theory of the source of stellar energies are monuments to the imagination of their author, comments upon this proposition in his delightful book, *Space, Time and Gravitation*[2] in the following words:

"If the earth is non-rotating, the stars must be going around it with terrific speed. May they not in virtue of their high velocities produce gravitationally a sensible field of force on the earth, which we recognize as the centrifugal field? This would be a genuine elimination of absolute rotation, attributing all effects indifferently to the rotation of the earth, the stars being at rest, or to the revolution of the stars, the earth being at rest; nothing matters except the relative rotations. I doubt whether anyone will persuade himself that the stars have anything to do with the phenomenon. We do not believe that if the heavenly bodies were all annihilated it would upset the gyrocompass. In any case, precise calculation shows that the centrifugal force could not be produced by the motion of the stars, so far as they are known."

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[2] *Space, Time and Gravitation,* 1921, p. 153.

A second hypothesis, which we shall consider later in connection with Einstein's theory of space-time, postulates a vast quantity of unobserved world-matter. Beyond the fixed stars and the spiral nebulae there may be quantities of matter which we have never beheld, the unseen influence of which penetrates to the laboratory of our little earth in the form of a modification of our customary ideas of space and time. This is obviously not the place to explore this mystical postulate.

A third alternative is that the earth actually has an absolute rotation independent of the other matter of the universe, and that this rotation takes place with respect to a great ocean of ether which pervades all space and permeates all matter. The ether we shall find to be the seat of the kinetic energy of the earth, which is very great. One does not realize what a tremendous storehouse of energy is involved in this speculation. Assuming an average specific gravity of 5.5 for the earth, we may calculate by a simple formula that the total kinetic energy of rotation of the earth is approximately $10^{30}$ foot-pounds. To put this statement into more understandable terms, this amount of energy would produce the total electrical output of the United States for about a trillion ($10^{12}$) years.

### 3. *A Discursion on Energy.*

Let us pause to ask a question about this energy. The general formula for kinetic energy seems to tell us that kinetic energy is not absolute but relative, since it can be calculated only by comparing the velocity of one body with another. Thus, in the above calculation, the angular velocity which appeared in the formula used was calculated from the time that elapses between one sunrise and another. Is it not possible, therefore, that this great store of energy is merely a mathematical fiction, and if the sun and stars were blotted from our vision by clouds, might it not cease to exist and in an instant that whole sum total of energy vanish like the ghost of Banquo? As Macbeth mused,

> "Can such things be,
> And overcome us like a summer's cloud,
> Without our special wonder?"

The Foucault pendulum seems to deny this possibility, but take away the concept of the ether and the energy explanation becomes one of great difficulty. Potential energy, from its definition, is the energy of position; kinetic energy is the energy derived from change of position. But the fundamental question is, position with regard to what?

Consider for a moment the properties of a piece of matter. A stone a few feet from the top of the table is certainly a different thing from a stone resting upon the table's surface. The first possesses an attribute which we call potential energy. If we let it drop, this potential energy at once transforms into kinetic energy which, by definition is equal to half of the product of the mass by the square of the velocity. The stone strikes the table and comes to rest. In a moment the kinetic energy is gone and with it the potential energy which it once possessed. We now know, though the knowledge became the general property of science as late as 1843 through the work of James P. Joule (1818-1889), that this useful energy has disappeared in heat and in the internal energy of the atoms which have been disturbed by collision with the table top.

In the foregoing statement of the history of the falling stone have we used only words? Did we get at the reality behind the exchanges of energy from the energy of position to the energy of heat? What was the source of the original energy? Where was it stored? Did it reside in the stone or in the medium surrounding the stone?

In this connection there is much to think about in a simple experiment first performed by Benjamin Franklin. A Leyden jar is merely a glass jar coated inside and out to within a short distance of the top with metal foil, the inner coat being connected with a conducting knob. This device possesses the valuable property of acting as a reservoir, in which a charge of electricity can be stored. Let us perform the experiment of first charging the jar and then removing from it all of the metal foil. Take the glass from one place to another, use it in other capacities, finally reconvert it into a Leyden jar again by replacing the metal foil, and connect the conducting knob with the outer coat. The resulting spark is evidence that the energy has not been dissipated by the removal of the foil. But where during the history of the glass jar was the energy of the electricity stored? Was it in an elastic strain set up within the glass material by the electrical energy, or did it reside in the surrounding medium?[3]

Another, but more elementary example, is furnished by the following experiment. Let a powerful steel spring be tightly coiled and bound by a piece of copper wire. Then, let the whole be immersed in a bath of hydrochloric acid. In a short time the steel spring will have been dissolved without uncoiling, however, since the copper is not attacked by the acid. Where, then, went the potential energy of the spring? Invoking the indestructibility of energy, one knows that it must have been absorbed by the fluid. This would be confirmed by a rise in temperature just large enough to account for the disappearance of the potential energy. In other words, this energy has reappeared in the kinetic energy of the molecules of the acid.

It is a fact worthy of much reflection that the total amount of kinetic and potential energy in an isolated system which does not store up internal energy, is always constant. An example of this is found in the journey of the earth around the sun. Why does the earth never slow down, but continue in its path year after year in remarkable opposition to those who deny the possibility of perpetual motion? This, we say, is because the sum of the kinetic and potential energies is a constant, and if we draw too near the sun, thus losing part of our potential energy, we immediately hasten our footsteps and draw away again. A pendulum at the bottom of its swing has converted its entire supply of potential energy into kinetic energy and would continue to oscillate indefinitely were not a certain amount of the total constantly dissipated in friction both with the air and at its point of support.

There is a remarkable theorem in the theory of electrical radiation which was discovered in 1884 by John Henry Poynting (1852-1914). This theorem had to do with the transfer of energy in an electro-magnetic field. Everyone is familiar in a general way with what happens when the terminals of a battery are connected with a motor or a heater or some other electrical device to which energy is to be conveyed. The electricity flows along the wire in a current and energy disappears from the battery to reappear at the motor.

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[3] This experiment has been repeated with a Leyden jar made out of paraffin and the electrical charge was found to be removed with the metal coverings. A similar result was obtained for glass which had been thoroughly dried, but the mystery of where the energy resides is by no means elucidated by this discovery. See C. L. Addenbrooke, *Philosophical Magazine*, Vol. 43, 6th series, 1922, pp. 489-493.

But how is this transport made? It had been the common custom before Poynting's time, to think that the energy was carried from the battery along the wire by the current itself much in the same way as the energy of a stream of water is conveyed through a pipe. But in the electro-magnetic theory of Clerk Maxwell, the storehouse and vehicle of energy is the medium surrounding the wire, and it was Poynting's achievement to show that *the energy does not flow along the wire but into it at right angles to the current.*[4]

In this connection we shall quote a significant passage from the great book of H. A. Lorentz (1853-1928), *The Theory of Electrons,* in which the father of relativity reflects upon the meaning to be attached to the phrase: "A flow of energy":[5]

". . . the flow of energy can, in my opinion, never have quite the same distinct meaning as a flow of material particles, which by our imagination at least, we can distinguish from each other and follow in their motion. It might even be questioned whether, in electromagnetic phenomena, the transfer of energy really takes place in the way indicated by Poynting's law, whether, for example, the heat developed in the wire of an incandescent lamp is really due to energy which it receives from the surrounding medium, as the theorem teaches us, and not to a flow of energy along the wire itself. In fact, all depends upon the hypothesis which we make concerning the internal forces in the system, and it may very well be, that a change in these hypotheses would materially alter our ideas about the path along which the energy is carried from one part of the system to another. It must be observed however that there is no longer room for any doubt, so soon as we admit that the phenomena going on in some part of the ether are *entirely* determined by the electric and magnetic force existing in that part. No one will deny that there is a flow of energy in a beam of light; therefore, if all depends on the electric and magnetic force, there must also be one near the surface of the wire carrying a current, because, here, as well as in the beam of light, the two forces exist at the same time and are perpendicular to each other."

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[4] See J. H. Poynting: "On the transfer of energy in the electromagnetic field." *Trans. of the London Phil. Soc.* Vol. 175, 1884, pp. 343-361. See also H. A. Lorentz: *Theory of Electrons*, 1909, p. 23.

[5] P. 25.

### *4. The "Elastic Solid" Ether.*

The mystery of energy and the mystery of inertia are but phases of a greater problem: Is there an ether? The fundamental property of matter is extension, said Descartes, and this extension is characteristic also of a vacuum which is thus not to be regarded as entirely empty. But according to Leibniz, Descartes is not wholly right, because matter also possesses the inherent property of inertia. Were these savants, perhaps, after all, talking about aspects of the same thing, and is the problem of inertia the problem of the ether?

Is there an ether and if so, how can we define it? When we are thus required to represent an idea by a word and to reduce some profound and perplexing experiences of the race to language, we may well remind ourselves of the dictum of Humpty Dumpty who stated: "When I use a word it means just what I choose it to mean—neither more nor less."

What meaning shall we give to the word *ether* that will remove it from the realm of metaphysical speculation and introduce it into the world of real experience? It is obvious that one rational way to proceed is to set down a few postulates and then to subject them to the test of experimental evidence.

We shall begin with the concepts of the so-called "elastic solid" theory which was thoroughly explored in a long series of brilliant papers by the physicists of the nineteenth century. This intricate subject can be reviewed best, perhaps, by setting forth a few tentative postulates and investigating their consequences and contradictions.

We shall begin by making the following assumption.

*1. The ether is an isotropic, elastic medium of unlimited extension.*

By this proposition we commit ourselves to an analogy with ordinary matter. The ether is to be regarded as an elastic substance which differs from the material of sensuous experience only in degree and is possessed of the two fundamental properties of elasticity and density. The word *isotropic* very much simplifies the mathematical problem, because by it we assume that the ether is equally elastic in all directions. What this means mathematically is that instead of having to deal with the twenty-one coefficients which appear in the general theory of elastic bodies we shall be concerned with only two. The first of these, the *modulus of compression,* is a measure of the elastic behaviour of the material under tension and compression; the second, the *coefficient of rigidity*, describes the elastic properties under shearing forces which cause distortion. The material is then defined, as far as its ability to transmit energy by elastic vibrations is concerned, as soon as we know its density in addition to the two constants just described. The entire history of the theory of the "elastic solid" ether is an account of the efforts of mathematical physicists to adjust these three constants in a set of equations in such a way that all the phenomena of light propagation, its reflections and refractions, both in empty space and in crystalline matter, could be systematically accounted for.

We proceed to the second postulate which may be stated as follows:

*2. The constants measuring elasticity, rigidity, and density must be so chosen that no longitudinal vibrations will take place in the ether.*

The great quest started with a hunt for an equation. In what way may the vibrations of an isotropic medium be described? It was the good fortune of Claud Navier (1785-1836) first to formulate the correct equation governing the displacement of elastic particles, although in his work only the constants of rigidity and density appear.[6] His researches happily fell into the hands of the great analyist Augustine Louis Cauchy (1789-1857), who added to the equation of Navier the modulus of compression and put before the world the problem of adjusting these constants in such a way that all the phenomena of light might be properly described.[7]

Let us first ask ourselves the question: Can the ether be regarded as a very tenous gas, resembling air, and are the waves in it similar to the waves of sound? In an ether of this sort as was shown by the mathematician Simeon Denis Poisson (1781-1840) the vibrations would be both longitudinal and transverse in character, that is to say the energy would be transported both by expansions and compressions in the ether, and also by deformations perpendicular to the direction of motion.[8] Each wave would have its own velocity of propagation, and if the rigidity and density were small, the important wave would be the longitudinal one.

That the luminiferous ether cannot possibly be of this character is proved by a number of experiments which show that light vibrations are not longitudinal in character at all, but consist wholly of transverse motions of the medium. This transverse motion of light is beautifully illustrated by a very simple experiment performed with two crystals of tourmaline (see Fig. 1). Let a beam of light fall upon a plate of tourmaline which has been cut parallel to the axis of the crystal and examine the structure of the light which has been transmitted. In order to do this, place a second plate of tourmaline in front of the transmitted beam with its axis parallel to the first and then gradually turn it through an angle of ninety degrees. As the second plate is rotated the light gradually fades until at the maximum position when the two crystals are at right angles to one another it has been completely extinguished. The phenomenom is explained by saying that the vibrations which make up the beam of light are transverse and not longitudinal, for such an experiment would not affect vibrations of the latter sort. The beam of light that has passed through the tourmaline and thus acquired a two-sidedness is said to be *plane-polarized.* The phenomenon of polarization has been the greatest foe to the elastic solid theory.

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[6] *Mémoire de l'Académie de France*, Vol. 7, 1827, p. 375.

[7] *Exerçices de mathématiques*, Vol. 3, 1828. *Oeuvres*, Vol. 8, ser. 2, pp. 195-226.

[8] *Mémoire de l'Académie de France*, Vol. 8, 1828, p. 623.

At the very outset, therefore we see that we must choose the three constants at our disposal in such a way that longitudinal vibrations are excluded from the ether. To the skillful analyst, of course, this would not offer great difficulty, however much the method might displease the practical physicist, who thinks that he must always deal with reality in the form of mechanical models. Cauchy calmly settled the difficulty of longitudinal waves by

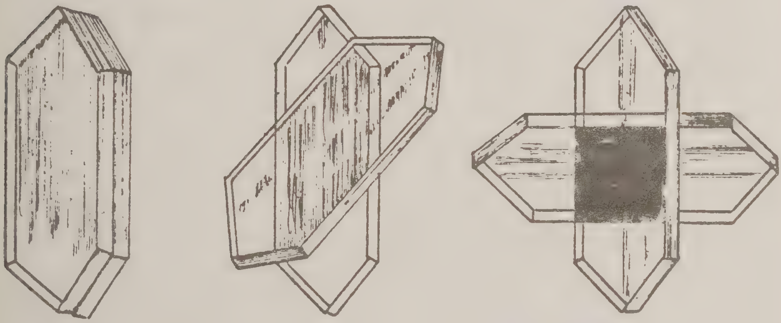

FIG. 1

Tourmaline crystals illustrating the polarization of light.

equating to zero the expression in his equation which gave rise to the longitudinal disturbance. To be sure the modulus of compression became a negative number, but in this assumption the *contractile* or *labile* theory of the ether was created. At first it seemed that the possibility of inventing a mechanical model was jeopardized by this assumption, but Lord Kelvin, who devoted considerable attention to this ether, was able to overcome most if not all of the theory in so far as it pertained to propagation through a uniform continuous medium.

A second possibility was suggested by James MacCullah (1809-1847) in a paper presented to the Royal Irish Academy in 1839.[9] This paper presented the possibility of an ether in which the potential energy depended only upon the rotation of the volume elements of the medium, and not, as in the case of ordinary elastic bodies, upon the change of the size and shape of the elements, that is to say, upon their compression and distortion. This idea was, in fact, a truly notable contribution, but its significance did not become evident until G. F. Fitz Gerald (1851-1901) called attention to it, nearly forty years later when Maxwell's theory had superseded the older elastic solid concepts. The fact is that MacCullah actually hit upon the truth, that is, in so far as Maxwell's ideas may be regarded as the truth, but he was unable to give any plausibility to an ether in which the coefficients of density and rigidity no longer had their ordinary dynamical significance.

It might be of interest at this point to mention calculations made by Lord Kelvin upon the value of the density of the ether. Lord Kelvin published his speculations in 1854 under the poetical title: "Note on the possible density of the luminiferous medium and on the mechanical value of a cubic mile of sunlight."[10] His conclusion was that the ether in a cubic mile at the surface of the earth contained not less than the billionth of a pound of matter. This statement, however, does not carry a high probability of belief because it cannot be verified experimentally. It rests, in fact upon the purely tentative estimate that the ratio of the velocity of vibration of the ether to the velocity of light does not exceed one-fiftieth. Making the further speculative assumption that the ratio of the amplitude of the energy in a ray of light to the wave length of the light is one one-hundredth, J. Larmour calculates the elastic modulus to be many times smaller than that of glass.[11] We might thus conclude that the ether is a very tenuous material with slight elasticity.

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[9] *Trans. Royal Irish Academy*, Vol. 21. *Collected Works*, p. 145.

[10] *Trans. of Royal Soc. of Edinburgh,* Vol. 21, pp. 57-61; *Comptes Rendus*, Vol. 39, pp. 529-534; *Phil. Mag.*, Vol. 9, pp. 36-40; *Collected papers*, Vol. 2, art. 67, pp. 28-33.

In another instance Lord Kelvin regarded the ether as a highly rarefied, though very rigid entity, and discussed the question of how the planets can then move through it without their motion being visibly affected. His answer was to call attention to the fact that Trinidad pitch is a brittle solid and yet is permeable, although slowly, to a solid body placed upon it. Thus he assumed that the ether may resemble this model and its rate of permeability to matter be merely greater than that of pitch.

"Luminiferous ether must be a body of almost extreme simplicity. It may perhaps be soft. We might imagine it to be a body whose ultimate property is to be incompressible; to have a definite rigidity for vibrations in times less than a certain limit, and yet to have the absolute yielding character that we recognize in wax-like bodies when the force is continued for a sufficient time. It seems to me that we must know a great deal more of the luminiferous ether than we do. But instead of beginning with saying that we know nothing about it, I say that we know more about it than we do about air or water, glass or iron—it is far simpler; there is less to know. That is to say, the natural history of the luminiferous ether is an infinitely simpler subject than the natural history of any other body."[12]

By these and other assumptions the concept of an elastic solid ether can be made to fit experimental evidence, but the troubles grow when we try to press the matter further by exploring the optical properties of solid substances in terms of the density and rigidity concepts. We come to another postulate, upon which lie the wrecks of many equations.

*3. Within the interior of solid media, the density of the ether varies but the rigidity remains constant.*

The battle-ground of the elastic etherists has been the behaviour of light in crystalline solids. These problems center around the assumptions that have to be made to account for certain technical polarization phenomena, for partial reflection, and for the peculiar property of *double refraction* possessed by all crystallized minerals except those whose fundamental form is the cube. This last mentioned phenomenon was discovered by Erasmus Batholinus (1625-1698) a Danish philosopher, about the year 1669.[13] While experimenting with a crystal of Iceland spar he made the interesting observation that a beam of light incident upon the crystal was broken up into two pencils, one of which obeyed the ordinary laws of refraction and the other a new law, for which no theory had been devised.

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[11] *Encyclopaedia Britannica*, Article on *Aether*, 11th ed., p. 292.

[12] *Baltimore Lectures*, page 12, 1884.

These subjects, unfortunately, are too technical to be reviewed here in detail. It will suffice to say that, while the assumption of a uniform rigidity in a labile ether of crystalline substances, combined with the hypothesis of an aelotropic density, seems to be adequate to explain the phenomena of partial reflections, it is wholly unable to account for double refraction. In fact, it seems as though the contrary assumption would have to be made, in which the density remains fixed but the rigidity varies for different directions.

We shall not pass judgment upon the elastic theory of the ether, but its present status seems to be that of a pageant of historical interest only. It may be that, in the revolution of physical ideas which we are now experiencing, we shall once more return to an ether to which substantial reality by analogy with elastic matter is to be given, but that hope today seems a very remote one.

### 5. *Maxwell's Ether and the Electro-Magnetic Theory of Light.*

We turn next to the ether of James Clerk Maxwell whose classical treatise on the theory of electricity and magnetism, published in 1871, clothed Faraday's tubes and fluxes of force in the garment of differential equations and led directly to the discovery of wireless communication. Maxwell was born near Edinburgh and attended both Edinburgh University and Trinity College, Cambridge, from which he came out second wrangler, the first place being held by E. J. Routh, the author of numerous works on dynamics. Maxwell then lectured at Cambridge, and later filled appointments as professor both at Aberdeen and King's College, London. He retired to private life in 1865, but in 1871 became professor of physics at Cambridge. During his short life of forty-eight years he made contributions of the very highest importance both in the kinetic theory of heat and the theory of electro-magnetism. He is, perhaps, the giant among the giants of that great age of physics.

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[13] *Experimenta cristalli Islandici disdiaelastici*, 1669.

The postulates of Maxwell's ether will now be stated:

1. *The ether is space-filling, that is to say, it is an entity of unlimited extension.*

2. *The ether is so extremely tenuous that it pervades all matter.*

3. *The ether is the seat of energy and conveys energy from place to place.* In Maxwell's immediate thoughts this energy was electro-magnetic in character, but the enlargement of the postulate to include all energy is certainly not contrary to the spirit of his conjectures.

Upon these postulates Maxwell erects the superb structure of his electro-magnetic theory of light. In a famous passage in his treatise Maxwell explains the concept in the following words:[14]

"When light is emitted, a certain amount of energy is expended by the luminous body, and if the light is absorbed by another body, this body becomes heated, showing that it has received heat from without. During the interval of time after the light left the first body and before it reached the second, it must have existed as energy in the intervening space.

"According to the theory of emission, the transmission of energy is effected by the actual transference of light-corpuscules from the luminous to the illuminated body, carrying with them their kinetic energy, together with any other kind of energy of which they may be receptacles.

"According to the theory of undulation, there is a material medium which fills the space between the two bodies, and it is by the action of contiguous parts of this medium that the energy is passed on, from one portion to the next, till it reaches the illuminated body.

"The luminiferous medium is therefore, during the passage of light through it, a receptacle of energy. In the undulatory theory, as developed by Huygens, Fresnel, Young, Green etc., this energy is supposed to be partly potential and partly kinetic. The potential energy is supposed to be due to the distortion of the elementary portions of the medium. We must therefore regard the medium as elastic. The kinetic energy is supposed to be due to the vibratory motion of the medium. We must therefore regard the medium as having a finite density.

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[14] *Electricity and Magnetism*, vol. 2, 3rd ed., 1904, p. 432.

"In the theory of electricity and magnetism adopted in this treatise, two forms of energy are recognized, the electrostatic and the electrokinetic and these are supposed to have their seat, not merely in the electrified or magnetized bodies, but in every part of the surrounding space, where electric or magnetic force is observed to act. Hence our theory agrees with the undulatory theory in assuming the existence of a medium which is capable of becoming a receptacle of two forms of energy."

In order to appreciate the full significance of these ideas one must familiarize himself with two very interesting quantities associated with matter which are called respectively the *coefficient of magnetic permeability* and the *dielectric constant.* The first of these quantities measures the susceptibility of matter to conducting lines of magnetic force. For example the magnetic permeability of iron is very much greater than that of air, because magnetic forces travel through iron much more easily than they do through air. The dielectric constant plays a similar role for electrical forces and is thus a measure of the capacity of a medium to conduct electrical lines of force. In more technical language the dielectric constant, or *inductive capacity,* as it is sometimes called, is that "quantity in an insulator which determines the charge taken by a conductor imbedded in it when charged to a given potential." In the so-called electro-magnetic units of physics the magnetic permeability for the ether is assumed to be unity and in electrostatic units the dielectric constant for the ether is unity.

The question to which Maxwell devoted his attention was this: Does any connection exist between the magnetic permeability and the dielectric constant of the ether? Upon the basis of his postulate of an ether, capable of storing and transmitting energy, Maxwell was able to show that *the square root of the reciprocal of the product of the two coefficients had the dimensions of a velocity.* But to what did this velocity belong and could any meaning be attached to such a statement?

It was the truly magnificent insight of Maxwell that led to the answer. If both the magnetic permeability and the dielectric constant are reduced to the same system of units, *the velocity just defined is numerically equal to the velocity of light.*

With this discovery the electro-magnetic theory of light was born! At last the dream of Faraday who wished to connect the mystery of light with the lines of electric and magnetic forces had become a reality. Can we wonder at the enthusiasm with which this great discovery was greeted by most of the physicists of his day? Alas, as in all other human activities, it is impossible to record unanimous accord, for it may be doubted whether Lord Kelvin himself ever gave the electro-magnetic theory his allegiance.[15] This identification of light with electro-magnetic forces in the metaphysical ether must rank with Newton's discovery of the gravitational origin of the tides. No higher encomium is possible than this.

But one very significant thing, in the light of the philosophical course which we pursue, must be pointed out here. The ether of Maxwell had lost a large part of its rational existence. Its density and its elasticity were no longer proper subjects of investigation. The same reason that prevented the ether of MacCullah from gaining adherents prevailed also for Maxwell's ether, with this difference that the approach to the concept had come through dielectric constants and magnetic permeabilities. It had historical dignity and conceptual reality from Faraday's tubes of force.

But the truth cannot be disguised. *Maxwell's ether is essentially Maxwell's set of equations;* its reality resides far more in the subjective image created by these formulas than in any attempt to give it objective existence by analogy with properties of matter. This set of equations, to be sure, is derived by picturing fluxes of Faraday's tubes of force through caps over conducting circuits, and by comparing the number of lines of electric and magnetic forces which enter a given region with the number which emerge, but do these images help us in any way to describe the reality of the ether? Upon this point there may be doubt.

### 6. *The Ether of Stokes.*

In the next chapter we shall consider the problem of the aberration of light which has been omitted from our introductory considerations. This is not quite the place to consider the theory of the ether as it relates to the motion of material particles through it. We can, however, record the postulates of an ether which was designed by George Gabriel Stokes (1819-1903) to explain phenomena connected with the motion of ponderable bodies relative to the ether entangled with them.[16] This ether had imperfections which were considered by H. A. Lorentz and Max Planck, both of whom suggested ways in which they should be modified.[17]

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[15] For an account of this interesting unwillingness of Lord Kelvin to accept the views of Maxwell see Thompson's: *Life of Lord Kelvin* (loc. cit.), p. 1024.

The postulates of Stokes' ether may be formulated as follows:

1. *The ether is a tenuous medium filling all space.*

2. *It is the seat of energy and conveys energy from place to place by vibrations analogous to displacements in an elastic body.*

3. *The ether near the earth is dragged along by it so that on the earth's surface there is an exact equivalence between the ether velocity and the velocity of the earth.*

4. *At some distance from the earth the ether is at rest.*

5. *The ether is irrotational.*

This hypothesis is a very important one because of its mathematical implications, but it is difficult to explain in simple language. It means that the infinitesimal particles of which the ether may be thought to consist do not rotate. Consider for example, that the iron of a magnet is composed of an immense number of tiny magnets which in a magnetic field are able to orient themselves independently of one another in the direction of the lines of force. If the magnet, thus conceived, is rotated in a magnetic field all the elementary particles will also rotate, and the motion of the elements of the bar will be rotational. On the contrary, there will be no individual motion of the particles, if no magnetic field be present, and the rotation of the bar, thus considered, will be irrotational.

6. *The ether has a variable density and at the surface of the earth is 60,000 times as dense as in empty space.*

This last postulate was added by Planck to replace the postulate, originally made by Stokes, that the medium was incompressible, which led to a contradiction with the fifth assumption. In commenting upon this rather remarkable postulate, Lorentz makes the following statement:[18]

"In this department of physics, in which we can make no progress without some hypothesis that looks somewhat startling at first sight, we must be careful not rashly to reject a new idea, and in making his suggestion Planck has certainly done a good thing. Yet I dare say that this assumption of an enormously condensed ether, combined, as it must be, with the hypothesis that the velocity of light is not in the least altered by it, is not very satisfactory. I am sure, Planck himself is inclined to prefer the unchangeable and immovable ether of Fresnel, if it can be shown that this conception can lead us to an understanding of the phenomena that have been observed."

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[16] "On the Aberration of Light," *Phil. Mag.* (3), Vol. 27, 1845, p. 9; *Mathematical and Physical Papers*, Vol. 1, pp. 134-140.

[17] "Stokes' theory of aberration in the supposition of a variable density of the ether." *Amsterdam Proceedings*, 1898-1899, p. 443; also see, *The Theory of Electrons*, p. 173.

[18] *The Theory of Electrons*, pp. 173-174.

### 7. *The Wraith-Like Ether of Lorentz.*

Perhaps the last formal attempt to endow an ether with consistent properties was that given by H. A. Lorentz in his *Theory of Electrons*, in which the Dutch physicist returns to the original ether of Fresnel, the stagnant ocean which forms the substratum of undulatory transfers of energy. According to Lorentz the ether may be thought of as endowed with the following properties:[19]

1. *The ether pervades all matter.* It "not only occupies all space between molecules, atoms or electrons," but "it pervades all these particles."

2. *The ether is always at rest.* "We can reconcile ourselves with this, at first sight, somewhat startling idea, by thinking of the particles of matter as some local modifications in the state of the ether. These modifications may of course very well travel onward while the volume elements of the medium in which they exist remain at rest."

3. *The ether is the seat of the electric and magnetic forces.*

4. *The ether has none of the properties of an elastic solid. It possesses neither mass, density, nor elasticity.*

5. *It has no structure.*

6. *It cannot be experienced sensuously except as it serves as a medium for the propagation of energy.*

7. *It is not a framework by means of which we can measure the velocities of material particles.*

When one reflects carefully on these postulates he is struck by their negative character. The material ether has disappeared and we are left with an approximately metaphysical entity. As one reflects on the gradual dissipation of the material properties of the ether from the vigorous elastic medium of Cauchy to the thin wraith of Lorentz, he is reminded of Mark Twain's description of the vanishing of the "Mysterious Stranger."

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[19] *The Theory of Electrons*, p. 11.

"He thinned away and thinned away until he was a soapbubble, except that he kept his shape. You could see the bushes through him as clearly as you see things through a soapbubble, and all over him played and flashed the delicate iridescent colors of the bubble, and along with them was that thing shaped like a window-sash which you always see on the globe of the bubble. You have seen a bubble strike the carpet and lightly bound along two or three times before it bursts. He did that. He sprang—touched the grass—bounded—floated along—touched again—and so on, and presently exploded—puff! and in his place was vacancy."

Is there any physical reality to the Lorentz ether? Does it exist in any sense? Have we the same degree of physical apprehension of it that we have for the concepts of entropy, or action? The only answer that the writer knows to this delicate question is best expressed in the following brief debate between two masters of modern physical science:

H. A. LORENTZ: (For the affirmative) "I cannot but regard the ether, which can be the seat of an electromagnetic field with its energy and its vibrations, as endowed with a certain degree of substantiality, however different it may be from all ordinary matter."[20]

L. SILBERSTEIN: (For the negative) "One fails to see what properties, in fact, it [the Lorentz ether] still has left to it, besides that of being a colorless seat (we cannot even say substratum) of the electro-magnetic vectors. . . . The ether, having been deprived of many of its precious properties, was at any rate already so nearly non-substantial, that the first blow it had to sustain from modern research knocked it out of existence altogether. . . Still, substantial or not, for the theory of Lorentz we are now considering, it is *something*, namely its unique system of reference. So long, therefore, as it was thought that there is such a unique system, Lorentz's all-pervading medium could continue its scanty existence."[21]

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[20] *The Theory of Electrons*, p. 230.

[21] *The Theory of Relativity*, 2nd ed., 1924, London, p. 42.

### 8. *The Ether as the Primary Atom.*

In the foregoing history of attempts to construct an ether we have held strictly to the physical point of view, in which our attention is sharply focused upon the problem of how energy gets from the sun to the earth through the high vacuum of interstellar space. It would not be entirely out of place to record another concept, which turns our thoughts toward chemical aspects of the problem.

This novel picture of the ether was given by the distinguished discoverer of the periodic properties of the elements, Dmitri Ivanovich Mendeléeff (1834-1907). However novel his speculations may seem, we must remember that Mendeléeff's reputation rests upon his power of prophecy, and one cannot wholly discredit the speculations of such a man, founded as they are upon his rich experiences during the great years of the nineteenth century.

Mendeléeff was born at Tobolsk, Siberia, the youngest of seventeen children. At the age of twenty-two he went to St. Petersburg to study natural science and from thence to Heidelberg in 1860, where he started a laboratory of his own. Returning to St. Petersburg the following year he became professor of chemistry in the technological institute of that city in 1863. In 1890 he resigned his professorship to become director of the Bureau of Weights and Measures three years later, a position that he held until his death, in 1907, the same year that saw the passing of Lord Kelvin.

Mendeléeff's chief renown rests upon his discovery of the periodic law of the elements. He had noticed that when the elements were arranged according to the size of their atomic weights they naturally fell into groups and series, those in each group having similar properties. Thus the chemical properties of lithium, sodium, and potassium as one triad, or chlorine, bromine, and iodine as another, are very similar, and the atomic weight of the middle element in each is approximately the average of the atomic weights of the other two. A. E. B. de Chancourtois, who anticipated Mendeléeff in some aspects of the periodic theory, thought of the elements as being arranged on a helix, those in each group having similar positions upon it. The great achievement of Mendeléeff was his prediction and description of three new elements to fill vacancies in his table. These elements, to which he gave the names ekaboron, ekaaluminum, and ekasilicon, were discovered within fifteen years after their prediction, gallium in 1871, scandium in 1879, and germanium in 1886. The greatest book written by Mendeléeff was his *Principles of Chemistry*, which has had many editions and has been translated into numerous languages.

The ether of the Russian chemist grew out of a strong conviction that ether was a material entity and the primary element of nature. He did not mean by this that the ether was to be regarded as "the constituent principle out of which the chemical atoms are formed," but rather as itself the first of the elements, indivisible and untransmutable. Mendeléeff was somewhat loath to publish this novel idea, and it was not until 1902, when advancing years forced him to the issue, that he undertook to give a systematic development of his postulate.[22]

His position is explained in the following quotation:

"If ether were producible from atoms and atoms could be built up from ether, the formation of new unlooked-for atoms and the disappearance of portions of the elements during experiment would be possible. A belief in such a possibility has long been held in the minds of many by force of superstition; and the more recent researches of Emmens to convert silver into gold, and those of Fittica (1900) to prove that phosphorus can be transformed into arsenic, show that it yet exists. In the fifty years during which I have carefully followed the records of chemistry, I have met with many such instances, but they have always proved unfounded. It is not my purpose to defend the independent individuality of the chemical elements, but I am forced to refer to it in speaking of the ether, for it seems to me that, besides being chemically invalid, it is impossible to conceive of ether as a primary substance, because such a substance should have some mass or weight and also chemical relations—mass in order to explain the majority of phenomena proceeding at all distances to the infinitely great, and chemical relations in order to explain those proceeding at distances infinitely small or commensurable with the atoms."[23]

The ether of Mendeléeff has the following properties:

*1. It is a gas, like helium or argon, incapable of chemical combination with any element.*

If the ether is a gas it must have weight, but its power of penetrating all matter is so great that it is impossible to determine its mass experimentally. "It does not require the recognition of a peculiar fourth state (of matter) beyond the human understanding (Crookes). All mystical, spiritual ideas about ether disappear."

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[22] *Principles of Chemistry, Appendix 3,* "An Attempt towards a Chemical Conception of the Ether," Third English edition, 1905, Vol. 2, pp. 509-529.

[23] P. 512.

2. *It is the lightest gas with a high penetrating power and an extremely high velocity.*

Mendeléeff does not attempt to place his new element in the series with hydrogen but invents a new series of zero-th order and calls the ether element the unknown "$x$." He calculates its atomic weight to be between $9.6 \times 10^{-7}$ and $5.3 \times 10^{-11}$, where the atomic weight of hydrogen is assumed to be unity. He discards the second value in favor of the first for the very interesting reason, in view of our recent quantum theory of energy, that "it would in some measure answer to a revival of the emission theory of light." Adopting the first value, an atomic weight approximately one millionth that of hydrogen, the atoms of the new element would have the considerable velocity of about 2250 kilometers (1400 miles) per second.

Can these views be thought of as only those of a lively imagination, colored by a periodic view of the elements of nature, or is there any basis for scientific credence in this age of exciting mystery? It appears to the writer that there is something quite prophetic, when we reflect upon the quantum theory and cosmic radiation, in the following passage of the noted chemist:[24]

"It seems to me that the optical and photo-radiant phenomena, not to mention the loss of electrical charges, indicate a material flow of something which has not been weighed, and it appears to me that they might be understood in this manner, for peculiar forms of the entrance and egress of ether atoms should be accompanied by such disturbances in the etherial medium as give the phenomena of light."

### 9. *Is There An Ether?*

We have now surveyed hastily and without the aid of mathematics, some of the most important theories of the ether. These theories, excepting that of Mendeléeff, have been subjected to the searching scrutiny of calculation and laboratory data. From them, one by one, their material properties have been gradually removed until but a shadow connects them with sensuous experience. The ether of Lorentz preserves its existence as an imperative of human reason, which demands that something must exist for the conveyance of energy from place to place and for the possibility of conceiving lines and tubes of force.

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[24] *Loc. cit.*, p. 528.

The aim of this book is to exhibit the character of the postulates of material science. Certainly the baffling problem of the ether forces this sharply upon us. During the early years of the nineteenth century there was a certain tangible reality in the ether because the difference between its properties and the properties of ordinary matter was merely one of degree. Today scarcely a vestige of this shadowy reality remains, and yet the physical entities that led to its invention cry out for some kind of rational explanation, which appears to be denied them.

Do we want to believe in action at a distance or forces which work through the medium of a complete vacuum? Do we want to believe that energies flow from place to place without the intervention of any kind of transmitting medium? What shall we say about the potential energy of the lifted stone or the electrical energy of the Leyden jar? Is the curious concept of action which the earth minimizes in its annual swing around the sun, to be regarded as a thing without a real existence? What shall we say about the Foucault pendulum and its evidence of the existence of a cosmic inertial frame?

Are we seeing mysteries where there are none? It would appear not. The problem may be resolved, perhaps, by assuming a new definition of reality. Is it necessary for a physical entity, in order to have reality, to be sensuous, or even to have an analogy with material things? If we are willing to answer this question in the negative, then we have committed ourselves to what may be regarded as a metaphysical philosophy. In this, of course, we must look to the methods of the mathematicians and safeguard our concepts by a lower bound beyond which we shall not go, else this admission will involve us in all manner of esoteric mysticisms.

Let us state, for example, as a basis for the new metaphysics the postulate that physical entities may exist which are of transcendental character in the sense that they cannot be explained on the basis of analogies with material objects or other models of sensuous character. Let us require, however, that any proposition in the new physics which has no sensuous basis for reality must be capable of a mathematical formulation the implications of which can be reduced to terms that can be checked against the experimental evidence. In other words, let us assume that the ether of Maxwell is identical with the equations of Maxwell and has reality only in so far as these equations check with the data of experience. Newton's action at a distance is Newton's potential function, out of which flow the long formulas of Leverrier and Simon Newcomb, with their prediction of the motions of the planets.

Have we asked too much, or is there still a craving of the human spirit for a closer bond with the mysterious forces and the radiant energies of the phenomena? In the next chapter we shall explore further the postulate of an ether as it relates to the problem of matter in motion, and perhaps we can see through the glass a little less darkly than at this stage of our inquiry.

# CHAPTER 4.

# THE NATURE OF LIGHT

"What a beautiful belt you've got on!" Alice suddenly remarked. . . . "At least," she corrected herself on second thought, "a beautiful cravat, I should have said—no, a belt, I mean—I beg your pardon!" she added in dismay, for Humpty Dumpty looked thoroughly offended, and she began to wish she hadn't chosen that subject. "If I only knew," she thought to herself, "which was neck and which was waist!" Lewis Carroll in *Through the Looking-Glass.*

## 1. *Huygens and the Wave Theory of Light.*

Sir Francis Bacon (1561-1626) in his essay on "Truth" paraphrases Lucretius in the following words:

"It is a pleasure to stand upon the shore and to see ships tossed upon the sea; a pleasure to stand in the window of a castle, and to see a battle and the adventures thereof below: but no pleasure is comparable to the standing upon the vantage ground of Truth, and to see the errors, and wandering, and mists, and tempests, in the vale below."

In the pages of the last chapter, from the vantage point of time, we have reviewed the heroic struggle of those who would endow the ether with the properties of a material substance. At present these efforts have seemed to be without avail. Like the action at a distance of Newton, the action of Maupertuis, and the entropy of Clausius, the reality of this physical entity is elusive. On the assumption of an elastic ether every postulate with regard to density, elasticity, and rigidity led to contradiction with experiment. The ether of Stokes and Planck had to be 60,000 times as dense at the surface of the earth as it was in empty space; the ether of Maxwell was merely the seat of electro-magnetic forces and their energy; its reality consisted in a set of equations; the ether of Lorentz had become a sort of spectral entity the tangible properties of which were its extension and its ability to convey energy from place to place.

In this chapter we shall look at the problem from another angle. Let us postulate an ether, and endow it with but a single property, namely that of undulation. We confess that this leads us face to face with a dilemma. We may either conceive of a thing that has no substantive being but yet has the ability to propagate a wave, or we may postulate that energy gets about according to mathematical laws of our subjective creation. The second possibility has been repellent to rational philosophy until recently, but on the assumption that one can get used to anything, there seems to be a more general acceptance of this metaphysical position.

In the last chapter little was said about the rival theories of light propagation and the optical properties of moving bodies. It was, in truth, the struggle between the undulatory theory of the Dutch physicist Christian Huygens (1629-1695) and the corpuscular theory of Sir Isaac Newton that forced the concept of the ether into scientific thinking. Has the struggle ceased? Today the effort of those who seek to match their formulas against the evidence of the laboratory is to decide between the corpuscular theory (now called the quantum theory) and the wave theory of energy. It is a strange contest, as we shall show in more detail later, in which radiations assume some of the aspects of particles, while particles in some respects behave like waves. But neither appears to be both at the same time. We are witnessing, perhaps, two facets of the same thing, but the complete gem is not yet visible to the eye.

Huygens, who was born at The Hague, received his early education from his father, later studied at Leiden under Frans Van Schooten, and completed his formal discipline in the juridical school at Breda. Some of his early work fell into the hands of Descartes, who predicted a brilliant future for him. In 1660 and 1663 he went to Paris and to London and in 1666 he accepted a flattering offer from Louis XIV to make his residence in Paris. From then until 1681 he resided in the stimulating atmosphere of the Bibliothèque du Roi, where many of his ideas were developed. He returned to his native city in 1681 partly from considerations of health and partly because of the revocation of the Edict of Nantes. The interests of Huygens were many and varied. He improved the telescope of Galileo by partly overcoming chromatic aberration, and with it discovered the sixth satellite of Saturn and explained the varying appearance of the planet as due to a ring. He studied the nebula of Orion and invented the pendulum clock. His magnum opus was the *Horologium oscillatorium*, published in 1673, but his chief renown rests upon a little volume entitled a *Traité de la lumière*, completed in 1678 but not published until 1690. In this beautiful little treatise he elaborates the undulatory theory of light which had already been advanced by Robert Hooke (1635-1703) in 1668 and by Ignace Pardies (1636-1673) in 1672, and states that fundamental law of optics which is now known under his name.

The theory of the undulatory character of light assumes that light is propagated in waves. In the early history of the theory, as has already been explained in the preceding chapter, the waves were regarded as similar to those of sound which are longitudinal. But in the later theory they were thought of as transverse vibrations such as may be created in the deformation of rigid elastic bodies. The old question persists: In what are these waves moving?

Huygens, the chief exponent of the wave theory in the early history of the subject, explains his postulates in the following words:

"It is inconceivable to doubt that light consists in the motion of some sort of matter. For whether one considers its production, one sees that here upon the earth it is chiefly engendered by fire and flame which contain without doubt bodies that are in rapid motion, since they dissolve and melt many other bodies, even the most solid bodies; or whether one considers its effect, one sees that when light is connected, as by concave mirrors, it has the properties of burning as the fire does, that is to say it disunites the particles of bodies. This is assuredly the mark of motion, at least in the true philosophy, in which one conceives the causes of all natural effects in terms of mechanical motions. This, in my opinion, we must necessarily do, or else renounce all hopes of ever comprehending anything in physics.

"And as, according to this philosophy, one holds as certain that the sensation of sight is excited only by the impression of some movement of a kind of matter which acts on the nerves at the back of our eyes, there is here yet one reason more for believing that light consists in a movement of matter which exists between us and the luminous body.

"Further, when one considers the extreme speed with which light spreads on every side, and how, when it comes from different regions, even from those directly opposite, the rays traverse one another without hindrance, one may well understand that when we see a luminous object, it cannot be by any transport of matter coming to us from this object, in the way in which a shot or an arrow traverses the air; for assuredly that would too greatly impugn these two properties of light especially the second of them. It is then in some other way that light spreads; and that which can lead us to comprehend it is the knowledge which we have of the spreading of sound in the air."[1]

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[1] *Treatise on Light*, translated by Silvanus P. Thompson, 1912, pp. 3-4.

Huygens' principle which seems to have come unscathed through the modern scrutiny of first principles merely asserts that each element of a wave front progressing through a uniform medium is to be itself regarded as the origin of a secondary wave, and the wave front which represents the total disturbance is the envelope of all these secondary disturbances. This happy thought allowed Huygens to explain refraction and reflection, where his contemporaries had failed.

### 2. *The Corpuscular Theory of Light.*

The corpuscular theory, on the other hand, assumes that light consists of tiny pellets or corpuscles shot out with the velocity of 186,000 miles per second from the radiating source. Today we have slightly changed the theory by calling the corpuscles "photons" and we regard them as bits of energy rather than as small material particles. The subject is now known as the *quantum* theory of light.

The chief exponent of the corpuscular, or emission theory, of light was Newton, who, however, curiously enough, postulated the existence of an ether at the same time that he rejected the undulatory theory of Hooke and Huygens.[2] With Newton there existed an all-pervading ether capable of propagating vibrations; its density was conceived of as variable, being greatest in inter-stellar space. But the vibrations cannot be thought of as light, because of difficulties which he saw in explaining rectilinear propagation on this hypothesis. Therefore he conceived of light as being "something of a different kind propagated from lucid bodies. They that will, may suppose it an aggregate of various peripatetic qualities. Others may suppose it multitudes of unimaginable small and swift corpuscles of various sizes, springing from shining bodies at great distances one after another; but yet without any sensible interval of time, and continually urged forward by a principle of motion, which in the beginning accelerates them, till the resistance of the etherial medium equals the force of that principle, much after the manner that bodies let fall in water are accelerated till the resistance of the water equals the force of gravity. But they that like not this, may suppose light any other corporeal emanation, or any impulse or motion of any other medium or etherial spirit diffused through the main body of ether, or what else they can imagine proper for this purpose. To avoid dispute, and make this hypothesis general, let every man here take his fancy; only whatever light be, I suppose it consists of rays differing from one another in contingent circumstances, as bigness, form, or vigour."[3]

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[2] Extracts from Newton's *Opticks* are reproduced by T. Preston in his *Theory of Light*, 3d ed. London, 1901, pp. 20-21, to "show how much more closely than is generally supposed" Newton's theory of light resembles the undulatory theory now accepted.

### 3. *The Phenomenon of Interference.*

The eighteenth century saw the almost complete triumph of the emission theory, supported as it was by the authority of Newton. But by the end of that century, and during the early years of the next, a revolution had started, which caused a complete reversal of the situation. Thomas Young (1773-1829), a native of Somersetshire, England, and trained in medicine, commenced to advance the wave theory in a series of papers dating from 1799. The famous contribution made by Young was his suggestion of the phenomena of interference.

Borrowing an illustration used in a similar connection by Newton, Young explains his ideas in the following historic passage:

"Suppose a number of equal waves of water to move upon the surface of a stagnant lake, with a certain constant velocity, and to enter a narrow channel leading out of the lake; suppose then another similar cause to have excited another equal series of waves, which arrive at the same channel, with the same velocity, and at the same time with the first. Neither series of waves will destroy the other, but their effects will be combined; if they enter the channel in such a manner that the elevations of one series coincide with those of the other, they must together produce a series of greater joint elevations; but if the elevations of one series are so situated as to correspond to the depressions of the other, they must exactly fill up those depressions, and the surface of the water remain smooth. Now I maintain that similar effects take place whenever two portions of light are thus mixed; and this I call the general law of the *interference* of light."[4]

The phenomenon of interference can be witnessed by anyone who will drop two stones into a smooth body of water. The waves from the two sources spread out with a certain velocity in two circles, which finally join and break up into crests and valleys of varying heights and depths. There is in point an interesting note taken from the diary of William Thomson, later Lord Kelvin, which was written while on a continental tour when he was sixteen years of age:[5]

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[3] *Royal Society,* Dec. 9, 1675.

[4] Young's *Works,* Vol. 1, p. 202.

"Reached the bar at the mouth of the Maas, near Brill, at about 4½ o'clock in the morning, where we had to lie until 10. The vessel rolled greatly from side to side, but the rolling was intermittent, as every two or three minutes it calmed down and then rose again with perfect regularity. This probably arose from two sets of waves of slightly different lengths coming in in the same direction from two different sources."

Those who have ever had the hardihood to take a canoe out on a lake when it is rough will know that the skill in the sport consists in being able to guess the time and amplitude of those larger rollers which are the culmination of a series of little waves. And just as water waves tend to amplify and annul one another when crests and valleys coincide, so also will light waves (if such things really exist) intensify and interfere with one another.

The accompanying diagram (Figure 1) will illustrate the principle of interference more precisely. A train of plane waves of light approaches the grid represented by the heavy black line broken by a number of equally-spaced openings. Each of these apertures becomes the center of a new spherical wave, since by the principle of

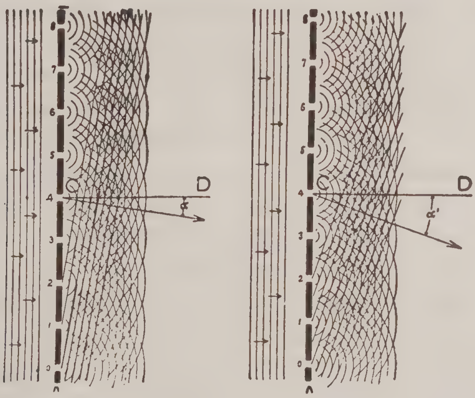

Figure 1. Diagram of the diffraction of waves by a grid.

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[5] *Life of Lord Kelvin* (loc. cit.), vol. 1, p. 16.

Huygens every element of a wave is itself the origin of a secondary wave. The new waves, proceeding from each of the openings, interfere with one another, exactly as in the case of water waves, and we find positions to the right of the grid where the energies attain maximum and minimum values. A photographic plate placed in the path of the advancing waves will be affected by these variations in light intensity, and the resulting photograph exhibits what is called a *diffraction pattern.*

The ideas of Young did not meet with cordial response in England, and a fierce attack launched in the *Edinburgh Review* by Henry Brougham, afterwards Lord Chancellor of England, successfully discredited the new theory of interference.

The case was otherwise in France, however, for Young's concept was thoroughly explored in a series of brilliant memoirs by Augustin Fresnel (1788-1827). The story is one of the dramas of the history of science. Fresnel was opposed politically to Napoleon and when the latter returned from the island of Elba in 1815 for his celebrated hundred day sojourn in the empire, Fresnel was given a period of enforced idleness. O solitude, "the best nurse of wisdom"! how little we appreciate your virtues! Swift's paradox that a "wise man is never less alone than when he is alone" has seldom been realized by those who might somewhere endow sanctuaries consecrated to seclusion. It is well occasionally to reflect that Newton's greatest work was done when the plague drove him away from Cambridge.

During this period of arrest, Fresnel busied himself with the wave theory of light and in an early memoir advanced a theory of diffraction similar to that of Young. By a diffraction phenomenon, as we have said above, we mean an optical phenomenon caused by a deviation in the rectilinear propagation of light. The first observations on diffraction were made by F. M. Grimaldi (1618-1663), who had noticed that there was some illumination within the geometrical shadow of an opaque body.[6] Newton had repeated with a little variation the experiment of Grimaldi and had observed the iris-colored bands which bordered the image of two knife edges set very close together, between which light is transmitted to a screen.

Young had attempted to explain diffraction on the basis of his theory by assuming that there was interference between the light which passed very close to the knife edges and that which was reflected from them. Fresnel also fell into this error in his first paper,[7] but not satisfied with his explanation he reëxamined the theory and in 1816 presented to the Academy a supplement to his original paper, in which the real explanation was made,[8] namely, that the diffraction phenomena are due to the interferences between the secondary waves emitted from the unobstructed part of the original wave. His calculations were made on the basis of Huygens' principle.

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[6] In a posthumous work entitled: *Physico-mathesis de lumine, coloribus, et iride.* Bologna, 1665. In particular, see Book 1, prop. 1.

### 4. *The Diffraction of Light.*

In the meantime the corpuscular theory had gained new vigor from an explanation on dynamical principles of the double refraction phenomenon in Iceland spar, this explanation having been advanced by no less a person than Pierre Simon Laplace (1749-1827) who was then at the height of his mental vigor and influence. The Academy, hoping to establish the corpuscular theory beyond further criticism, proposed "Diffraction" as the subject of the prize to be awarded in 1818.

Fresnel, under this spur, developed his theory with great vigor and presented his ideas in a large memoir in 1818 for the prize of the Academy.[9] The judges, however, were three ardent advocates of the corpuscular theory, Laplace, Poisson, and Biot, and to face such a committee with a manuscript following the tradition of Huygens and Young seemed like a joust with windmills. However, the mathematical ability of Poisson in reality won the victory for the young theorist. While reading the manuscript Poisson noticed that the methods of Fresnel could be extended to the case of diffraction by an opaque disk, and he easily showed that if the path of a ray of light were occulted by such an obstacle there would appear in the exact center of the shadow a white spot whose intensity would equal the intensity of the incident ray.

What a breathless moment in the history of science! Was not this *reductio ad absurdum* of the great Poisson the very weapon to turn upon the hosts of the corpuscular army? The experiment was made by Arago, and there in the center of the shadow of the disk was the white spot predicted by Poisson. The prize was given to Fresnel.

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[7] *Annales de Chemie* (2), i, 1816, p. 239; also *Oeuvres complètes d'Augustin Fresnel*, Paris, 1866, in three vols. with introduction by Émile Verdet (1824-1866), Vol. 1, p. 89.

[8] *Oeuvres*, Vol. 1, p. 129.

[9] *Mémoire de l'Académie*, Vol. 5, 1826, p. 339; *Oeuvres*, vol. 1, p. 247.

The two patterns appearing in Plate III were made by light which diffracted around steel spheres placed in the path of the incident beam.[10] The white spot which so discomfited the proponents of the corpuscular theory is clearly exhibited in the center of the discs. The second set of pictures (Plate IV) shows a photographic transparency (a) from which was made a diffraction photograph (b) in which the usual lens of the camera was replaced by a steel ball. How the early theorists would have marvelled to know that some day actual photographs would be made with a ball of steel placed directly across the path of the beam of light.

The consequences of diffraction have, of course, been thoroughly explored in the later history of the subject. Eugene von Lommel (1837-1899), in a series of classical papers, reduced many of the diffraction problems to calculation and was able to show the exact correspondence between mathematical principles and objective experiment.[11]

The question always lurks in the background of physical speculation, however, that if matters could only be pushed a few decimal places further, then the discrepancy between the theory and its application would be revealed. In Newton's theory of universal gravitation the behaviour of the planet Mercury has been a disturbing element. Why should that smallest of the planets, swinging in its rapid course about the sun, present an anomaly difficult to account for on the hypothesis of Newton's laws? Why does its perihelion advance 43 seconds per century, so that the time of Mercury's transits across the face of the sun fails to accord with the exact calculations of Leverrier and Newcomb, based upon the mathematical theory into which the slight perihelion correction has not been introduced? While this discrepancy exists, unexplained by the discovery of the planet Vulcan or by some other plausible disturbance within the

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[10] M. E. Hufford: "Some New Diffraction Photographs," *Physical Review*, Vol. 3, 2nd ser., 1914, pp. 241-243.

[11] "Die Beugungserscheinungen einer kreisrunden Öffnung und eines kreisrunden Schirmchens," *Abb. der K. Bayer, Akad. der Wissenschaften*, Vol. 15, 1884, pp. 233-328.

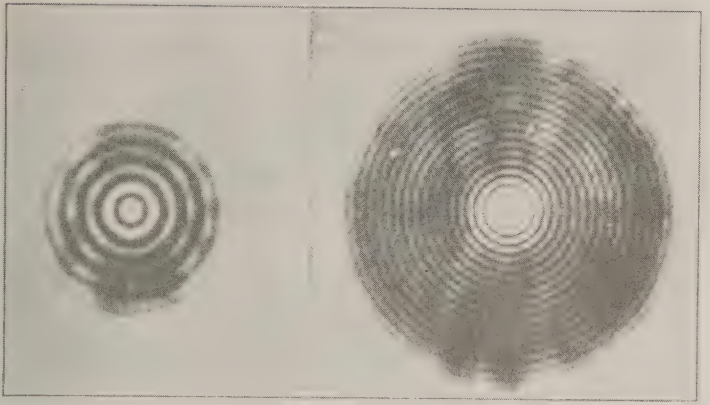
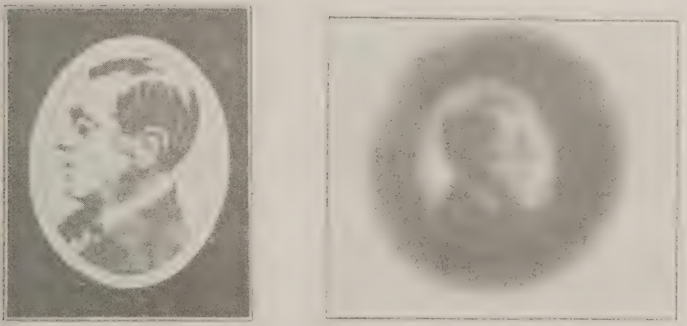


PLATE III

Patterns made by light diffracted around steel spheres. The Arago white spot is to be seen in the center of each pattern. From the *Physical Review*, Vol. 7, 1916, p. 548.



(*a*) (*b*)

PLATE IV

Picture (*b*) is a reproduction of (*a*) made by diffracting the light around a steel ball. From the *Physical Review*, Vol. 3, 1914, p. 242.

reach of Newton's theory, there will always be a reasonable doubt as to the limits to which his assumptions can be applied.

What is the story with regard to diffraction and the wave theory of light? Calculations have been made to check the radii of a pattern of seventy rings, made by diffracting light through a circular orifice.[12] The details of the experiment are very simple. A tiny pinpoint of monochromatic light is placed at one end of a tube 106 feet in length and the resulting beam is allowed to fall upon a photographic plate at the other end. Midway between the source and the screen is set a metal plate containing a hole slightly more than an inch in diameter, through which the beam passes and from whose edges it is diffracted. The beautiful pattern with its seventy rings

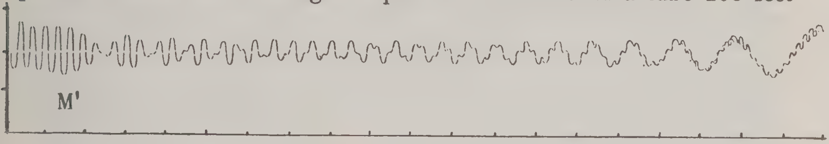


Figure 2.

Diagram showing the structure of a diffraction pattern. The horizontal axis corresponds to the radius of the photograph with the center at the point zero. The vertical axis measures the intensity of the light and dark bands of the pattern.

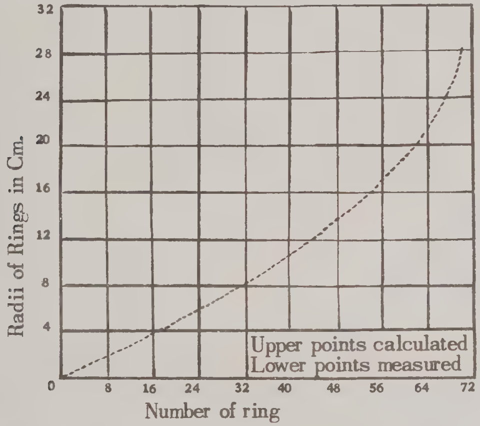


Figure 3.

Diagram showing the striking agreement between theory and experiment in the case of diffraction. The measured and calculated radii of the diffraction rings of the pattern form a single line.

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[12] "The Diffraction of Light by a Circular Opening and the Lommel Wave Theory," M. E. Hufford and H. T. Davis, *Physical Review*, Vol. 33 (2nd ser.), 1929, pp. 589-597.

is reproduced in the accompanying photograph, Plate V. But the question in which we are interested is this: Do the position and intensity of the rings accord with the theory of Fresnel? The answer is contained in the two accompanying diagrams (Figures 2 and 3). In the first of these the intensity on the vertical axis is computed for every value of the radius of the plate on the horizontal axis. That is to say the value marked zero corresponds to the center of the diffraction pattern and the last value recorded on the horizontal axis corresponds to the final darkening of the plate at the outer edge of the ring system. The vertical axis measures the intensity of the light creating the rings, a dark band on the diffraction photograph corresponding to a crest of the irregular curve and a light band corresponding to a hollow. The tiny undulations which make up the later rings are evidence of the fine structure of the pattern, although these small variations in the intensity are masked in the actual photograph because of the long exposure required to produce the picture.

And what is the answer to our question? Does the Fresnel picture of the wave correspond with the actual facts investigated over so long a range? The second diagram shows the exceptional agreement between the radii measured from the actual diffraction pattern and the radii computed from the crests and hollows of the intensity diagram. We reach the inevitable conclusion that the wave theory of diffraction, regarded as a purely subjective creation, agrees with remarkable exactness with the objective experiments.

But the old question persists: Does this prove that there is an ether? Was Lorentz correct when he said that, for all the arguments of his contemporaries, he still felt that there was an objective reality to the ether concept? Does diffraction prove it or does diffraction only increase our bewilderment, as we try to organize the phenomena of the world on a basis agreeing with rational intuitions? Can the ether, robbed of all of its substantive properties and endowed with a transcendental function in which our measure of belief merely depends upon our inability to believe anything else, be regarded as belonging to the domain of reality?

### 5. *The Puzzle of Aberration.*

Having discussed the controversy between the wave theory and the corpuscular theory of light, we shall now review briefly several experiments which have had a profound influence upon the progress of physical science.

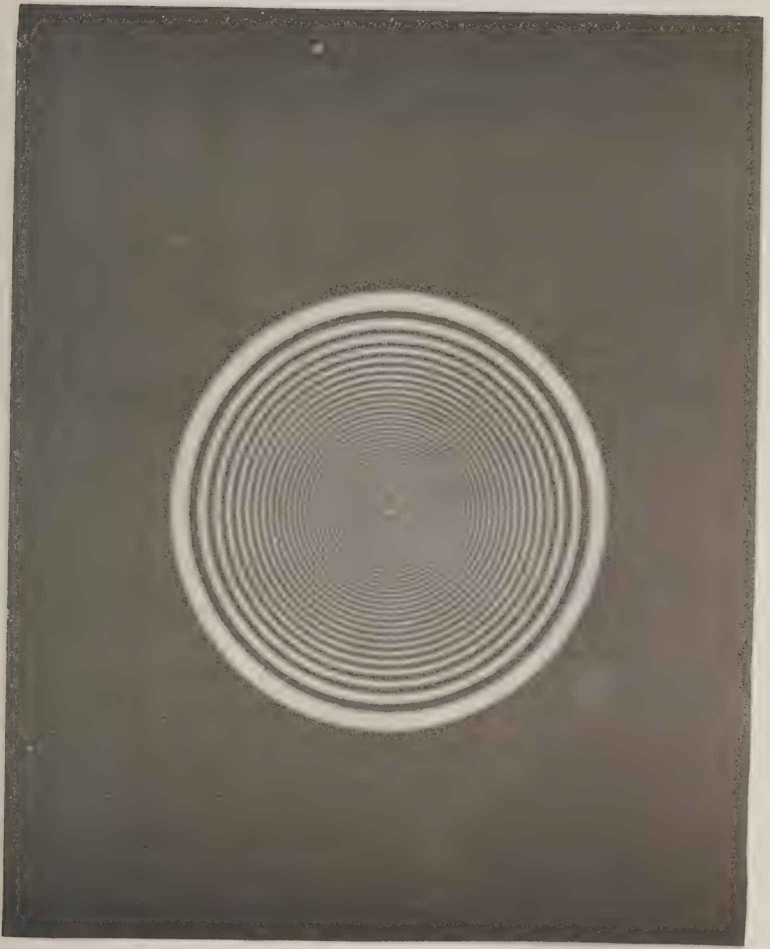

PLATE V

A diffraction pattern made by diffracting light through a circular orifice.

The first of these was the discovery in 1728 by James Bradley (1692-1762) of the *angle of aberration.*[13] If one goes out in a rain when there is no wind, he must, nevertheless, hold his umbrella in front of him to keep from getting wet. The drops of rain, which are in reality falling straight down, appear to be approaching at an angle due to the fact that one is in motion. As a matter of fact if the velocity of the falling rain drops were known one could actually calculate his walking speed from the angle at which the rain appears to strike his umbrella. This is the phenomenon of aberration which is often illustrated by the picture of a ball thrown through the window of a moving car. If the car is moving fast enough the ball will enter a window in the front and will pass out through a window in the rear. To a passenger its path will appear to have been a diagonal across the car.

In exactly the same way a ray of light from a distant star appears to come into the atmosphere of the earth at an angle with the star's true position due to the motion of the earth through space. Of course, if the earth had not been moving in an ellipse, this fact would never have been discovered, because the angle of aberration would always have been a constant and would have forever eluded our efforts to detect it. It was only discovered when Bradley found that the star Gamma in the head of the constellation of the Dragon which had appeared in one direction at Christmas time was in a different direction in July.

At first Bradley was much perplexed by this discovery of the apparent displacement of the stars at different times in the year, until he casually observed a flag floating at the mast head of a ship; when the ship changed its course, the flag flew in a different direction. Hence the explanation of the cause of aberration as due to the elliptical motion of the earth in space occurred to him.

This illustration of the flag is an example of the force of little things and the power of elementary analogies. One of the impressive facts in science is this ability of the race to make progress in the explanation of mysterious phenomena by models of simple things. Lord Kelvin devised his vortex theory of the atoms, one of the excitements of the latter years of the last century, by observing the behaviour of rings of smoke. The story of Archimedes and his discovery of the specific gravity of metals by his loss of weight in the bath tub is familiar to everyone. How strange that someone in the early days of science did not reflect upon the vortical behaviour of water flowing out of a bowl! He might well have wondered how the elementary parts of the fluid were able to decide which way to whirl. The situation would have appealed to him as similar to that of Buridan's ass which starved between two equal mounds of hay. Reflections based upon this very elementary experiment might have led to the discovery of the rotation of the earth.

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[13] "Account of the new discovered motion of the Fix'd Stars," *Phil. Trans.*, Vol. 35, 1728, p. 637.

Let us pause here just a moment to see what a remarkable fact this discovery of aberration is. In the last chapter it was shown how Galileo might have proved an absolute rotation of the earth by means of a pendulum suspended from the dome of a cathedral and set in vibration over a piece of ruled paper on the floor. What a marvellous fact he could have deduced from Bradley's constant of aberration. Focusing his telescope upon Polaris in December and again in July he would have been able to bring incontrovertible evidence to show that the earth moved about the sun. Perhaps some of the humiliation of that early science could have been avoided by this one fact.

In the early days much weight was thrown against the Copernican theory that the earth moved around the sun because the so-called parallax of the stars could not be measured. By the annual or heliocentric parallax we mean the angle $\odot\ S\ E$ where $\odot$ is the sun, $S$ the star and $E$ the earth. This angle is so exceedingly small that a stellar parallax was not measured until 1838, when F. W. Bessel found one for 61 Cygni.[14] The largest known parallaxes are those of Alpha Centauri and Proxima Centauri. According to the most recent measurements, the first has a parallax of 0".756 and the second one of 0".762. Thus, excepting the sun, Proxima Centauri is the nearest star to the earth. A simple calculation shows that these parallaxes correspond to a distance of 4.3 light years. The angle of aberration, on the other hand has a magnitude of 20."47, which, as one observes, is of an entirely different order of magnitude from the angle of parallax. Parallax depends upon the relative distance between two bodies; the angle of aberration depends upon an absolute revolution only. Let us suppose that the sun, the moon, the planets and all the stars were blotted from the sky, but let one beam of light be started somewhere in the universe. From this one external source of energy, Galileo might have demonstrated that the earth had something corresponding to an absolute revolution in space. This is a poetic thought and one perhaps as mysterious as the absolute rotation which we discussed in the last chapter. In other words, we are again faced by the question of revolution about what? Can it be that these phenomena of the revolution and rotation of the earth are but evidences of a mysterious relationship of matter with a seat of energy which we have called the ether?

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[14] See a letter from Professor Bessel to Sir J. Herschel, Oct. 23, 1838, *Monthly Notices of the Royal Astronomical Soc.*, Vol. 4, pp. 152-161.

In the days of Bradley the corpuscular theory of light was then the accepted theory and stellar aberration was easily explained by the analogies just cited. When the wave theory commenced to gain the upper hand, under the researches of Young and Fresnel, the situation assumed a different aspect, and the question was immediately asked how a ray transmitted by vibrations in the ether could be affected by the elliptical motion of the earth. The analogy of the man in the rain storm no longer had force. The answer to the question may be stated thus: If we are willing to admit a stagnant ether, that is to say, an ether which never moves and is unaffected by the motion of the earth through it, then it is possible to explain aberration just as well on the wave theory as on the corpuscular postulate. Unfortunately the proof of this statement is mathematical and must be omitted. We should carefully note this fact, however, that to explain aberration we are partly committed to what is called the "stagnant ether" theory. In the last chapter when we were not concerned with the optical properties of moving bodies this postulate was not advanced, but the phenomenon of aberration brings it clearly into view. Young has picturesquely put the concept in the following words:[15]

"Upon considering the aberration of the stars, I am disposed to believe that the luminiferous ether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees."

In commenting upon this statement, Silberstein remarks: "This picturesque analogy fitted altogether the case of air, which behaves very nearly like a vacuum, but not glass or water, for which the 'grove of trees' had to be replaced by a rather dense thicket."[16]

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[15] "Experiments and Calculations relative to physical optics," *Philosophical Transactions*, 1804, pp. 1-16, in particular p. 12.

Hence the critical phenomenon of aberration when applied to the movement of the earth as determined from the light of the stars could be explained equally well on either the wave theory or the corpuscular postulate. Any one left to choose between the two theories on the basis of this phenomenon alone would, we believe, have chosen the latter, since it is so much easier on intuitional grounds to think of light in terms of rapidly moving material particles than as a transverse vibration in a metaphysical medium.

### 6. *Does Matter Drag the Ether?*

However, an unexpected and brilliant victory was soon to be won by the forces under the banner of the wave theory. The question was asked: What would happen to the aberration angle if one used a telescope the tube of which was filled with water rather than with air? As anyone knows who has ever tried to spear fish, light bends when it enters an optical medium which is denser than air. Should there not be found, therefore, a difference in the angle of aberration due to the fact that the speed of light in water is less than it is in air? This very reasonable view led to experimentation by Arago, who found no change in the aberration constant for the case of glass,[17] and in 1871 by Sir G. B. Airy (1801-1892), royal astronomer at Greenwich, who tested the water telescope with the same result.[18]

What strange phenomenon was this? Arago, as a member of the corpuscular party, was much perplexed and his attempt at an explanation drove him to a curious hypothesis. He conceived that the source of light sends out particles with an infinity of different velocities but that out of these none but one endowed with a certain velocity has the power of exciting our sense of sight. Hence, if a swarm of particles fell upon the telescope full of water, they would all be retarded to a certain extent, and those particles which had been traveling too rapidly in the air telescope to affect our sense of sight would now move down into the visible region. This strange hypothesis soon involved Arago in a maze of difficulties from which he could not extricate himself, and he wrote a letter to Fresnel. Fresnel's reply is one of the famous documents in physical literature and part of it is quoted below:[19]

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[16] *Theory of Relativity*, 1924, p. 35.

[17] See Biot: *Astro. Phys.*, 3d ed., v, p. 364.

[18] "On a supposed alteration of the amount of Astronomical Aberration of Light produced by the passage of the Light through a considerable thickness of Refracting Medium," *Proc. Royal Soc. of London*, Vol. 20, pp. 35-39.

"Through your beautiful experiments upon the light of the stars, you have proved that the movement of the earth has no sensible influence upon the reflection of rays which come from the stars. . . .

"You have asked me to examine the question whether the result of these observations can be reconciled more easily with the system which makes light consist of vibrations in a universal fluid. It is all the more necessary to give the explanation in this theory, since it ought to apply equally well to terrestrial objects; for the velocity with which the waves are propagated is independent of the movement of the bodies from which they emanate.

"If one admits that our globe impresses its movement on the ether in which it is enveloped, one would easily conceive why the same prism always refracts light in the same manner whatever side it arrives from. But it seems impossible to explain the aberration of the stars on this hypothesis; up to the present time at least I have been able to conceive that phenomenon clearly only by supposing that the ether passes freely through the globe, and that the velocity communicated to this subtle fluid is only a small part of that of the earth; it does not exceed the hundredth part, for example.

"However extraordinary this hypothesis may at first sight appear, it is not in contradiction, it seems to me, with the idea that the greatest physicists have made concerning the extreme porosity of matter. One may demand, in truth, since a very thin opaque body intercepts light, how it happens that there exists a current of ether through our globe. Without pretending to reply completely to the objection, I can remark, however, that these two types of movements are of a nature too different for us to be able to apply to the one that which we observe relative to the other. The movement of light is not a current, but a vibration of the ether. One can imagine that the small elementary waves into which light is divided in passing through a body are, in certain cases, out of phase when they reunite, by reason of the difference in routes passed over or from the unequal hindrances which they encounter in their journey; this it is which obstructs the propagation of the vibrations, or changes their nature in such a way as to remove the property of illumination, such as we find in a very striking way in black bodies; whereas the same circumstances do not prevent the establishment of a current of ether. One increases the transparency of hydrophane by moistening it, and it is evident that the placing of the water between the particles, which favors the propagation of luminous vibrations, should be, on the contrary, an additional small obstacle to the establishment of an ether current; this demonstrates the great difference which exists between these two types of movements.

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[19] "Lettre d'Augustin Fresnel à François Arago, sur l'influence du mouvement terrestre dans quelques phénomènes d'optique," *Annales de Chem. et de Phys.*, Vol. 9, p. 57, cahier de septembre, 1818; *Oeuvres*, Vol. 2, Paris, 1868, pp. 627-636. See also Silberstein's *Theory of Relativity*, pp. 60-62.

"The opacity of the earth is not a sufficient reason to deny the existence of an ether current between the molecules, and one may suppose sufficient porosity in order that it may communicate to this fluid only a very small part of its movement.

"By aid of this hypothesis, the phenomenon of aberration is as easily conceived in the theory of undulations as in that of emission; for there results some displacement of the telescope while the light is moving through it: or, by my hypothesis, the luminous waves do not partake sensibly of the movement of the telescope, which I suppose directed toward the true place of the star: hence the image of this star finds itself behind the thread placed at the focus of the lens by a quantity equal to that passed over by the earth while the light passed through the telescope.

"It is now my affair to explain, on the same hypothesis, why refraction does not appear to vary with the direction of light rays due to terrestrial movement."

The concluding passages of this remarkable letter give a technical account of the consequences of his hypothesis, answering the question of Arago, and explaining why no change in the angle of aberration is discovered with a telescope filled with water or some other medium denser than air. In order to make his explanation Fresnel introduced the concept of a partial *ether drag* for optical media which has been the subject of much discussion and experiment since that time.

Putting the matter as simply as possible, Fresnel's theory assumes that the excess, and only the excess, of the ether contained in any ponderable body over that in an equal volume of free space is carried along with the full velocity of the body; the remainder of the ether within the space occupied by the material, like the whole of the free ether outside, is stationary with respect to the fixed stars.

Stating this proposition in other words, Fresnel believed that matter in some way imprisons a small amount of ether over and above what it is entitled to as a geometrical body in free space, and the ether thus entrained or captured belongs to and travels with it. This theory can be put a little more technically by saying that matter contains within it as its own ether an excess equal to $n^2-1$ of the total ether displacement, where $n$ is the index of refraction. Thus for air, where $n=1$, the excess of the ether is zero; for water the index of refraction equals $4/3$; hence excess ether imprisoned within this fluid is $16/9-1=7/9$.

### 7. *Fizeau's Proof of Fresnel's Theory.*

It was natural that careful experiments should be made to test the theory of Fresnel, and during the years 1871-1872 Sir G. B. Airy in the observatory of Greenwich made measurements on Gamma Draconis with a water telescope, as has already been related. His calculation of the constant of aberration agreed within experimental error with the constant determined by means of the air telescope. This, of course, was only an indirect proof of the correctness of the hypothesis of Fresnel.

We turn next to the beautiful experiment performed by H. Fizeau in 1851[20] and later repeated under very careful conditions with certain modifications by Michelson and Morley in 1886.[21] This was a period of intensive research in light and other electro-magnetic phenomena. The year 1886 saw the publication of the classical papers of Lommel on diffraction, which have already been mentioned above. Michelson and Morley were already engaged with the details of their famous experiment on ether-drift, which we shall describe in the next section.

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[20] "Sur les hypothèses relative a l'éther lumineux, et sur une expérience qui parait démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur interieur," *Comptes rendus*, Vol. 33, 1851, pp. 349-355; *Annales de Chemie*, Vol. 57, 1859.

[21] "Influence of Motion of the Medium on the Velocity of Light," *American Journal of Science*, Vol. 31, pp. 377-386. See also, Michelson: *Light Waves and their Uses*, Chicago, 1907, p. 155, and: "Relative motion of the earth and the luminiferous ether," *American Journal of Science*, Vol. 34, 1887, pp. 333-345; *Phil. Mag.*, Vol. 24, 1887, p. 449; *Journ. de Phys.*, Vol. 7, 1888, p. 444.

Two years later, in 1888, H. R. Hertz (1857-1894) announced his epoch-making discovery of the actual creation in the laboratory of the wireless waves predicted mathematically by Clerk Maxwell.[22] The foundations were being laid for the great events that were to come in the modern age of science.

Returning to the experiment of Fizeau let us see whether or not the ether is actually entangled in matter. Referring to Figure 4 we see that a beam of light from a narrow slit *O* was allowed to fall upon a plane mirror and hence was reflected in two beams, a lower and an upper one, around the indicated circuit, by means of a system of mirrors and lenses. The beams after their journey were reunited at the point *S* where they could be observed. Now in the course of their travels the two rays passed through a jacket through which water was forced at a high rate of speed. In the experiment the water ran at a rate of about eight meters per second or approximately 19 miles an hour. It is clear from the diagram that one of the beams would always go with the current and the other always against it, so that if the ether were actually dragged along by matter according to the law of Fresnel we should find the wave fronts arriving at different times at the point *S*. This fact would be exhibited by a set of interference fringes.

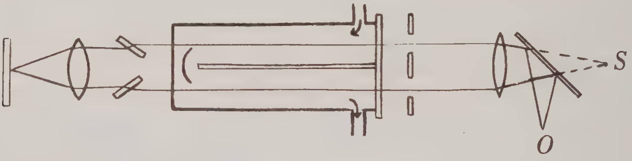

Figure 4.

If water has no effect upon the propagation of light, then the two beams of light will reunite at *S* without interference. If, however, Fresnel is correct and water exerts a drag upon the ether, then interference should be produced at *S* and we should find the plate crossed by interference fringes. The answer is that Fizeau obtained a sensible displacement when water moved two meters per second, and a measurable displacement for seven meters. Using eight meters per second and calculating the dragging coefficient for comparison, Michelson and Morley obtained the value 0.434 which is sufficiently close to the calculated value of 0.438 to vindicate the Fresnel theory.

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[22] *Wiedemann's Annalen,* Vol. 34, 1888.

## 8. *Is there an Ether Wind?*

With this background we are now upon the threshold of the momentous experiment from which the theory of relativity finally emerged. This experiment bears the names of A. A. Michelson (1852-1931) and E. W. Morley (1838-1923) who announced their results in the year 1887. The object of the experiment[23] was to determine whether or not the motion of the earth through the stagnant ether of Fresnel and Young could be determined. The analogy with a boat makes the object of the experiment clear. If one is upon a boat in the middle of Lake Michigan and wishes to determine whether or not he is in motion the simplest suggestion is to throw a log overboard and interpret its drift as a measure of the velocity with which the water is slipping past the vessel. The log used in the experiment about to be described was nothing but an interference fringe determined in the following manner.

From a source $S$ (Figure 5) a beam of light is sent into the ether in the direction of the motion of the earth. At the point $D$ part of the light passes through a small hole and part of it is reflected in a direction perpendicular to the motion of the earth. By means of mirrors the two parts of the beam are reunited at $D$ and reflected and transmitted to the point $O$, where they form an interference figure. By means of a very simple calulation in algebra one can show that the difference in length between the two paths is equal to $D \times v^2/c^2$, where $D$ is the distance of each reflecting mirror from the point

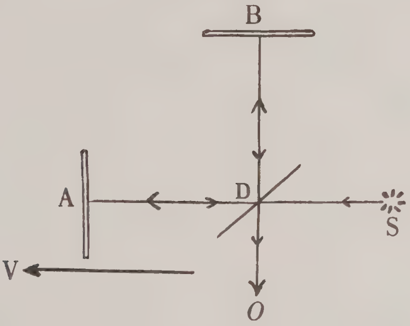

Figure 5.

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[23] "On the Relative Motion of the Earth and the Luminiferous Ether," *Philosophical Magazine*, Vol. 24 (5), 1887, pp. 449-463.

where the beams were divided, $v$ the velocity of the earth, and $c$ the velocity of light. If the entire apparatus is rotated so that the beam of light goes in a direction opposite to the motion of the earth, then the direction of the interference shift is changed and the total displacement is equal to $2Dv^2/c^2$.

In the original experiment the apparatus which we have described was set upon a massive stone floating on mercury placed in a cast-iron trough which was cemented into a low brick pier. The main difficulties encountered in the experiment were the distortion of the apparatus produced by rotating it and its extreme sensitiveness due to vibration. The latter was so great that the interference fringes were seen only at brief intervals even when the observers worked at two o'clock in the morning.

The result was as follows: Using as a measure the length of a wave of sodium light, the value of $2D$, about 22 meters, was equal to $3.7 \times 10^7$. Also the value of $v^2/c^2$ was $10^{-8}$. Hence the expected value of the shift was $3.7 \times 10^7 \times 10^{-8} = 0.37$ of a fringe width. In no case did the actual displacement of fringes exceed 0.02, and probably it was less than 0.01, which was less than one-fortieth of the expected value.

The experiment was repeated by E. W. Morley and Dayton C. Miller in 1905[24] with considerably increased accuracy, and they concluded that, if there were a shift of any kind, it was something like 0.0076, which is not greater than two one-hundredths of the computed value.

A much more elaborate set of experiments on this interesting question was undertaken by Miller in the years from 1921 to 1925 both at the famous observatory on Mount Wilson and at Cleveland.[25] Miller had not been satisfied with the conclusion reached in 1905 and was not entirely convinced that the small positive shift discovered was due entirely to experimental error. From these very intricate new measurements Miller concluded that there exists a positive shift, much smaller, however, than that demanded by the stagnant ether theory, but nevertheless a shift which varies both with the time of day and with the altitude at which the experiment is performed.

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[24] Extract from a letter dated Cleveland, Ohio, Aug. 5th, 1904, to Lord Kelvin, *Phil. Mag.*, Vol. 8 (6), 1904, pp. 753-754; also, "An Experiment to detect the FitzGerald-Lorentz effect," *Phil. Mag.*, Vol. 9 (6), 1905, pp. 680-685.

[25] "Ether-Drift Experiments at Mount Wilson," *Science*, Vol. 61, 1925, pp. 617-621; *Proc. of the Nat. Academy of Science*, vol. 11, 1925, pp. 306-314; "Significance of the Ether-Drift Experiments of 1925 at Mount Wilson," *Science*, Vol. 63, 1926, pp. 433-443.

Miller's experiment was an exceedingly arduous one, the variations studied very minute, and the results difficult to interpret. From a statistical study of the proper motions of the stars astronomers have arrived at the conclusion, subject of course to considerable error, that the solar system has a drift in space which is approximately twelve miles per second in the direction of the constellation Hercules. If this velocity is compounded with the earth's velocity of 18.6 miles per second in its orbit a change in the total drift of the earth is found at different times in the year, and this change should have revealed itself through variations in the interference fringes. The results obtained by Miller showed no such annual change; the assumption was made by him that astronomers were in error both with regard to the direction of solar drift and its absolute magnitude. Miller's explanation was, in fact, that the earth had a cosmic drift perhaps twenty times as great as that calculated statistically from the proper motions of the fixed stars, and this drift is toward the constellation of the Dragon rather than toward Hercules. The magnitude of this great velocity is thus sufficiently large to mask the slight variation that would be occasioned by compounding it with the smaller orbital motion of the earth.

Miller's investigation of the effects of altitude upon his experiment is not conclusive and perhaps somewhat contradictory. It will be recalled from the preceding chapter that the theory of Stokes and Planck assumes the ether to be stagnant in outer space but in motion at the surface of the earth. On this hypothesis we should expect to find the ether drift increasing with altitude. In 1925 Miller reached the conclusion: "The ether-drift experiments at Mount Wilson during the last four years, 1921 to 1925, lead to the conclusion that there is a relative motion of the earth and the ether at this observatory of approximately nine kilometers per second, being about one-third the orbital velocity of the earth. By comparison with the earlier Cleveland observation, this suggests a partial drag of the ether by the earth, which decreases with altitude."[26] This interesting conclusion, confirmatory of the postulates of Stokes and Planck, was denied the following year when Miller stated: "The evidence now indicates that the drift at Mount Wilson does not differ greatly in magnitude from that at Cleveland and that at sea-level it would probably have the same value."[27]

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[26] *Science*, 1925, p. 621.

The last statement of the preceding paragraph is also in agreement with an experiment made in 1925 by R. Tomascheck, at an altitude of 11,342 feet on the Jungfrau in the Alps.[28] Tomascheck was repeating the experiment originally performed in 1903 by Trouton and Noble,[29] who sought to determine a motion of the earth through the ether by measuring the torque exerted by the ether wind upon a suspended electrical condenser. His results, like those of the original experiments, were negative. The elusive ether refused to reveal itself in anything so tangible as a velocity with respect to moving matter. Further corroborating evidence of this conclusion is furnished in the negative results obtained in 1928 by R. J. Kennedy working under the direction of Michelson, who repeated his original experiment at the astronomical observatory at Mount Wilson with a much more sensitive apparatus than that originally used. The source of the small positive shift obtained by Miller was not revealed in this experiment.[30]

### 9. *Optical Proof that the Earth Rotates.*

One must not imagine, however, that all attempts to measure velocities fail, for there is one striking class of phenomena for which one can always expect to get positive results. These phenomena involve accelerated velocities. We have already examined in the last chapter the surprising fact that the absolute rotation of the earth can be detected within the closed walls of a building; we have tentatively advanced the proposition that the explanation of this cosmic phenomenon may be intimately connected with the reality of the energy of rotation. We have seen in the present chapter how the orbital motion of the earth is revealed in the constant of aberration where again we are dealing with a phenomenon in which a large sum of kinetic and potential energy is involved. Any experiment which has for its goal the absolute measurement of accelerated velocity appears always to give a positive result wherever such accelerations actually exist. It is a striking fact that the Foucault pendulum at the equator is powerless to detect the rapid velocity of matter upon this rim of the whirling earth, but at any other latitude reveals the velocity of the particles. An important experiment, which depended for its result upon the relative acceleration of particles at different latitudes was performed by A. A. Michelson and H. G. Gale at Chicago and reported in 1925.[31]

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[27] *Science*, 1926, p. 443.

[28] *Annalen der Physik*, Vol. 78, 1925, pp. 743-756.

[29] F. T. Trouton and H. R. Noble: "The Mechanical Forces Acting on a Charged Electric Condenser Moving Through Space," *Phil. Trans. of the Royal Soc.*, (A), Vol. 202, 1904, pp. 165-181.

[30] For this evidence and a comprehensive conference on the entire problem of ether drift see: "Conference on the Michelson-Morley experiment," *Astrophysical Journal*, Vol. 68, 1928, pp. 341-402.

The Michelson and Gale experiment was performed by sending beams of light in different directions through a tube constructed in the form of a rectangle. The tube which was more than a mile in total length was first reduced to a partial vacuum. The two beams of light were then reflected around the rectangular circuit in opposite directions and reunited so as to form an interference fringe.

Now this experiment might be expected to give a negative result, if the Stokes-Planck ether described in the preceding chapter were the true picture of reality, for the ether trapped within a heavy pipe buried in the earth might very well be dragged along with the system. Rays of light in such a stagnant medium would reunite without interference. On the other hand if the postulate of an ether wind passing through even the most solid of material substances were the correct one, then we might expect, were it not for the Michelson-Morley experiment, a positive result exhibited by an interference fringe, because the light beam in one direction would be retarded over the other, due to passing over equal paths in different latitudes. At the lower latitude the slower beam would move against the velocity of the earth and at the higher latitude with it. This differential, though small, was sufficiently large to yield a theoretical shift of 0.236 of a fringe or 1/200,000 of an inch, an amount within experimental reach.[32]

The value actually found by the experimenters was 0.230 of a fringe, that is to say this figure was approximately the average of the 300 odd readings made. Those, however, who would see in this experiment a contradiction with the original Michelson-Morley result must reflect that in the Michelson-Gale experiment the velocity measured was one associated with the acceleration of matter, since the effect made use of was the change of velocity with latitude. In other words *the result achieved was another vindication of the statement*: *"the earth rotates," and not in any sense a demonstration* of the existence of an ether wind.

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[31] "The Effect of the Earth's Rotation on the Velocity of Light," *The Astrophysical Journal*, Vol. 61, 1925, pp. 137-145.

[32] For a detailed mathematical analysis of the considerations which enter here see Silberstein's *Theory of Relativity* (loc. cit.), p. 376 et seq. See also, *Phil. Mag.*, Vol. 8 (6), 1904, p. 716 and *Journal of the Optical Society*, Vol. 5, 1921, p. 291.

### 10. *The Paradoxical Ether.*

We are now upon the threshold of relativity. This theory was originally devised to explain the interesting paradox which we have exhibited in the preceding paragraphs. The constant of aberration is clear proof that the earth has a velocity with respect to the framework of the fixed stars; the Michelson-Morley experiment is similar evidence that the earth is at rest with respect to the postulated ocean of ether which conveys the vibration of light.

In order to extricate ourselves from the difficulty we might postulate an ether dragged by the earth and stagnant in the interstellar spaces, but to make this concept acceptable to rational intuitions we must add the irrational assumption that the density at the surface of our planet is 60,000 times as great as that remote from gravitating matter. "Truth," said Mark Twain, "is the most precious thing we have; therefore let us use it sparingly." But even if this fanciful density were to be assumed, the experiment of Michelson and Gale, with their pipes buried in the ground, or the experiment of Tomascheck in the high Alps, is strong evidence that the dragging theory of the ether must be abandoned.

Fresnel enjoys the uncanny reputation of almost flawless prediction and his theories have withstood the careful scrunity of those who have erected the structure of relativity. With Fresnel the ether was a stagnant ocean, particles of which flow unimpeded through solid matter as easily as wind rushes through the branches of a tree, save that matter itself imprisons as part of its corporeal substance a quantity of this universal entity. This theory is in contradiction with no experiment except that the ether wind eludes discovery.

Do we yet believe in the reality of the ether? Whenever we try to give it material properties, weigh it, calculate its density, endow it with elasticity and rigidity, measure its flow through the pores of solid matter, we are completely baffled. But when we look with reflective glance upon the beautiful rings of a diffraction pattern and view a photograph taken within the shadow of a ball of steel, is our disbelief not greatly shaken? How can we expain these mysteries?

Does one now believe that physical science is confined to the pursuit of decimal points and the exploration of a tangible reality by instruments of precision? Does one not see in this mysterious impasse a bond with metaphysics? Is one justified in stating that ether is or is not a real thing because its substantive properties have seemed to vanish and only that single property of undulation remains by means of which one explains the phenomenon of diffraction?

This scrutiny of the assumptions of physics has been attempted recently by several philosophers, but none of them essays the task more ably than N. R. Campbell, whose treatise entitled *Physics, the Elements* published in 1920, is singularly free from either formulas or description of experiments. Campbell puts the matter squarely in the following paragraph:[33]

"But is it true that metaphysics can be avoided wholly in an attempt to probe to the foundations of science? (Now and henceforth I propose to use the word metaphysics, not as a mere term of abuse, but to denote the study which those who accept the status of metaphysician think valuable. So far as I can make out, the study consists in the investigation of reality and existence.) At some stage in our inquiry we must stop and accept judgments without argument; is it certain that these judgments will not be found to be metaphysical? Or again, are we sure that the process of reasoning by which we develop our conclusions from these fundamental judgments does not depend on the acceptance of doctrines that are distinctively metaphysical?"

Campbell accepts the general opinion that science does not in any way depend upon metaphysical concepts, and his answer may very probably be predetermined by his definitions. But the concept of the ether, existing for Maxwell in a set of mathematical equations, and for those who look at diffraction, existing as a scaffolding for the application of the principles of Huygens, Young, and Fresnel, and for those who look upon the negative result of the Michelson-Morley experiment, existing as an historic wraith haunting the laboratories of light, this concept of the ether presents the most elegant of metaphysical questions. Who would want to settle the delicate problem and either give to ether an objective reality or deny it?

Perhaps these speculations leave you feeling like Alice. "Fan her head!" the Red Queen anxiously interrupted. "She'll be feverish after so much thinking."

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[33] *Physics, the Elements.* Cambridge, 1920, p. 11.

# CHAPTER 5.

## TIME, SPACE, AND THE FOURTH DIMENSION

"A downright paradox, no doubt,
A mystery remains alike to fools and sages."
Goethe's *Faust.*

"She's in that state of mind," said the White Queen, "that she wants to deny *something*—only she doesn't know what to deny!"
Lewis Carroll in *Through the Looking-Glass.*

### 1. *The Ether Paradox.*

In the last chapter we arrived at a very perplexing place in the development of physics. We reflected upon the phenomenon of aberration and were led to the belief that light was an undulatory motion in the luminiferous ether and that the ether itself was so tenuous that it passed through solid matter like wind through the branches of a tree. We were told by Fresnel that ether condenses within atoms of ponderable matter and is carried along by them in a manner that can be calculated as soon as we know the value of the coefficient of diffraction. We then discovered that the most delicate instrument in the world could not detect a significant amount of relative velocity between the moving earth and the stagnant ether. Well do we understand the lamentations of Odysseus "for on the one hand lay Scylla, and on the other mighty Charybdis in terrible wise sucked down the salt sea water."

Let us state the problem otherwise. Suppose that we are sailing in a vessel on Lake Michigan during a rain storm and as we tack across the lake first in one direction and then in another we notice that the rain appears to come from different quarters. One would naturally interpret this to mean that our boat is in motion first in one direction and then in another, even if we have no other means of determining its motion. But in order to have a check upon our conclusion suppose we toss a log overboard and watch it fall behind the ship. If the log doesn't show a relative motion with respect to the boat we will be very much surprised and will experience the same perplexity that science felt in the years following the Michelson-Morley experiment. The constant of aberration was almost conclusive evidence that the earth was in motion with respect to the ocean of ether postulated for the undulation of light, but the experiment of Michelson and Morley appeared to show that the earth was at rest with respect to this same ocean of ether surrounding it.

How can we explain the paradox of an earth at rest with respect to the ether ocean and yet in motion? If we are to be true scientists in the sense of the dictum of Poincaré we must seek for that "harmonious order of the parts . . . which a pure intelligence can grasp." In another place he says that if there is one explanation of a given body of natural phenomena, then there is an infinite number of such explanations. "But every proposition may be generalized in an infinity of ways. Among all the generalizations possible we must choose, and we can only choose the simplest. We are therefore led to act as if a simple law were, other things being equal, more probable than a complicated law. Half a century ago this was frankly confessed, and it was proclaimed that nature loved simplicity; she has since too often given us the lie. Today we no longer confess this tendency, and we retain only so much of it as is indispensable if science is not to become impossible."[1]

The most classical example of the postulate of simplicity is the warfare between the followers of Ptolemy and those of Copernicus with regard to the motion of the planets. Ptolemy asserted that the earth was the center of the universe and that all the planets and the sun moved around it. However, in order to explain their motions he was led into complex paths. The motion of the sun around the earth would appear to be an ellipse, but the motion of Jupiter would seem to consist of a series of loops. The apparent, or geocentric motion, of the planets is a combination of two motions, the first of which is that of a body moving once a year upon the circumference of an ellipse of small eccentricity equal to the earth's orbit, and the second that of the center of the ellipse carried around the sun in the real orbit of the planet and in the planet's period as shown in Figure 1. A motion of this kind is called epicycloidal.

The most powerful argument in favor of the Copernican system is not the criterion of reality, but that of simplicity. Its appeal rests upon the fact that the complexities of the Ptolemaic picture disappear when we assume that the sun is the center of the planetary system and all the planets move about it. The complicated epicycles become ellipses in the familiar Copernican system.

But suppose that we ask which of these systems is the correct one. Naturally we choose the simpler because there is something in the rational point of view which makes us want to assume that nature is fundamentally simple. Without the check of the angle of aberration which we discussed in the last chapter, or a knowledge of the parallaxes of the stars, it would be difficult to maintain that one system was necessarily more rational than another, without first invoking the postulate of simplicity. As a matter of historical fact Tycho Brahe was led to choose the Ptolemaic system rather than that of Copernicus, first on the basis of scriptural authority, and second because the parallax of the stars was beyond the refinements of his telescope to observe.[2] Today, as Poincaré would say, we should be under the necessity of choosing the simplest of the hypotheses among all those that might be made which were not in contradiction with experiment.

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[1] From "Science and Hypothesis" in *The Foundations of Science* (loc. cit.), p. 120.

It is a matter of more than casual historical interest to observe that, long before Ptolemy and eighteen centuries before Corpernicus, Aristarchus (c. 280 B. C.) had advanced the heliocentric theory of the solar system. This was rejected by Archimedes and his contemporaries for the sound reason that no shift was observed in the position of the fixed stars. It was rejected by the philosophers on other less tenable grounds, namely, that the theory seemed to deny to the earth, and thus to man himself, a central place in the universe. Thus we read in Plutarch:[3]

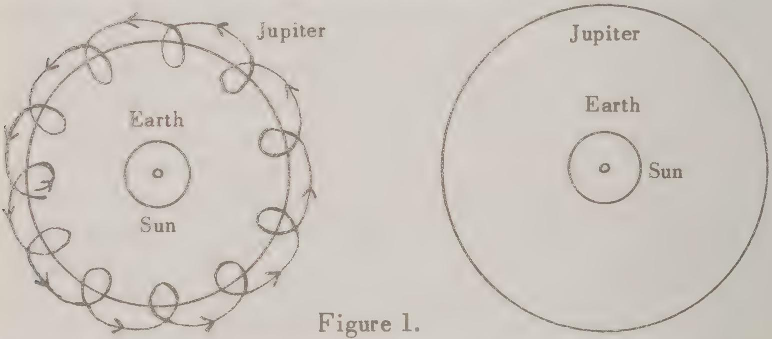

Figure 1.

Comparison between the representations of the orbit of a planet as viewed from the heliocentric theory of Copernicus and from the geocentric theory of Ptolemy.

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[2] To be quite accurate we must observe that the system finally adopted by Tycho was a mixture of those of his predecessors. The sun moved around the earth in his cosmology but the other planets moved around the sun. See Russell-Dugan-Stewart's: *Astronomy* (Revision of Young's *Manual*), 1926, Vol. I, p. 245.

[3] *On the Face Appearing in the Orb of the Moon*, Sec. 6.

"Only do not, my good fellow, enter an action against me for impiety in the style of Cleanthes, who thought it was the duty of the Greeks to indict Aristarchus of Samos on the charge of impiety for putting in motion the Hearth of the Universe, [i. e., the earth], this being the effect of his attempt to save the phenomena by supposing the heavens to remain at rest and the earth to revolve in an oblique circle, while it rotates, at the same time, about its own axis."

The remarks that have been set down above with regard to the formation of hypotheses may seem trivial to some, but they contain, nevertheless, an important point of view. The toilers in the laboratory are on the road to the experimental truth about phenomena, but experiments are difficult to make and can be brought to bear only upon certain ideal phases of the problem. Hypotheses are necessary as supplementary aids to truth, guides, so to speak, to further investigation. Their implications, often wholly unsuspected in the original statement, must be investigated by mathematical equations. Consider, for example, the problem of the age of the earth, about which there is now a reasonable accord between physicists, mathematicians, astronomers, and geologists. Lord Kelvin, one of the eminent mathematical physicists first to investigate the problem, assumed that the earth was a molten ball with a temperature throughout of 3,900 degrees centigrade, and made other auxiliary hypotheses with regard to thermal coefficients.[4] On the further assumption that the earth's surface was maintained at approximately zero temperature he sought to answer the question how long a time would elapse before the crust would be in a sufficiently stable condition for the oceans to form. His problem was to find from his hypotheses how long it would take the earth to cool to that point at which the geothermal gradient would have its observed value of one degree in 27.76 meters. Was Lord Kelvin's answer of 100,000,000 years an incorrect one? By no means! This conclusion followed inevitably from his hypotheses; the answer could be modified only by manipulating the original assumptions. Simplicity and physical intuitions guided Kelvin's choice. In this later period, with new assumptions as to the radio-activity of the earth's crusts, and other concepts of energy, a different answer to the problem has been obtained. The assumptions in the one case made the guess of Kelvin; the assumptions in the other lead to our modern estimate.

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[4] *Math. and Physical Papers*, Vol. 3, p. 295. See also *Smithsonian Report*, 1897, p. 337. For a resume consult Becker: *Smithsonian Miss. Coll.*, Vol. 56, No. 6, 1910.

### 2. *The Contraction Hypothesis.*

With this introduction into the philosophy of explanations, let us examine the motion of the earth with speculations as to a possible explanation of the fundamental disagreement between aberration and the Michelson-Morley experiment. Wherein is the flaw in our hypotheses? Is it in our notion of an ether wind or is it bound up more fundamentally with our concepts of velocity?

The simplest explanation that presents itself is to deny the evidence of the original ether drift experiments, but the skepticism of Miller has not been justified by his subsequent repetition of the experiment. The critique of this argument made in the last chapter reduces to a small figure the probability that assumption of experimental error is the ultimate explanation.

A second argument, advanced somewhat facetiously to be sure, is that nature is irrational. Why should we not expect to reach contradictions in nature? What makes us so certain that there is a rational explanation for phenomena? When we read the long history of human thought in its struggle against superstition it is clear that man in the mass is more emotional than rational. Perhaps there is a basic cause for this behaviour. As a matter of fact is seems as difficult to explain why we expect rationality in the phenomena of nature as it is to explain our belief in the laws of probability. There is an intimate connection between these two things. Our belief that the sun will rise tomorrow is a rational belief merely because the records of history as far back as we can trace them contain references to this phenomenon. The probability that the sun will not rise upon a particular day in the not too distant future is certainly incalculably small. And yet who would want to predict for a million years hence? We look upon the records of the sky and we find there occasional references to those great bursts of light caused by the appearance of cosmic cataclysms. The astronomical journals merely state that a "nova" has been observed. The new star flares for a few days and is gone, but there is indelibly written upon our minds a personal application in that magnificent spectacle. Who knows the mystery of our sun?

The third explanation was made early by G. F. Fitz Gerald (1851-1901) and H. A. Lorentz and is called the contraction hypothesis.[5] From it the latter derived his famous equations of the Lorentz transformation, which were important stepping stones that Einstein used in reaching his celebrated theory.

Hendrik Antoon Lorentz was born in Arnhem, Holland in 1853 and died in 1928. His *magnum opus* was his mathematical study of the phenomena of the electron, that tiny particle with its negative charge of electricity which is thought to be one of the ultimate constituents of matter. The electron is so small that it has to be studied by means of the mathematical interpretation of certain experiments on the behaviour of light, X-rays, wireless radiations, etc. One of these experiments explained by Lorentz was the so-called *Zeeman effect.* If a source of light is placed in a strong magnetic field and the spectrum examined, it will be found that lines which were solid in the unmagnetized spectrum, have now been divided into several parts or components. The great achievement of Lorentz was to trace this phenomenon to its source and to show how a modification in the vibrations of the electrons of the radiant matter was created by the magnetic field.[6] These investigations shared with the work of Zeeman the Nobel prize for physics in 1902. His contributions to scientific literature are the following: The theory of electrical and optical phenomena in moving bodies; the theory of electrons; and eight volumes of lectures on modern physics.

What was the explanation of Fitz Gerald and Lorentz? It was very simple. When matter is in motion with respect to the ether it becomes shorter than when it is at rest. Nature is in a great conspiracy to prevent us from discovering our motion through the ethereal framework of space. We move and the atoms of our substance shrink together in such a way as to defy the detection of absolute velocity. The hypothesis can be stated in terms of a simple formula which says that if a body at rest in the ether has a length $L$, its length while moving with a velocity $V$ is equal to $L\sqrt{1 - V^2/c^2}$, where $c$ is the velocity of light. What a strange hypothesis! And why the mysterious appearance of the velocity of light? Just because our original experiments were performed by investigating the motions of this elusive entity, is that sufficient reason why its velocity should play such a fundamental part in the conspiracy of nature? Does this hint at a fundamental constant in the structure of the universe? Are there numbers, perhaps, which bound other activities of nature just as 460 degrees below zero bounds the heat content of material bodies? Are we justified in suspecting that this apparently limitless universe of time and space and matter may, conceivably, be bounded in other of its material dimensions?

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[5] Sir Oliver Lodge communicated the suggestion of Fitz Gerald to *Nature*, Vol. 46, 1892, p. 165. It was adopted by Lorentz in a communication to the Amsterdam Academy. *Verslagen d. Kon. Ak. van Wetenschappen*, 1892-3, p. 74. See also, *Theory of Electrons*, p. 195.

[6] See *Theory of Electrons*, Chap. 3.

Let us examine the contraction hypothesis further. The appearance in the formula of the velocity of light explains why this phenomenon has escaped the observation of early experimenters. The solar system with its velocity in space, probably not greater than fifty miles a second, would not be appreciably affected by the contraction, but for an electron with a velocity in the neighborhood of half that of light, the contraction would amount to an eighth of its total length. It is thus a matter of relative velocity.

### 3. *The Lorentz Transformation.*

Lorentz pushed these ideas to their logical conclusion and derived what are called the equations of the Lorentz transformation. These equations are really quite simple when one understands them, which leads one to the remark that such a statement is merely a critique of understanding. It seems to be a perversity of the human mind to befog difficulties by calling them easy. There are numerous examples where mathematicians start with the following encouraging remark: "It is easily proved," or perhaps, if the difficulty is particularly bad, "it is very easily proved" or "it is obvious." Laplace, sometimes called the French Newton, was not noted for the clarity of his writing. J. B. Biot who assisted him in revising his celestial mechanics for the press once appealed to him to clear up some obscure analysis in a proof, and after trying in vain to reconstruct his argument, Laplace is reputed to have said: "I am confident that the result is correct; let us therefore say that the conclusion follows obviously from the premises." (Il est facile de voir).[7]

In order to explain the Lorentz transformation let us first consider such an object as the top of a table and let us give it a location in space and time. This naturally leads us to the concept of a *fourth dimension* where the fourth dimension is to be regarded as time.

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[7] See F. Cajori: *A History of Mathematics*, 2nd ed. New York, 1919, p. 262.

This idea is not to be looked upon as wholly a contribution of the present century, for it goes back a long distance into the history of scientific thought.[8] One of the first references to time as the fourth dimension is found in the article on "Dimension" written by J. d'Alembert (1717-1783) for Diderot's famous *Encylopédie.* In this article d'Alembert remarks:

"I have said above that it is not possible to conceive of more than three *dimensions.* A witty man of my acquaintance believes that one may, however, regard duration as a fourth *dimension*, and that the product of time by solidity, will be in some manner a product of four *dimensions*; this idea can be debated, but it has, it seems to me, some merit, even if it serves only that of novelty."

It has been conjectured that the "homme d' esprit" mentioned by d'Alembert may have been J. Lagrange (1736-1813), the great analyst, who was then, however, only eighteen years of age. At any rate it is significant that Lagrange later, in making application of his theory of analytic functions to the subject of mechanics, gave a fruitful interpretation of the geometrical character of time which modern science now employs. Lagrange remarked:[9]

"Let us now employ the theory of functions in mechanics. Here the functions are essentially with respect to time, which we shall always designate by $t$; and as the position of a point in space depends upon the three rectangular coordinates $x$, $y$, $z$, these coordinates, in the problems of mechanics, are regarded as functions of $t$. Thus one is able to look upon mechanics as a geometry in four dimensions, and the analysis of mechanics as an extension of the analysis of geometry."

Returning to our consideration of the table top, let us place a clock upon it and thus endow it with a place in the four dimensional manifold which creates the framework of objective nature. The German mathematician H. Minkowski (1864-1909) called the tetrad of values $x$, $y$, $z$, $t$ a *world point*.[10] That is to say a point of the world is to be regarded as a material particle at a certain instant of time. Suppose that we watch this particle in its travels throughout the period of its existence. The variations in $x$, $y$, $z$, and $t$ would generate what we might call a *world line*. Suppose further that a large number of these material particles are collected together to form a body of finite dimensions in space and that we draw through each particle its world line. The collection of world lines thus made constitutes a *world tube*, or as Minkowski first called it, a space-time filament. It is interesting to try to visualize the world tube for such an object as a firecracker. If one sees these world lines in the four dimensional manifold before the match is applied they are all compact. But at the instant of explosion a great bubble is generated in the tube, whose film spreads farther and farther apart as we go along the time dimension of the figure and as the particles are scattered to the four winds. Those to whom these ideas are novel might reflect upon the rather neat way in which H. G. Wells has put the matter in his romance, *The Time Machine* (1898), where he makes his Time Traveler say: "*There is no difference between Time and Space except that our consciousness moves along it.*"

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[8] See for example an article by R. C. Archibald: "Time as a Fourth Dimension." *Bulletin of the American Mathematics Society*, Vol. 20, 1914, pp. 409-412.

[9] *Théorie des fonctions analytiques.* Paris, 1813, 1st section of part 3: "Application de la théorie des fonctions à la mécanique." *Oeuvres*, Vol. 9, 1881, p. 337.

Returning once more to our table top, let us try to calculate its world point. But this, we shall discover, is a difficult thing to do, because somehow or other we think that we must define the point in terms of something else, which for convenience we call axes. Let us explore the matter further. Suppose we first refer the position of the table top to the axis of the earth, whose position we know in something resembling an absolute way from the Foucault pendulum experiment. But we also know from the constant of aberration that the earth is revolving around the sun; perhaps it might be better to transfer our original axis to a more stable one passing through the diameter of the sun and perpendicular to the ecliptic or path that the earth follows. But even that axis won't do. As we make careful observation on the stars we are surprised to find that those in the direction of the constellation Hercules seem, on the average, to be moving in our direction, while the stars at the opposite pole are apparently receding. The observation is naturally interpreted to mean that the sun itself is not motionless in space, but has a drift in the general direction of the constellation Hercules. Hercules can be observed by those who care to know the direction of our cosmic passage just below the dragon and the little bear, according to the scheme shown in Figure 2.

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[10] "Raum und Zeit." *Physik. Zeitschrift*, Vol. 10, 1909, pp. 104-111; *Jahresberichte der Deutscher Math-Vereinigung*, Vol. 18, 1909, pp. 75-88; also *Werke*, Vol. 2, 1911, pp. 431-444.

We can't, therefore, use the sun as a fundamental axis so we turn to the frame-work of the fixed stars. Since these so-called fixed stars are themselves in motion, it again becomes a matter of statistical averages, based upon the proper motions of celestial objects, to locate a set of axes passing through the center of gravity of the stars. From present revelation in astronomy we believe that the stars belonging to our galaxy, that is to say all the stars visible to our telescopes, form a spiral nebula, the constituent particles of which are probably in motion with respect to a center of gravity. Let us take this center perhaps 33,000 light years from us, to use a recent estimate,[11] and erect a set of axes there. But unfortunately we have not yet reached

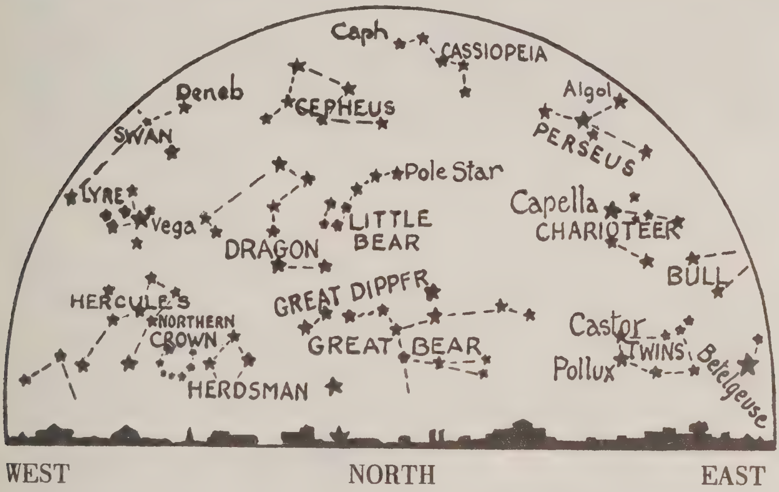

Figure 2.
Position of the constellations and the stars
about 8 o'clock on a November evening.

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[11] Harlow Shapley in his book *Star Clusters,* New York, 1930, p. 177 gave this center at approximately 52,000 light years from us. Recent estimates have reduced this figure to around 10,000 parsecs, or 33,000 light years approximately. See W. M. Smart: *Stellar Dynamics,* Cambridge, 1938, p. 389.

the solution of our problem because modern astronomers report the discovery of about a million such galaxies within the reach of our telescopes and certainly our galaxy is in motion with respect to them. Hence we must move again and erect our axes at the center of gravity of the super galaxy of the spiral nebulae. But this super galaxy may in turn be but an atom in another galaxy of higher order and so we must move on and on without getting anywhere.

We have had enough about absolute axes in space. Lorentz, however, thought of these axes as being fixed in the great ocean of ether. Let us bear with him for a while. Let us suppose that the ocean of ether is at rest with respect to the framework of the fixed stars. Let us suppose, further, that our table is fixed with respect to the axes that we have chosen. Let us place an object upon the table. We can then determine its world point to be *x*, *y*, *z*, and *t* with respect to the axes that we have chosen and the clock beside it.

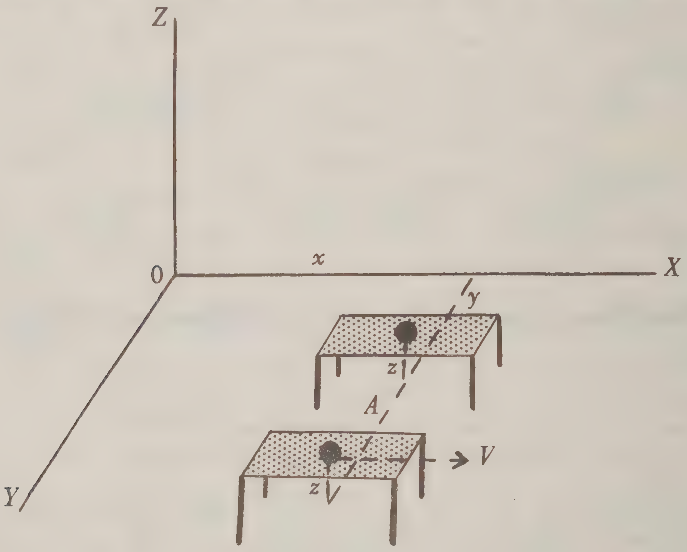

Figure 3.

Let us now place a second table beside the first and set it upon wheels so that it can be moved with whatever velocity we may choose to give it. Now suppose that we place a second object on the second table and a clock beside it. The two tables are identical in space and time except that one is slightly displaced, say a distance *A*, in the *y* direction, as shown in Figure 3. This displacement we shall disregard, since it has no bearing upon the argument of Lorentz. The second object has the same world point as the first excepting for $y$, that is to say $x' = x$, $y' = y + A$, $z' = z$, $t' = t$, where the primes refer to the second point.

Now suppose that the second table is set in motion with a velocity $V$ in the direction $x$ with respect to the first. We fix our attention upon the world point of the object on the table. What happens to it? Sir Isaac Newton, who was not concerned especially with these ideas, would say that the new world point of the second object is merely $x' = x + Vt$, $y' = y + A$, $z' = z$, $t' = t$. These simple equations merely state in the language of common understanding that the second object is in motion with respect to the first.

Not so, says Lorentz, for the time and space of the moving object contract according to the laws of the postulated conspiracy of nature. The new world point of the second object is not that of Newton or of rational intuitions, but is modified by the hypothesis of contraction into the following:

$$x' = \frac{x + Vt}{\sqrt{1 - V^2/c^2}}, \quad y' = y + A, \quad z' = z, \quad t' = \frac{t + Vx/c^2}{\sqrt{1 - V^2/c^2}},$$

where $c$ is the velocity of light.

What an amazing speculation this brings to view! What is the meaning of the constant $c$ which appears so strangely in these equations? What has the speed of light to do with the movement of one table top with respect to another through the mysterious ether? How can the speed of light be related to the framework of the fixed stars?

If we regard these transformations more closely we are surprised to see what would happen if we start the second table with the speed of light. The world point immediately jumps to infinity, which is merely a nice mathematical way of saying: "Hold on, you can't make that table go as fast as light, because nothing except light can go that fast." In other words *the speed of light is the ultimate relative velocity of matter.*

But see what has happened to time! If one thinks it strange that space should be related to the velocity of light, what opinion must one have of time? Time has become all tangled up with space and the mysterious velocity of light; the only way to return to normalcy is to set $V$ equal to zero and stop the table top. Of course all of this is difficult to believe, but we must reflect that a theory which would explain why the ether moved past the earth in one experiment and then did not move in another, would of necessity differ from our ordinary intuitions.

What Lorentz really meant was this. If we assume his relationship to exist between the space and time variables of two objects, one moving with respect to another, then the Michelson-Morley experiment can be explained as a contraction of matter in the direction of its motion. Two beams of light, one parallel to the motion of the earth through space and the other perpendicular to it, will be reunited without creating an interference fringe. In other words the distortion of matter, due to its motion through space and calculated according to the space and time transformations of Lorentz, is just enough to account exactly for the disappearance of the small amount of time which should have exhibited its existence as an interference fringe in the famous experiment.

A truly magnificent confirmation of the views of Lorentz is to be found in a mathematical experiment which can be made with Maxwell's equations of electromagnetic radiations. One will recall from the historical outline of the preceding chapter that Maxwell, years before the excitement arose regarding the Michelson-Morley experiment, had succeeded in defining a mathematical ether the reality of which was more in Maxwell's equations than in any substantial model. It is quite evident that no farseeing physicist of that day could have anticipated the theory of the Lorentz transformation in setting up his mathematical structure. The equations of Maxwell came directly and simply out of considerations of the fluxes of the Faraday tubes of electric and magnetic forces.

Let us now change these celebrated equations of the English physicist from one set of time-space variables to another.[12] In other words let us see what change in the experiments with electromagnetic radiations might be anticipated by an experimenter in a moving laboratory according to the equations of Maxwell. The mathematical problem is not difficult to perform, but the result has a profound implication. The Maxwell equations in terms of the new time and space are the same as the Maxwell equations in terms of the old four-dimensional manifold. In other words, the Lorentz transformation was lurking within these magical symbols all the time and might have been anticipated years before, had anyone thought to ask the question forced so spectacularly to attention by experimental evidence. This is a powerful argument in favor of the Maxwell theory and a mighty bridge from the old physics to the new. The optical properties of a laboratory moving with unaccelerated velocity through space are identical with those of a laboratory at rest, provided, of course, there can be any meaning attached to these common terms "moving" and "at rest."[12]

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[12] For a mathematical treatment of this invariance, see Silberstein: *Theory of Relativity*, Chap. 8, in particular note 1, p. 225.

### 4. *Einstein and the Subjective View.*

With these strange ideas before us, we now come to the fourth explanation. The man who conceived this explanation is Albert Einstein.

Einstein was born of Jewish parents at Ulm, Württemberg, May 14, 1879. His boyhood was spent at Munich, where his father, who owned electro-technical works, had settled. The family migrated to Italy in 1894, while Albert remained in Switzerland and attended a cantonal school in Aaron. He attended lectures while teaching mathematics and physics at the polytechnical school at Zurich until 1900. After a year as tutor at Schaffhausen he was appointed examiner of patents at the patent office at Berne, where, having become a Swiss citizen, he remained until 1909. He took his Ph. D. degree at Zurich and during this period began to publish his remarkable papers. It was in 1905 that his fundamental memoir, establishing the special theory of relativity, appeared.[13] In 1909 he was appointed extraordinary professor of theoretical physics at Zurich and in 1911 accepted the chair of physics at Prague, only to return to Zurich as full professor the following year. In 1913 a special position was created for him in Berlin as director of the Physical Institute. His relativity theory, begun in 1905, was accepted in Germany by 1912. It was followed in 1915 by that magnificent monument of human genius, the general relativity theory.[14] Although in some ways this appears to be the most significant of his theories, his work on the photo-electric and other phenomena of the quantum theory, in the opinion of many, has been equally fundamental and far-reaching. He has also contributed ideas of basic importance to the theory of the Brownian movement of molecular particles, and to the understanding of the nature of specific heats in the mysterious region of low-temperature phenomena. He was awarded the Nobel prize in 1921. In 1929 a short paper published by the Prussian Academy of Sciences attempted to bring the forces of electricity and magnetism within the geometrical scope of his general relativity theory.[15] But this proved to be unsuccessful. Another attempt to solve the riddle was published in 1950.

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[13] "Zur Electrodynamik bewegter Kõrpern," *Ann. der Physik*, Vol. 17, 1905, p. 891.

[14] "Die Grundlage der allgemeinen Relativitatstheorie," *Ann. der Physik*, Vol. 49, 1916, p. 769. With M. Grassmann: "Entwurf einer verallgemeinerten Relativitatstheorie und einer Theorie der Gravitation," *Zeitschrift der Math. Phys.*, Vol. 63, 914, p. 215.

In 1933 Einstein came to the United States as a life member of the Institute of Advanced Study in Princeton. In 1939 a letter by him on the desirability of encouraging research in the field of nuclear physics led the government to the appointment of an "Advisory Committee on Uranium," which resulted finally in the development of the atomic bomb. Einstein has thus lived to see the terrible vindication of his most famous equation, discussed later in this chapter, which unites matter and energy in a single concept. It is one of those strange ironies of fate that the man who expressed so deep-seated a hatred of war in his volume *Why War?*, published in 1933, should be the author of the theory which lies at the heart of the most fearsome weapon ever devised by man.

Pondering on the strange situation revealed by the Michelson-Morley experiment and dissatisfied with the *ad hoc* explanation offered by the contraction hypothesis of Fitz Gerald and Lorentz, Einstein in 1905 set forth a radically new idea, which is now generally known as the *special theory of relativity*. This is to distinguish it from the more comprehensive doctrine by means of which Einstein later offered an explanation of the mysteries of gravitation.

To many who approach the theory of relativity the basic assumptions often seem strange and mysterious. They are contrary to ordinary modes of thought. Popular writers on the subject have stressed too frequently the paradoxes and emphasized unrealities, but there is nothing essentially bizarre in the theory except its overthrow of conventional belief. If one has sufficient mental agility to take the equations of Maxwell as a definition of an objective ether, or if one does not find the reality of action too difficult to assume, then the postulates of Einstein are not beyond his reason. We must always remember that we are inquiring into the acts of nature with a mind which has evolved through ages of superstition and blind belief. Would it not be an amazing coincidence to find that intuitions evolved through these ages of ignorance should finally be found to accord with the verities of physical law? One would then surely incline to the pessimism of Mark Twain who concluded: "It is all a dream—a grotesque and foolish dream. Nothing exists but you. And you are but a *thought*—a vagrant thought, a useless thought, a homeless thought, wandering forlorn among the empty eternities!"

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[15] "Zur Einheitlichen Feldtheorie," *Sitzungsberichte der Preuss. Akad. der Wissen.*, Phys.-Math. Klasse, 1929.

In an earlier chapter we have broached this point of view, but it will be worthwhile to emphasize it further. As we look about us the universe steals in through the medium of our five senses and by some strange psychological process the world points and the world lines are catalogued in our brain. Out of this assemblage of points and lines we make an ordered arrangement which we think of as rational belief. We see a penny tossed and notice that it falls either heads or tails. Without making a single experiment we arrive at the conclusion that if 10,000 pennies were tossed, then approximately 5,000 of them will be heads. More than that, without performing a single experiment, we can predict with high probability that the limits of deviation will be 4,900 and 5,100. The point is this that we are projecting upon an objective world of pennies and penny-tossing a subjective creation. Without the necessity of experimental check we create a set of laws expressed in mathematical language which are to apply to the world of nature. How do we know that this mathematical picture gives us the true aspect of nature?

The answer to this question is that it doesn't. Paradoxical as it may seem, there are no laws in nature that rest upon a more secure foundation logically than the laws of chance. In one of the most beautiful documents in the literature of mathematical physics Clerk Maxwell has developed much of the subject of thermodynamics by the application of the laws of chance to a collection of spherical molecules in random motion. And yet, carried far enough, we find that the laws of chance lead us to the following belief, which we quote from the treatise of J. H. Jeans: *The Dynamical Theory of Gases.*[16]

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[16] 3rd ed., 1920, p. 181.

"To borrow an illustration from Lord Kelvin, if we have a bar of iron initially at uniform temperature, and subject neither to external disturbance nor to loss of energy, it is infinitely probable that, given sufficient time, the temperature of one half will at some time differ by a finite amount from that of the other half. Or again, if we place a vessel full of water over a fire, it is only probable, and not certain, that the water will boil instead of freezing. And moreover, if we attempt to boil the water a sufficient number of times, it is infinitely probable that the water will, on some occasions, freeze instead of boil. The freezing of the water, in this case, does not in any way imply a contravention of the laws of nature: the occurrence is merely what is commonly described as a 'coincidence,' exactly similar in kind to that which has taken place when the dealer in a game of whist finds that he has all the trumps in his hand."

The subjective creation which we have made without experiment and wholly dependent upon our human logic leads us to this strange belief. To amplify it a bit further, let us consider the case of playing the game of bridge. Four hands of thirteen cards are dealt. One should realize as he picks them up that he is faced by an unusual event. This is probably the first time in the history of the world, and perhaps it will be the only time in recorded history, that this particular distribution of cards in four hands has been played. It is only when we consider some particular arrangement, let us say the four perfect hands, where each player holds every card in a single suit, that the magnitude of the event strikes us. The numerical probability of such a distribution is $1: 2, 235 \times 10^{24}$. If all the people in the world dealt four hands per minute for the next million years the probability that the perfect distribution would appear by chance is still infinitesimal. In the case of the vessel of water the boiling or freezing depends entirely upon the distribution of the velocities of the constituent molecules and the rare event might happen where all of the swiftly moving molecules and all of the slowly moving ones became separated into two groups. Part of the water would then boil and the other part freeze. But as in the case of the perfect hands, the probability is so exceedingly small, that kettles placed upon fires will boil and not freeze even to the end of time.

So much for the mathematical structure by means of which we want to interpret the laws of nature. Nature always seems to avoid conformity to these rigorous models and the strange situation in physics with regard to the quantum theory of energy can be traced to a point at which energy fails to behave in a manner consonant with what is called the normal law of statistics. The details of this disagreement will be discussed in later chapters.

### 5. *The Velocity of Light as a Basic Constant.*

Let us return to Einstein's explanation of the fact that the earth moves through the ether according to aberration and yet does not move through the ether according to the ether-drift experiment of Michelson and Morley.

It will be illuminating to describe the theory of Einstein by means of a few postulates which, taken as a whole, may be said to define the special theory of relativity. The first of these can be stated as follows:

1. *In empty space the relative velocity of one material particle with respect to another cannot exceed the speed of light.*

There is much more in this postulate than would appear at first sight. For some mysterious reason the speed of light is a basic constant in the physical universe. This is an important consideration, because light-impulses are among the most fundamental contacts that we have with reality. If they did not escape us we should never lose the past; if we could move with greater velocity than light, time might be reversed and the peculiar "one wayness" of duration thus destroyed. To speak of direction, however, involves logical pitfalls, even when applied to the three dimensional continuum of space. At this instant we are certainly at one point in the universe, although it is impossible to give it an absolute location; in the next instant we are certainly not less than twenty miles from that point, and there is no irrefragable reason to believe that we ever shall or ever could return to that same point in space again.

Clerk Maxwell, whom we have exhibited in preceding pages as an eminent British physicist, has left behind a not insignificant amount of verse. But while Maxwell was not a Shelley, the physicist has put the problem of time picturesquely in the following lines:[17]

"What! has Time run out his cycle, do the years return again?
Are there treasure-caves in Dreamland where departed days remain?"

Let us reflect for a moment as to what would happen if we could actually move through space with a velocity exceeding that of light? Time, to the extent that it depends upon the propagation of light, would be reversed. Camille Flammarion (1842-1925), the imaginative French astronomer, in his charming volume *Lumen* makes an observer leave the earth with such a velocity. For him time has reversed itself and he sees the effect before the cause. The universe has a new basis which, from our point of view, is wholly irrational. Who, in such a situation, could meditate upon the reasons for the movements of history? How could he explain the phenomenon of Galileo before his inquisitors, after having first beheld the magnificent cosmology of Newton? A short quotation from Flammarion's novel furnishes both entertainment and food for reflection.

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[17] *Life of James Clerk Maxwell*, by Lewis Campbell and W. Garnett, London, 1882, p. 600.

"When I recognized the field of Waterloo, I saw at first a number of dead bodies stretched upon the ground. Beyond them I saw Napoleon arriving *backwards* holding his horse by the bridle. Then I saw the dead soldiers come suddenly to life and spring to their feet, the horses came to life again at the same time and their riders sprang into the saddle. As soon as two or three thousand men were thus resuscitated, they gradually reformed their ranks. The two armies began to fight with fury. In the center of the French army I perceived the Emperor, surrounded by his soldiers. The Imperial Guard had come to life again! At the end of the day not a single man was killed or even wounded—not a uniform was torn. Two hundred thousand corpses, come to life, marched off the field in perfect order. And the result of this strange battle was not to vanquish Napoleon, but on the contrary to restore him to the throne."

As one might readily surmise this idea of the reversal of time is not strictly a modern one and antedates by many years the theory of relativity. For example, we may cite the following whimsical passage from *Through the Looking-Glass,* which Lewis Carrol published in 1871:

" 'Living backwards!' Alice repeated in great astonishment. 'I never heard of such a thing!'

" '. . . but there's one great advantage in it, that one's memory works both ways.'

" 'I'm sure mine only works one way,' Alice remarked. 'I can't remember things before they happen.'

" 'It's a poor sort of memory that only works backwards,' the Queen remarked.

" 'What sort of things do *you* remember best?' Alice ventured to ask.

" 'Oh, things that happened the week after next,' the Queen replied in a careless tone. 'For instance, now,' she went on, sticking a large piece of plaster on her finger as she spoke, 'there's the King's Messenger. He's in prison now, being punished: and the trial doesn't even begin till next Wednesday: and of course the crime comes last of all.' "

Well, at any rate, we have admitted more than we first realize in admitting the first postulate of Einstein. One consequence is that we have shut out from our material world a tremendous number of events. Suppose, for example, that an explosion happened ten years ago on Polaris. Does that event belong to the history of this planet? Is it a past event? By no means. Since it takes light perhaps three centuries to make the journey from Polaris to the earth, that event has not yet happened. We do not mean figuratively, but actually. Since nothing can travel faster than light, we are bound to a limited universe of events. The thing becomes more vivid if we talk in terms of sound instead of light. If our only means of communication with reality were through sound, our active past would be greatly limited. We should always dwell in the present by merely moving in one direction with the speed of sound. The significance of this point of view is made manifest in the well-known story of the soldier who was asked about his feelings when he heard his first shell. "Well," said he, "I heard that shell twice. Once when it passed me and once when I passed it." That shell was once in his active past and once again out of it.

The relativists frequently speak of a four-dimensional cone in which we live. The axis of the cone is time and the generatrix is the path traced by a beam of light. Without mathematics this statement is only a pleasing fancy; with mathematics it becomes an analytical device of power.

The whole matter may, perhaps, be summarized in a modern version of a Mother Goose rhyme:

Hi diddle diddle, the cat and the fiddle,
The cow jumped over the moon,
Going faster than light, she sped through the night,
And returned on the preceding noon.

### 6. *The Geometry of Light Rays.*

Another postulate of the new theory states:

*2. A straight line is the path of a beam of light.*

If one is willing to admit this statement then he has committed himself to an objective rather than to a subjective geometry. To many this will seem a startling innovation, since hitherto this noble branch of mathematics has been generally regarded as the peculiar domain of pure thought. It is introduced early in the educational system as perhaps the most perfect means by which the youthful mind may be instructed in the processes of logical thinking. Its origins are to be found in the earliest antiquity and its formulation in the *Elements* of Euclid, which dates from about 300 B. C., was so perfect that this treatise remained a general textbook for more than twenty-two centuries.

Let us review for a moment some of the salient points of the history of geometry, which may show the spectacular change in modes of thought that are implied in the postulate stated above. Euclid began his *Elements* with the definition: "That which has position but not magnitude is called a point." This initial statement immediately places the subject in the realm of pure abstraction; it removes geometry from all experimental possibilities since the point so defined cannot in any way be identified with any material unit, even though it be of the order of magnitude of the atom. The logical consequences of this concept of a point will be explored more adequately in another chapter.

Euclid then continued with the following definition:[18]

"That which has position and length but neither breadth nor thickness is called a line. A line which lies evenly between points on it is called a straight line."

Thus Euclid makes geometry a pure creation of the mind. Two straight lines cannot intersect in more than one point. A finite straight line may be produced at either extremity to any length.[19]

Einstein on the other hand makes geometry a discovery of nature. "A straight line is the path of a beam of light." We study geometry, not with a pencil and paper or by contemplative introspection of our own minds, but by means of instruments of scientific precision. By experiment alone we can discover something about the nature of the space of the real world, and this may or may not have any relationship with the space which we have created out of the subjective points of Euclid. That this idea, so spectacularly thrust upon us by the theory of relativity, is not entirely a modern one may be readily understood by a statement made by K. F. Gauss who said: "If number is merely a product of our mind, space has also a *reality beyond* our mind of which we cannot fully foreordain the laws *a priori*."[20]

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[18] *Elements of Geometry.* Book 1, definitions 2 and 6.

[19] *Elements,* Postulates 1 and 4.

Many centuries after Euclid, in the period of mathematics made famous by the researches of Leonhard Euler (1707-1783), J. L. Lagrange (1736-1813), and their successors, a new aspect of straight lines was explored which had far-reaching consequences in geometry. A *straight line* then became *the shortest distance* between two points. In order to broaden the speculation the straight line when regarded from this point of view was called a *geodesic*. On the surface of the spherical earth this shortest distance was no longer a straight line in the narrow sense of Euclid, but became the arc of a great circle. The concept of space itself broadened and doubts began to appear that the space of the real world would actually conform to the *flat-space* defined by the Euclidean geometry. In order to discuss this interesting matter and other *related problems* an analytical method of great power was invented called the *calculus of variations*. *A straight line* was now identified with a mathematical expression which made the distance between two points a minimum. But this generalized length itself was no longer unique since it depended in an essential manner upon the nature of the space in which the two points were embedded. And in this formulation space inself was no longer limited to the three dimensions apprehended by the mind, for the number of variables required to define a point could be increased at will. Just as the surface of a sphere requires the concept of three dimensions if it is to be comprehended, so also a manifold in three dimensions requires four dimensions, and a space of four dimensions requires five. What a magnificent speculation thus suddenly has emerged from this removal of the narrow limitations of the older definition!

The proposition that a straight line is the path of a beam of light has focused immediate attention upon the nature of physical space. We now have given us the solution of the analytical problem of the geodesic and from it the nature of the space which leads to this solution can be inferred. The answer is one as strange as anything in the new physics. For if we picture the addition of velocities geometrically by means of the familiar vector diagram, we can use no longer the parallelogram of Euclid's geometry. Instead the non-Euclidean geometry of Lobachevski and Bolyai must be employed.[21]

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[20] In a letter to F. W. Bessel in 1829.

This new point of view reveals quite clearly the deep gulf which separates the pure mathematician from the pure physicist. We can but compare the statement of Oliver Heaviside, master of electromagnetic waves, that "mathematics is reasoning about quantities," with the toast once proposed by H. J. S. Smith (1826-1883), who found fame in the theory of numbers, "Pure mathematics: may it never be of any use to any one."[22] When we are dealing with an objective theory, geometry can become the subject of laboratory investigation; when we are dealing with pure reason, on the basis of a set of consistent postulates, we need have nothing at all to do with objective reality. The introduction of an external world may, in fact, prove to be a disturbing element, if allowed to intrude upon the delicate thread of pure reason.

### 7. *Light and Time.*

Let us now turn to a third postulate inherent in Einstein's philosophy. It should be remarked first, perhaps, that no attempt is being made, in this enunciation of the foundations of the special theory of relativity, to reduce the system of postulates to a minimum, and some will be found to overlap others previously stated. Striking consequences of the new doctrine may thus be exhibited separately.

3. *If a man A is separating from a second man B, with any uniform velocity whatsoever, provided of course, that it does not exceed the speed of light, then a beam of light observed by both men will appear to have the same velocity.*

This strange remark can be put rather picturesquely by assuming that *A* and *B* are replaced by separating beams of light. Then the velocity of one with respect to the other is still no greater than the speed of light. A little more sense appears in this remark when one reflects that time stands still for a beam of light. Since, by our first postulate, the speed of light is the absolute boundary of velocity, it is clear that a remarkable consequence must follow when this velocity is actually attained.

Look out at the stars on some clear evening and then reflect: How very old the light from some of them must be! The light has been travelling for hundreds of years and hence is hundreds of years old. But this is not so. Light never loses its youth. The light that arrives tonight from Polaris is just as young as the day when it was radiated from its electron.

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[21] See Section 10.

[22] F. Cajori: *History of Mathematics*, (Loc. cit.), p. 442.

In this connection G. D. Birkhoff (1884-1944) tells the story of a youth who started out upon a journey to one of the distant stars in a projectile which moved with the speed of light.[23] For him time stood still and he was forever young. Approaching the star he makes use of its gravitational field to reverse his direction and hence returns to earth again. When he arrives he is still young while his comrades have become old men. The nature of relativity is revealed in the explanation of this paradox. Suppose that the young man lost sight of the earth in his flight and looked out of his projectile through a window. Would he be conscious of his great velocity? Suppose that he sets up a Michelson-Morley experiment and tries to detect his velocity through the ether. Would he find any? As far as he is concerned he is motionless in the ether. His clocks go on in the same manner as before. His age increases and he is unconscious of his great velocity. Only when he looks back at the earth he will say: How rapidly the earth is receding from me. The clocks there seem to be standing still. My comrades there are perpetually young while I am growing old. It is all, however, a relative matter. For it is only when the young man turns around again that things will begin to happen. Events on the earth will seem to speed up. The years will roll by with incredible rapidity and his comrades will grow older, until, when he arrives again he will find that all of them have grown old together. The explanation of the paradox is thus seen to lie in the *reversal* of the direction of motion. The special theory of relativity does not inquire into the nature of the geometry of fields of acceleration, which would certainly have to be invoked in any question of the reversal of direction.

### 8. *The Postulate of Relativity.*

We proceed to another postulate, which may be stated as follows:

4. *If a man A is observing the activities of a man B, who is in relative motion with a velocity V, the clock of B will seem to be telling the time* $T\sqrt{1-V^2/c^2}$, *where T is A's time and c is the*

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[23] *The Origin, Nature, and Influence of Relativity*, New York, 1925, pp. 102-3.

*velocity of light, and the units of length will appear to be* $L\sqrt{1-V^2/c^2}$, *where L is the length used by A. Conversely B will have the same opinion of the faulty behaviour of the time and length of A.*

This strange postulate is one of the most basic assumptions of the special theory of relativity. It means that there is no such thing as an absolute velocity; nature will not recognize such a thing. We are taught by astronomers to believe here on the earth that we are moving with an orbital velocity of 18.6 miles per second around the sun and that the solar system as a whole has a drift of approximately twelve miles per second in the direction of the constellation Hercules.

"Absurd," says Nature, "there isn't any such thing as velocity. You only imagine these things."

"But look at aberration," you reply indignantly, "of course we are in motion."

"Wrong again," says Nature, "that doesn't prove that you are in motion. It only shows that you are turning around. It measures the kinetic energy of revolution. It shows that you have different relative positions with respect to the sun, but not that you have an absolute velocity. There isn't any such thing as motion in the absolute sense; of course you don't move. You only think that you do and you wouldn't think that if it weren't that the sun's gravitational field leads to that illusion."

In other words, there isn't anything with respect to which you can be in absolute motion. Is it the ether? If you knew that there existed an ether it might be with respect to it, but the reality of the ether is a delicate philosophical question. Is it the fixed stars? We know that they are not fixed. Is it the center of gravity of the galaxy? We suspect that that isn't fixed with respect to the centers of gravity of the totality of other spiral nebulae. Is it the center of gravity of the mega-galaxy? We don't seem to find any stopping place when we seek a point through which to place our axes. It makes us feel a little bit like Captain Stormfield on his trip to heaven as recorded by Mark Twain: "So, as I said, when I had been tearing along this way about thirty years I began to get uneasy. Oh, it was pleasant enough, with a good deal to find out, but then it was kind of lonesome, you know. Besides, I wanted to get somewhere."

Nature in a convincing way says that velocity is an illusion. Since there are no axes in the heavens to which we can refer velocity, of course we should not expect velocity to be absolute.

### 9. *The Concept of Space-Time.*

Another postulate may be stated as follows:

5. *The distance between two points must be measured in four dimensions of which one is time. The hypotenuse of a right triangle in four dimensions is unaffected by relative velocity.*

One of the most important theorems in elementary geometry is the Pythagorean theorem which asserts that the sum of the squares of the legs of a right angle triangle is equal to the square of the hypotenuse. The many proofs, well over a hundred, which have been given of this proposition attest the significance which has been attached to it. The legend goes that Pythagoras was so jubilant over the discovery of a proof of his theorem that he sacrificed a hundred oxen to the gods.[24] In symbols the theorem may be written: $L^2 = X^2 + Y^2$, where $L$ is the hypotenuse and $X$ and $Y$ the two legs of the triangle.

But Pythagoras in his considerations was dealing with a subjective triangle based upon postulates which were later incorporated into the foundations of the geometry of Euclid. In more recent analysis, the theorem is invoked in a fundamental way in defining the distance between two points in Euclidean space. And it is this application which interests us here since, as we have seen in the statement of the fourth postulate, the concept of length has lost some of its older meaning and changes with the relative motion of those who seek to measure it. To Pythagoras and to Euclid no idea occurred of introducing the concept of velocity into their geometry. Triangles were compared by placing one upon the other, but the actual motion involved in this matching, whether it consisted of a simple translation or a rotation in space or both, was never considered either by them or by their successors.

But in this later age when the behaviour of objective lengths focuses the attention upon velocity, we are led to ask what would happen if we tried to teach the Pythagorean theorem upon a moving platform to a class at rest? To the class the length would seem to change according to the law stated in the fourth postulate. Length in the two dimensional manifold of the surface of the blackboard fails to have that absolute quality which we must have if we are to demonstrate the truth of our theorem.

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[24] F. Cajori: *History of Mathematics*, p. 18.

Now, strangely enough, there is a length, if we wish to stretch this term into the fourth dimension of time, which preserves its identity unchanged by relative motion. This novel length may be written in symbols, $L^2 = x^2 + y^2 - c^2 T^2$, where $c$ is the velocity of light. This is the modern version of the theorem of Pythagoras. The hypotenuse of the space-time triangle does not alter with velocity. In terms of mathematics it is an *invariant* with respect to the Lorentz transformation, as may be proved by merely substituting the values of $X$, $Y$, and $T$ from the equations of the transformation previously given. This length is the reality; the other is but a beautiful subjective creation.

Although the entire content of the fifth postulate is contained in the simple fact that the expression for $L^2$ is invariant with respect to the Lorentz transformation, some additional explanation may prove illuminating. A position in space can never actually be designated by the use of space variables alone since time itself enters into the specification. And as a consequence the distance between two points cannot be measured by the traditional units of length. That this remarkable aspect of nature failed to be discovered earlier is readily understood from the fact that time behaves as a variable measured in what we are accustomed to call an imaginary direction. This strange relationship between the variables of space and the variable of time has led to the statement of the following dimensional equation: 186,300 miles $= \sqrt{-1}$ seconds.[25]

H. Minkowski, to whom these ideas are due, wrote, in an inspired moment, the following:[26]

"The views of time and space, which I have set forth, have their foundation in experimental physics. Therein is their strength. Their tendency is revolutionary. From henceforth space in itself and time in itself sink to mere shadows, and only a kind of union of the two preserves an independent existence."

### 10. *The Law of Addition of Velocities.*

We proceed next to the explanation of several significant conclusions which are derived as direct consequences of the relativity postulates. The first of these is the frequently quoted theorem relating to the addition of velocities. It may be stated thus:

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[25] See H. Minkowski: "Raum und Zeit", *Werke*, vol. 2, 1911, p. 441.

[26] "Raum und Zeit", *Physik. Zeitschrift*, vol. 10, 1909, p. 104; *Werke*, vol. 2, p. 431.

1. *If A's velocity with respect to B is U and B's velocity with respect to C is V, A's velocity with respect to C is not* $U+V$, *but* $(U+V)/(1+UV/c^2)$, *where c is the velocity of light.*

To put the matter a little more simply, the conclusion states that if a man is walking at the rate of four miles per hour down the aisle of a railway coach which is moving at the rate of 45 miles per hour, his velocity relative to the ground is not 49 miles but a small fraction less.

It is very interesting to test the matter in the case of separating beams of light. Suppose we calculate how fast opposite points on a spherical wave of light are separating. Setting $U$ and $V$ in the formula both equal to $c$ we reach the interesting conclusion that the sum of the two velocities is equal to the original velocity $c$. In other words the two points on the spherical waves, moving in opposite direction with the speed of light, are separating with the speed of light.

A similar conclusion is obtained if we set one of the velocities, let us say $U$, equal to the velocity $c$. We then find on the basis of the new law that the velocity of $A$ with respect to $C$ is not equal to $V+c$, as we should infer from the earlier mechanics of Newton, but is exactly equal to $c$. If one wishes to reinterpret the meaning of the symbol for addition to take account of the situation thus presented, one may then write the two remarkable equations: $c+c=c$, and $c+V=c$. Additional light will be thrown upon this interesting situation when, in a later chapter, we consider the concept of infinity.

The difficult problem of the addition of velocities has been beautifully explained in terms of the non-Euclidean geometry of Lobachevski and Bolyai. One will recall from an earlier discussion that the characteristic of this geometry was its assumption that more than one line can be drawn through a point so as to be parallel to another line. One conclusion to which this assumption leads is that the sum of the angles of a triangle is less than the sum of two right angles; this departure from Euclid is determined by the lengths of the sides of the triangle. It is clear that the law of the addition of velocities does not accord with our ordinary Euclidean geometry, where the sum of two velocities, regarded as vectors, is assumed equal to the vector joining their two extremities so as to form a Euclidean triangle the sum of whose angles equals a hundred and eighty degrees. All of this is changed in the new philosophy. The vector diagram of velocities now must form a triangle in non-Euclidean space the sum of whose angles is less than two right angles. For the case of the orbital velocity of the earth the departure from Euclidean geometry, measured in terms of the so-called angle of parallelism, is exactly equal to the angle of aberration, namely 20".6. Unfortunately this beautiful representation cannot be further explored here because of its technical difficulties, but this fact at least can be appreciated. Through many years we have been committed so thoroughly to the geometry of Euclid that we have come to look upon it as absolute representation of objective nature.[27] But careful experimental scrutiny of the meaning of velocity forces us to the conclusion that there is a slight departure from the postulates of Euclid when we examine the geometry of the mathematical representations of this elusive concept. The equally logical, but intuitively more difficult, subjective creation of Lobachevski and Bolyai is a closer representation of objective discoveries.

It is a curious fact that there seems to be no indication in earlier scientific writings of the revolutionary ideas presented by the theory of relativity. This is particularly true with respect to the relationship between velocity and space-time. However, there is a curious passage in the fantasy of Lewis Carroll which may indicate that its author had been indulging in a speculation not far removed from the one which we have just set forth. It is left to the judgment of the reader whether there is a discernible relationship between the ideas of Einstein and those of the poetical mathematician as exhibited by the following passage:

"Alice never could quite make out, in thinking it over afterwards, how it was that they began: all she remembers is, that they were running hand in hand, and the Queen went so fast that it was all she could do to keep up with her: and still the Queen kept crying 'Faster! Faster!', but Alice felt she *could not* go faster, though she had no breath left to say so.

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[27] As a matter of accuracy it should be pointed out that the non-Euclidean geometry of velocities is applied to what has been called the addition of *rapidities*, i. e. $a$ (the rapidity) $=$ arc tanh $v/c$, where $v$ is the relative velocity and $c$ the speed of light. See Silberstein's: *Theory of Relativity*, (loc. cit.), Chap. 6, in particular, p. 178.

"The most curious part of the thing was, that the trees and the other things round them never changed their places at all: however fast they went, they never seemed to pass anything. 'I wonder if all the things move along with us?' thought poor puzzled Alice. And the Queen seemed to guess her thoughts, for she cried 'Faster! Don't try to talk!'

"Not that Alice had any idea of doing *that*. She felt as if she would never be able to talk again, she was getting so much out of breath: and still the Queen cried 'Faster! Faster!' and dragged her along. 'Are we nearly there?' Alice managed to pant out at last.

"'Nearly there!' the Queen repeated. 'Why we passed it ten minutes ago! Faster!' And they ran on for a time in silence, with the wind whistling in Alice's ears, and almost blowing her hair off her head, she fancied.

"'Now! Now!' cried the Queen. 'Faster! Faster!' And they went so fast that at last they seemed to skim through the air, hardly touching the ground with their feet, till suddenly, just as Alice was getting quite exhausted, they stopped, and she found herself sitting on the ground, breathless and giddy.

"The Queen propped her up against a tree, and said kindly, 'You may rest a little, now!'

"Alice looked round her in great surprise. 'Why, I do believe we've been under this tree all the time! Everything's just as it was!'

"'Of course it is,' said the Queen. 'What would you have it?'

"'Well in *our* country,' said Alice, still panting a little, 'you'd generally get to somewhere else—if you ran very fast for a long time as we've been doing.'

"'A slow sort of country!' said the Queen. 'Now, *here*, you see, it takes all the running *you* can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!'"

### 11. *Mass and Velocity.*

Another conclusion which is implied by the postulates of the special theory of relativity, exhibits a remarkable relationship between mass and velocity. This conclusion may be stated as follows:

2. *Mass increases with velocity in accordance with the expression,* $M/\sqrt{1 - V^2/c^2}$ *, where* $V$ *is the velocity of the moving object,* $M$ *is its mass at rest, and* $c$ *is the velocity of light.*

The discovery of the variability of mass with velocity antedates the enunciation of the theory of relativity. The history of the development of this idea is one of unusual interest since it is essentially the history of what Lorentz has called "the important subject of the *electromagnetic mass* of the electron." In his treatise on the *Theory of Electrons*, published in 1909, Lorentz states that "the electron moves as if it had two different masses . . . the first of which comes into play when we are concerned with an acceleration in the line of motion and the second when we consider the normal acceleration . . . . In order to distinguish [them] we can apply the name of *longitudinal* electromagnetic mass to the first, and that of *transverse* electromagnetic mass to the second of these coefficients."[28]

This statement of Lorentz was occasioned by attempts made in the early years of the present century to account for the mass of an electron on the assumption that it is derived from the magnetic field which is created by the motion of the electric charge which it carries. The problem was relatively simple when slowly moving electrons were considered, but its solution encountered unusual difficulties when the motion was assumed to be a significant fraction of the velocity of light. Not only were the distributions of the electric and magnetic fields about the electron complicated by the motion, but the actual shape of the moving electron became a matter of fundamental significance in the problem.

The first attempt to solve the problem was made by M. Abraham, who assumed in his theory that the electron maintained a spherical form for all velocities.[29] Abraham attained expressions for the longitudinal and transverse masses and achieved the spectacular conclusion that these components of the mass of the electron increased with its velocity.

Two years later A. H. Bucherer published a similar investigation in which, however, he assumed that the diameter of the electron shortened in the direction of motion, but increased in the direction perpendicular to the motion.[30] The ratio of one diameter to the other introduced the now famous quantity, $1/\sqrt{1 - V^2/c^2}$ , where $V$ is the velocity of the electron and $c$ the velocity of light. Bucherer also derived two expressions for the mass components, but these differed from the ones previously obtained by Abraham.

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[28] *Theory of Electrons*, pp. 38-39.

[29] *Göttingen Nachrichten*, Vol. 3, 1902, p. 20; *Annalen der Physik*, Vol. 10, 1903, p. 105.

[30] *Math. Einführungen in die Elektrontheorie*, Leipzig, 1904, p. 58.

At the same time H. A. Lorentz made a third investigation of the problem. Directing his attention to the negative results of the Michelson-Morley experiment, which, as we have seen earlier, he had explained on the assumption of a contraction of matter in the line of motion, Lorentz assumed that the diameter of the electron shortened only in the one direction and remained unchanged in the other. He then found the following expressions for the transverse mass ($M_t$) and the longitudinal mass ($M_l$) respectively:

$$M_t = \frac{M}{\sqrt{1 - V^2/c^2}}, \quad M_l = \frac{M}{\sqrt{(1 - V^2/c^2)^3}},$$

where $M$ is the rest mass of the electron.

Here then, one year before Einstein's publication of the special theory of relativity, there existed three solutions of the problem. All agreed that the mass increased with velocity and all separated the mass into a longitudinal and a transverse component. The solutions differed only in the mathematical expressions for these components.

For several years the question remained in doubt as to which of the formulas should be accepted as the correct interpretation of experimental facts. Extensive investigations of the variability of mass with velocity were made by W. Kaufmann[31] in 1906 and A. H. Bucherer[32] in 1908. The uncertainty which enveloped the subject is clearly indicated by the fact that Lorentz in 1909 midway through his work on the *Theory of Electrons* said regarding the experiments of Kaufmann: "His new numbers agree within the limits of experimental errors with the formulas of Abraham," and hence Lorentz rejected his own hypothesis. On the last page of his treatise, however, apparently added just before the book went to press, Lorentz finally concluded: "Later experiments . . . have confirmed the formula [his own] . . . for the transverse electromagnetic mass, so that, in all probability, the only objection that could be raised against the hypothesis of the deformable electron and the principle of relativity has now been removed."

More recent experiments have given overwhelming confirmation of the statement of Lorentz and the formula has been tested over and the process of oxidation, as well as the *difference in color of the two kinds of blood*, must be on the whole less in the torrid zones than in colder regions."

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[31] "Über die Konstitution des Elektrons," *Annalen der Physik,* Vol. 19, 1906, pp. 487-553.

[32] "Messungen an Becquerelstrahlen," *Phys. Zeitschrift,* Vol. 9, 1908, pp. 755-760.

His reflections upon this phenomenon finally led him to the proposition that "an invariable quantitative relation between heat and work is a postulate of the physiological theory of combustion." What a tremendous leap from the red blood of European visitors in Java to a great cosmic principle. How one marvels at the power of imagination "which bodies forth the form of things unknown."

Not less interesting is the story of the reception of Joule's theory when it was first presented to the English scientists. The story is given as follows in Joule's own words.[35]

"It was in the year 1843 that I read a paper "On the Calorific Effects of Magneto-Electricity and the Mechanical Value of Heat" to the chemical section of the British Association assembled at Cork. With the exception of some eminent men, among whom I recollect with pride Dr. Apjohn, the president of the section, the Earl of Rosse, Mr. Eaton Hodgkinson, and others, the subject did not excite much general attention; so that when I brought it forward again at the (Oxford) meeting in 1847 the chairman suggested that, as the business of the section pressed, I should not read my paper, but confine myself to a short verbal description of my experiments. This I endeavored to do, and discussion not being invited, the communication would have passed without comment if a young man had not risen in the section, and by his intelligent observations created a lively interest in the new theory."

This young man was William Thomson, later Lord Kelvin, with whom Joule had a long and lasting friendship. At almost the same time Helmholtz, who had just announced the same principle in a masterly paper entitled: "On the Conservation of Force," was struggling for recognition against the skepticism of the leading physicists of Germany.

The principle of the conservation of energy asserts that within a closed region of space the total supply of energy is neither increased nor destroyed but remains constant. It may change its form, and it may become inaccessible, as when the supply of free energy in a thermal system runs down its temperature gradient, but the energy is never lost. It has merely concealed itself in the internal movements of the individual molecules of matter.

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[35] *Scientific Papers*, vol. 2, p. 215; see also Thompson's *Life of Lord Kelvin*, vol. 1, p. 263.

But what a revolution is seen in the equation of Einstein. Mayer, Joule, and Helmholtz showed that work and heat energy were equivalent; Einstein shows that energy and matter are the same. There are not two laws, one conserving matter and the other energy, but one law wherein the totality of energy and matter is conserved.

Mass and energy are, therefore, essentially alike; they are only different expressions for the same thing. The tremendous possibilities in this relationship can be pictured by means of a few figures. Let us consider a single gram of mass which is a very small quantity, since it would require nearly half a thousand grams to make a single pound. Replacing the velocity of light in the equation by its proper numerical value we obtain the astonishing answer that this insignificant amount of matter is equivalent to 6.63 times $10^{13}$ foot-pounds, or $9 \times 10^{20}$ ergs, of energy. Stating this otherwise, we mean that this tiny bit of matter is the equivalent of the amount of energy which would run a four thousand horse-power engine for an entire year. A simple calculation based upon these figures shows that a mass of the order of thirteen pounds, if converted completely into energy, could produce the entire amount of electrical energy generated by the electric power industry of the United States in the year 1950.

The energy lost by the sun through radiation in a single year, according to the Einstein formula, may also be computed without difficulty, and we find that it is equivalent to an amount of matter weighing $1.5 \times 10^{20}$ grams, which is approximately equal to one fifty-millionth the mass of the earth.[36] On this estimate the sun could continue to radiate at its present rate for fifteen trillion years before its total supply of energy would be exhausted. Do we wonder at the power of the atomic bomb which, as we shall see later, makes use of this fundamental transformation of matter into energy!

Where in all of this is the role of kinetic energy (K. E.), which from the time of Newton has been written in the form: K. E. = $\frac{1}{2}MV^2$, where $V$ is the velocity of a particle of mass $M$? Under the impact of the new ideas introduced by the special theory of relativity, however, where mass is no longer a constant entity in the formula, this older concept must be modified in an essential manner.

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[36] See Sir James Jeans: *Astronomy and Cosmogony,* Cambridge, 1928, p. 108 and Silberstein's: *Theory of Relativity,* p. 250.

The new physics, which thus merges mass and energy in a single concept, requires the new formula:

$$\text{K.E.} = Mc^2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1\right],$$

where $c$, the velocity of light, now makes its mysterious appearance. For velocities which are small with respect to light, this formula may be shown to reduce essentially to the older expression. But when velocities approach the velocity of light, as they do in the case of high-speed electrons, then the situation is quite different.

A few figures will show the startling nature of this idea. If the mass of a particle were converted completely into energy, this energy, as we have indicated above, would be equal to $Mc^2$. It is customary to call this the *rest energy* of the particle. But if the particle should attain a velocity equal to 99 per cent that of light, not unusual in modern atomic physics, its kinetic energy would multiply this already fantastic value by a factor in excess of six. A velocity 87 per cent that of light would give an energy just equal to the rest energy of the particle.

We thus see again in these figures the revolutionary nature of the ideas that have been advanced in the new physics and the remarkable role played by velocity in the mystery.

### 13. *The New Metaphysics.*

In the special theory of relativity we catch a glimpse of the new reality. Physics, with its galvanometers and its mirrors, has found itself facing the vision of a vast metaphysical doctrine. Time and space, which were formerly but thoughts for the philosopher in his subjective musing, now assume the most important aspects of reality. They are fused together into a four-dimensional framework which is not to be regarded as space alone nor as time alone, but as the single entity of space-time.

And what shall we say about the objective reality of this picture? We have created a material geometry whose elements are rays of light. No longer may we build our mathematical picture of the objective world on logical principles alone. In so far as we shall want to talk about the geometry of nature, the delicate question of choice between the three branches of geometrical postulation is no longer one of inclination but is forced upon us by the evidence of laboratory experiment.

But whether or not the new philosophy repels us by its difficulty and by its disregard of what we think of as rational intuition, there is certainly one virtue that it preëminently possesses. It has exhibited the power of synthesizing isolated theories. The conservation of matter and the conservation of energy are revealed as but two aspects of a single problem. Time and space no longer enter our equations as unrelated variables, but are henceforth to be regarded as parts of a single thing.

In the light of the new knowledge, if you would become a student of physics you must first become a geometer. Wise and far sighted, indeed, was Plato who put above the entrance to his school: "Let no one ignorant of geometry enter my door."

# CHAPTER 6.

## WHAT IS GRAVITATION?

"On another occasion I wished to jump across a lake. When I was in the middle of it, I found that it was much larger than I had imagined at first. So I at once turned back in the middle of my leap, and returned to the bank I had just left to take a stronger spring. The second time, however, I again took off badly, and fell up to my neck. I should, beyond any doubt, have come to an untimely end, had I not, by the force of my own unaided arm, lifted myself up by my pig-tail, together with my horse, whom I gripped tightly with my knees."

*The Life and Adventures of Baron Munchausen.*

### 1. *The Problem of Knowledge.*

In the last chapter we investigated an interpretation of two experimental facts, the first of which told us, through the constant of aberration, that the earth moved in space, and the second, through the ether-drift experiment, that the earth was at rest with respect to the postulated medium by means of which radiant energy is propagated. This explanation involved us in a speculation as to the relationship between objective nature and the baffling concepts of space and time. For the first time in the history of thought these twin entities emerge as objective things. Subjective geometry begins to assume an aspect of reality. But there must already have occurred to the reader the thought that these new ideas are thrusting upon us with renewed force the old question of epistemology: What is knowledge?

This question is not wholly divorced from racial characteristics, as has been pointed out by Poincaré. "They [the English] are always to hold, so to speak, one foot in the world of senses, and never burn the bridges keeping them in communication with reality. . . . The Latins seek in general to put their thoughts in mathematical form; the English prefer to express it by material representation. . . . For a Latin, truth can be expressed only by equations; it must obey laws simple, logical, symmetric and fitted to satisfy minds in love with mathematical elegance. The Anglo-Saxon to depict a phenomenon will first be engrossed in making a *model,* and he will make it with common materials, such as our crude, unaided senses show us them. He also makes a hypothesis, he assumes implicitly that nature, in her finest elements, is the same as in the complicated aggregates which alone are within the reach of our senses. He concludes from the body to the atom."[1]

What is knowledge? We feel that it can be gained only through those sensations which enter through our five senses. Reason is merely a bridge from one sensation to another. It is a guide to the interpretation of relations between sensations. We acquire a set of data from the laboratory and translate it into the language of mathematics; the transformations suggested by reason lead us to conclusions which we must test by new experiments with sensuous apparatus. The theory of relativity gained general recognition only when the predicted deflection of light past the limb of the sun had been found by astronomers from observations during an eclipse. One must reflect with thanksgiving that the Creator of the universe has given the earth a moon so fortunately situated that its shadow periodically obscures the entire disk of the sun. What a cosmic calamity it would have been had the orbit of the moon been but a few thousand miles greater than it is! There are some who wish to make us believe that Venus, the "star of evening," may be inhabited by intelligent beings, whose development may be greater than our own. But we must first reflect upon their great handicap living within the perpetual clouds of a planet which has no moon to reveal the secrets of the universe.

Will it be necessary for us to make a new definition of knowledge and will there always be a nebulous region between that which is known and that which is unknown so that boundaries can never be established? Are we satisfied to regard the ether as something forever barred from sensuous experience but with a kind of reality exhibited in the fact that the forces and energies which it was designed to carry are completely defined in terms of a set of equations? Must we look upon the atom as something material, round, perhaps, and solid like a marble of infinitesimal size? Must we know its weight and its radius, its velocity and its inertia? Must we be able to count its numbers in a given quantity of matter and think of it merely as a primary substance, differing only in dimension from the material with which we are in daily contact? Are we satisfied to give up this substantial image for a set of equations? Are we willing to let the reality of nature disappear in a manner mindful of the philosophies of Hume and Berkeley and preserve its objective existence in the partially subjective equations of logical intuitions? "What is Truth?" said jesting Pilate; and would not stay for an answer.

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[1] Preface to "*The Foundations of Science,*" p. 4.

There is at least a partial reply to these difficult questions in the following quotation from Poincaré: [2]

"Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt a reality completely independent of the mind which conceives it, sees or feels it, is an impossibility. A world as exterior as that, even if it existed, would for us be forever inaccessible. But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all; this common part, we shall see, can only be the harmony expressed by mathematical laws. It is this harmony then which is the sole objective reality, the only truth we can attain; and when I add that the universal harmony of the world is the source of all beauty, it will be understood what price we should attach to the slow and difficult progress which little by little enables us to know it better."

### 2. *Newton and Gravitation.*

We turn from these delicate considerations to one of the most perplexing mysteries of the universe. What is gravitation?

Gravitation is traditionally defined as a force which is exerted upon each particle of matter by every other particle of matter, according to the famous law of inverse squares. In other words every particle of matter attracts every other particle of matter directly as the product of the masses and inversely as the square of the distances between them. The discovery of this law is attributed by universal consent to Newton, although it existed somewhat nebulously in the minds of a few of his contemporaries and immediate predecessors. For example Sir Francis Bacon (1561-1626) had advanced the theory that gravitation decreased both outward and inward from the surface of the earth and had proposed to check this idea by placing clocks run by weights at the top of a high steeple and in the bottom of a deep well and comparing their rates with a clock run by an iron spring at the surface of the earth.[3] This experiment was actually performed two centuries later by P. von Jolly (1809-1884).[4] Robert Hooke (1635-1703) had attempted as early as 1662 to detect the loss of weight which would be sustained by an object lowered with a piece of thread from a convenient place upon Westminster Abbey, but the delicacy of the experiment masked his results and he reported a negative result. In 1682, five years before the publication of Newton's *Principia*, Hooke read before the Royal Society a paper entitled "A Discourse on the Nature of Comets," in which he considered gravitation to be a principle of all matter, propagated through a universal medium, and varying inversely as the square of the distance.

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[2] *Foundations of Science*, p. 209.

[3] *Novum Organum*, Book 2, Section 36.

The genesis of the idea of gravitation in the mind of Newton makes an interesting story. The following account is given by Voltaire, who learned it from Catherine Barton, a favorite niece of Newton, and is the source of the famous "apple story" so frequently related in this connection.[5]

"But being retired in 1666, upon account of the Plague, to a solitude near Cambridge," says Voltaire, "as he was walking one day in his garden, and saw some fruits fall from a tree, he fell into a profound meditation on that gravity, the cause of which had so long been sought, but in vain, by all the philosophers, whilst the vulgar think there is nothing mysterious in it. He said to himself, that from what height soever in our hemisphere those bodies might descend, their fall would certainly be in the progression discovered by Galileo; and the spaces they run through would be the square of the times. Why may not this power which causes heavy bodies to descend, and is the same without any sensible diminution at the remotest distance from the center of the earth, or on the summits of the highest mountains, why, said Sir Isaac, may not this power extend as high as the moon?"

Now it happened that Johannes Kepler (1571-1630), after a profound study of the data on the motions of the planets collected by Tycho Brahe, had come to three important conclusions which are known in astronomy today as Kepler's laws:

The first of these asserted that *the planets move sensibly in ellipses with the sun at the focus;* the second that *the line connecting*

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[4] "Die Anwendung der Waage auf Probleme der Gravitation," *Abh. Bay. Akad. Wiss.* Cl. 2, vol. 13, *Abth.* 1, pp. 157-176; vol. 14, *Abth.* 2, pp. 3-26.

[5] *Éléments de la philosophie de Newton,* Part 3, Chap. 3.

*a planet with the sun passes over equal areas in equal times;* the third that *the cubes of the mean distances of any two planets from the sun are to each other as the squares of their periods.*

The first law is clearly an addition to the statement of the Copernican theory of planetary motion. The sun is at the center of the system and the planets move about it in ellipses. The second law, often referred to as the *law of areas,* accounts for the acceleration of the velocities of the planets at certain times in the year. For example, in the winter the earth moves more rapidly in its orbit than during the summer, because of its closer approach to the sun and the necessity for greater velocity in order that the requirement of the law of areas should be fulfilled. The third law, called the *harmonic law,* was especially pleasing to Kepler and he expresses himself in the following extravagant words. Can we not indulge him this ecstasy who first found in the tables of Tycho the magnificent relationship which binds all the planets together into a single system?

"It is not eighteen months since I got the first glimpse of light, three months since the dawn, very few days since the unveiled sun, most admirable to gaze upon, burst upon me. Nothing holds me; I will indulge my sacred fury; I will triumph over mankind by the honest confession that I have stolen the golden vases of the Egyptians to build up a tabernacle for my God far away from the confines of Egypt. If you forgive me, I rejoice; if you are angry, I can bear it; the die is cast, the book is written, to be read either now or by posterity, I care not which; it may well wait a century for a reader, as God has waited six thousand years for an observer."[6]

The problem that confronted Newton was to find a single hypothesis which would exhibit these laws as aspects of a unifying principle. He thought that he had found this in the assumption of a cosmic force varying inversely as the square of the distance. But unfortunately, when Newton proceeded to apply his theory to the relationship between the earth and the moon, he encountered an arithmetical disagreement, which for sixteen years kept him from announcing his great discovery. In making his calculations of the relationships existing between the earth and its satellite, Newton used the value of sixty miles for the length of a degree of arc upon the earth's surface, which led to a contradiction between the calculated and observed value of the gravitational acceleration at the surface of the earth. This disappointment led him to lay "aside at that time any further thought of the matter" until 1684, when there was brought to his attention a new determination of the length of a degree of arc which had been made by Jean Picard near Paris. This new value was 69½ English miles instead of the 60 previously used; and when Newton made this substitution in his formulas, the two values, theoretical and observed, were found to agree. The Newtonian theory of gravitation was a reality.[7]

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[6] *Harmonicis mundi,* book v. Concluding sentence of the "Prooemium." For admirable accounts of the life and work of Kepler see, M. Bertrand: *Kepler: His Life and Works.* Translation by C. A. Alexander. *Report of the Smithsonian Inst.,* 1869, pp. 93-110; also Sir Oliver Lodge: *Pioneers of Science,* 1919, chap. 3.

Newton now devoted his entire attention to perfecting his work and during the next two years he was wholly absorbed in the task. Various stories are told of his preoccupation during this period, his absent-mindedness, even his forgetfulness of meals. At what cost are magnificent structures of thought evolved!

The curious incident of the discovery of Newton's monumental work is of perennial interest. According to the story [8] Sir Christopher Wren, Robert Hooke, and Edmund Halley had been discussing the subject of gravitation, and Wren offered the other two a prize of "a book worth forty shillings" if either of them could prove that a body moving under the law of inverse attraction would be an ellipse. The prize was not collected; Halley paid a visit to Newton and asked him if he knew how to approach this perplexing problem. Newton replied that he had already solved it, but in searching through his papers was unable at the moment to discover his solution. In a short time, however, he sent the paper to Halley together with other extensions of his ideas on motion.

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[7] An alternative explanation of the long delay between discovery and publication has been advanced by J. C. Adams and J. W. L. Glaisher. This explanation asserts that while the numerical verification of the theory was fairly complete in 1666, Newton hesitated to publish until he had proved the theorem that a spherical body attracts an external particle as though the entire mass were concentrated at its center. Glaisher remarks: "No sooner had Newton proved this superb theorem—and we know from his own words that he had no expectation of so beautiful a result till it emerged from its mathematical investigation —than all the mechanism of the universe at once lay spread before him." See F. Cajori: *A History of Physics*, Revised ed., 1929, pp. 66-67.

[8] See Sir David Brewster: *Life of Sir Isaac Newton*, chap. 11, second ed., p. 138.

Halley immediately made the Royal Society aware of what he had discovered and Newton was asked by the Society for permission to publish the manuscript. Newton gave his consent, and the Society put the matter in the hands of Halley, whose interest and insistence finally forced Newton to complete his great work and prepare it for publication. The first edition entitled *Philosophiae Naturalis Principia Mathematica* appeared in 1687 at the expense of Halley himself, who always regarded his discovery of the monumental contribution with peculiar pride and pleasure.

### 3. *Properties of Gravitation.*

Let us now turn our attention to some of the experimental properties of gravitational force.

1. *Weight is proportional to mass.*

This simple statement is more profound than it might at first appear to be. Why should weight, that is to say the gravitational force at the earth's surface, depend wholly upon inert mass? Even Aristotle thought otherwise, and for many centuries the universities, following his authority, taught that a heavy weight and a light one fell with different velocities. We have already recorded in another place how Galileo refuted his colleagues in the University of Pisa by dropping two objects of unequal weight from the famous leaning tower, but did this demonstration win admiration for himself and acceptance of the theorem? It merely served to demonstrate the courageous spirit of Galileo, because to drop those iron weights lost him more friends and put him in more jeopardy of life than if he had thrown them at random in the market place. One can but perpetually marvel at the attitude of the human mind toward newly discovered truth. The first reaction seems to be to decide whether one is for or against a new doctrine in the light of its immediate effect. The evidence must be examined only secondarily.

Are we wishing to see in this now commonly accepted fact of the equivalence of inertial mass and weight a new mystery or should it be an expected consequence of our rational concept of matter? Most people upon reflection will agree that the proportionality between weight and mass is a remarkable phenomenon. Why should a bar of lead, for example, possess exactly the same attractive force as a piece of glass of equal inertial mass? Is the mystery of inertia identical with the mystery of gravitation? Are they but aspects of the same thing?

Certainly this would appear to be the case if one gives close scrutiny to the three laws of motion upon which Newton built his mechanics.[9]

Law I. *Every body perseveres in its state of rest or of motion in a straight line, except in so far as it is compelled to change that state by impressed forces.*

Law II. *Change of motion is proportional to the moving force impressed, and takes place in the direction of the straight line in which such force is impressed.*

Law III. *Reaction is always equal and opposite to action; that is to say, the actions of two bodies upon each other are always equal and directly opposite.*

The concepts of force and inertia are inextricably entangled. Suppress one from your formulas and anything that you may say about the other is meaningless. Acceleration and inertia are but the active and passive aspects of the behaviour of matter in the continuum of space and time. The implications of these laws permeate the entire structure of dynamics. Force is proportional to acceleration and the factor of proportionality is mass. But weight, evidence of the reality of the attractive influence of gravitation, is a constant force and hence proportional to mass.

The truth of the law of Galileo has been investigated by the most accurate of modern instruments and recent measurements upon the possible variations in the gravitational constant between platinum, copper, water and asbestos have resulted in the conclusion that the variation can not exceed one part in twenty billion.

Another fact to which attention must be directed refers to the determination of the so-called constant of gravitation. This may be stated as follows:

2. *The gravitational pull between two spheres, each of one gram mass, and placed so that their centers are one centimeter apart, is equivalent to a force equal to* $6.658 \times 10^{-8}$ *of a dyne.*

In a treatise, the main object of which is the examination of the philosophical structure of modern physics, such a statement as that just written down may appear out of place. This is not a review of the specific numerical consequences of experimental evidence. But we are making an inquiry into the nature of physical reality, and it must be of the first order of importance to know whether the hypothetical force postulated by Newton is merely a fortuitous guess as to the geometrical behaviour of nature, or whether there is direct evidence to prove the real existence of the gravitational force.

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[9] *Principia,* Axiomata.

The reality of this force has been demonstrated by a number of early experimenters, a few of whom will be mentioned briefly. Pierre Bouguer (1698-1758) about 1740 undertook to determine the density of the earth by measuring the variation of the second's pendulum observed at Quito, Ecuador, approximately 9,000 feet in elevation and again at the Isle of Inca at sea level. Bouguer found that the variation did not follow exactly the inverse-square law of gravitation, and he attributed the difference to the pull exerted by the lofty plateau upon which Quito is built. Making an estimate of the density of the attracting rocks he arrived at a value for the density of the earth which was, when we consider the difficulties of measurement, naturally considerably in error.[10]

Later Bouguer made another attempt to show the magnitude of the force of gravitation by determining the horizontal pull of Chimborazo, a mountain 20,000 feet in height. His method was to find the deflection caused in a plumb line due to the proximity of the mountain, the deflection to be measured by the zenith transit of a star observed in the neighborhood of the mountain and again at a distance from it. As in the other experiment the results attained exhibited a significant error. Much distress was experienced by the experimenters from the weather conditions near the mountain. Bouguer wrote: "I shall not speak of the cold and the other discomforts we had to put up with; snow covered our tent and all the ground around as far as 800 or 900 toises (a toise = 6.4 feet) below us, and we lived in fear of being buried under its weight. It needed constant vigilance in order to avoid it."[11] The achievement accomplished by these hardy adventurers was to show that the force of gravitation is not a subjective concept, but existed as a real measurable property of rocks.

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[10] An account of his long labors is published in *La Figure de la Terre*, Paris, 1749. A translation of section 7 is given by A. S. Mac Kenzie in *The Laws of Gravitation*, New York, 1909, pp. 23-44.

[11] From *La Figure de la Terre*, sec. 7.

A similar attempt to estimate the density of the earth by computing the constant of gravitation was made in 1774 by Nevil Maskelyne, (1732-1811), who measured by a zenith telescope the deflection of a plumb line on the north and south sides of Schihallien in Perthshire, Scotland.[12]

More exact determinations of the density of the earth were made in 1854 by Sir G. B. Airy, (1801-1892), who employed for his purpose the variations in a pendulum clock lowered to the bottom of a mine shaft. The experiment was carried out in England at the Harton pit near South Shields, the effective depth being 1250 feet.[13] A similar experiment was also tried in 1882-1883 at the Adalbert shaft at Pribram in Bohemia and in 1885 at the Abraham shaft near Freiberg by R. von Sterneck, but the results were necessarily rendered inaccurate by the great difficuty of estimating the density of the different strata through which the shafts were cut.[14]

The most effective way of determining the gravitation constant is by the direct measurement of the attraction between two carefully constructed masses in the laboratory. Here the difficulty of the experiment is on the other foot. The masses are exactly known, but the force is very small; in the case of the mountain the force is comparatively large, but the density is difficult to determine.

The first experiment of this kind was carried out by Henry Cavendish (1731-1810), prince of experimenters, who perfected an apparatus originally designed by Rev. John Mitchell.[15] This device consisted of a wooden arm made so as to combine great strength with little weight. The arm was suspended by a small wire in a horizontal position, and at each extremity there was hung a lead ball approximately two inches in diameter. Close to these two weights were suspended two larger lead balls eight inches in diameter and the whole apparatus was then enclosed in a sealed room and observed from without by means of a telescope. When the relative positions of the balls were varied by means of a device controlled from outside the room, the variation in attractive force caused a vibration in the system composed of the smaller weights and these vibrations, interpreted by a torsion formula, were then used to determine the force of gravitation.

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[12] "An Account of observations made on the mountain Schihallien for finding its attraction," *London Phil. Trans.*, 1775, pp. 500-542.

[13] *Monthly Notices of the Royal Astr. Soc.*, Vol. 15, 1855, pp. 35-36, 46, 125-6; *Ann. de Chim. et de Phys.*, 1855, Vol. 43, (3), pp. 381-3; *Arch. des Sc. Phys. et Nat.*, 1855, Vol. 29, pp. 188-191.

[14] "Untersuchungen über die Schwere im Innern der Erde," *Mitth. Mil.-Geog. Inst. Wien*, Vol. 2, 1882, pp. 77-120; Vol. 3, 1883, pp. 59-94; Vol. 4, 1884, pp. 89-155; Vol. 5, 1885, pp. 77-105; Vol. 6, 1886, pp. 97-119.

[15] "Experiments to Determine the Density of the Earth," *London Phil. Trans.*, 1798, part II, pp. 469-526.

This experiment has been frequently repeated, a sensible improvement being effected in 1895 by C. V. Boys,[16] who replaced the metal wires of a Cavendish apparatus by delicate quartz fibres. His experiment, subsequently repeated in 1896 by K. Braun,[17] with results agreeing with those of Boys to four significant figures, is today regarded as giving the most accurate determination of the gravitational constant.

R. von Eötvös in 1896 perfected an apparatus, later much used by geologists in the practical application of the torsion balance to mineral and oil explorations.[18]

The conclusion to be drawn from these facts is that we are dealing in the subject of gravitation with a measurable force the objective existence of which can be demonstrated in the laboratory. It is not akin to the mystical creation of the ether, whose substantial property is an elusive necessity of reason. The force of gravitation will affect the torsion balance; the ether will react upon no material instrument; the force of gravitation is not a creation of Newton's imagination, with reality only in the equations which determine the orbits of the planets; the ether derives its principle right to a place in science from the equations of Maxwell.

Let us turn to a perplexing question. Why does gravitation fail to have any substantial attribute except that exhibited by the torsion balance? Why does neither heat, cold, chemical structure, nor the screening influence of other matter have the slightest effect upon this mysterious quantity? Are we after all justified in assigning more reality to gravitation than to the ether? The ether is the postulated medium by means of which a vibrating electron on the surface of the sun is able to influence a radiometer in our laboratory; gravitation is a measurable force acting upon two material bodies. Neither appears to have any other substantial characteristic. Are we justified in assuming reality for the one and denying it to the other?

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[16] "On the Newtonian Constant of Gravitation." *Trans. Royal Soc. of London* (A), 1895, Vol. 186, pp. 1-72.

[17] Die Gravitations Constante, die Masse and mittlere Dichte der Erde nach einer neuen experimentellen Bestimmung," *Denkschr. Akad. Wiss. Wien. Math.-naturw. classe,* Vol. 64, 1896, pp. 187-258-c.

[18] "Untersuchungen über Gravitation und Erdmagnetisms," *Wied, Ann.* Vol. 59, 1896, pp. 354-400. A careful account of the history and developments in this subject up to 1900 is given in *The Laws of Gravitation* by A. S. MacKenzie.

Let us examine these negative characteristics of gravitation a little more closely.

3. *The force of gravitation is unaffected by temperature.* This is a surprising conclusion, since gravitation is essentially a property of matter and the form and structure of matter are intimately associated with its heat content.

Delicate measurements have been made to test this negative characteristic of gravitational force. J. H. Poynting and P. Phillips counterpoised a mass of 208 grams on a balance, varied its temperature from −186° to 100° centigrade, and found that if there were a change it was less than one part in a billion per degree change for the range from 0° to 100° and less than one in ten billion for the negative range.[19] More recent measurements made in 1916 by P. E. Shaw are somewhat in disagreement with these figures. Working with a range of temperature from 15° to 250° centigrade, he found a positive temperature effect on gravitation, which is, however, not larger than twelve parts in a million per degree.[20]

These experiments are necessarily extremely difficult because of the minuteness of the difference sought for and the many sources of experimental error. A safe conclusion to draw is that the temperature effect, if it exists at all, is very small.

Another significant fact about gravitation is this:

4. *Matter is transparent to gravitation.*

This is to say there is no appreciable effect upon the gravitational constant if a material screen of any sort be placed between two gravitating objects. There is an interesting speculation in the entertaining story of H. G. Wells: *The First Men in the Moon* as to what would happen if a screen opaque to gravitation could be discovered. Mr. Cavor, the scientist of the tale, had found a method for manufacturing such a screen.[21]

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[19] *Proc. of the Royal Soc.* Vol. 76 (A), 1905, p. 445. A similar result was obtained by Southerns in 1906. *Ibid.* Vol. 78.

[20] Consult: "The Newtonian Constant of Gravitation as affected by Temperature." *Transactions of the Royal Society,* Vol. 216 (A), 1916, pp. 349-392.

"He showed me by calculations on paper which Lord Kelvin, no doubt, or Professor Lodge or Professor Karl Pearson, or any of those great scientific people might have understood, but which simply reduced me to a hopeless muddle, that not only was such a substance possible, but that it must satisfy certain conditions. It was an amazing piece of reasoning. Much as it amazed and exercised me at the time, it would be impossible to reproduce it here. "Yes," I said to it all, "yes. Go on!" Suffice it for this story that he believed he might be able to manufacture this possible substance opaque to gravitation out of a complicated alloy of metals and something new—a new element I fancy—called, I believe, *helium*, which was sent to him from London in sealed stone jars. Doubt has been thrown upon this detail, but I am almost certain it was *helium* he had sent him in sealed stone jars. It was certainly something very gaseous and thin. If I had only taken notes. . . . There seemed no limit to the possibilities of the scheme; whichever way I tried, I came on miracles and revolutions. For example, if one wanted to lift a weight, however enormous, one had only to get a sheet of this substance beneath it and one might lift it with a straw. My first natural impulse was to apply this principle to guns and iron-clads and all the material and methods of war and from that to shipping, locomotion, building, every conceivable form of human industry. The chance that had brought me into the very birth chamber of this new time—it was an epoch, no less—was one of those chances that come once in a thousand years. The thing unrolled, it expanded and expanded."

The facts in the case seem to be these. By careful experimentation it has been found that the absorption of terrestrial gravitation by a sheet of lead five centimeters in thickness, if any, is less than two parts in a hundred billion. If the effect had been at all appreciable we should long since have observed it, for the screening effect of the earth upon the gravitational pull of the moon would have made itself evident in the tides on that side of the earth opposite to the moon.

### 4. *Mechanical Theories of Gravitation.*

The following questions long ago forced themselves upon scientists: Why should gravitation exhibit these negative characteristics? Why should gravitation be wholly inert to experiment except as the one important property of attraction is invoked?

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[21] Page 18.

Many theories have been propounded in the past to explain gravitation and like those which we reviewed for the ether they are based upon material models. That desire of the human reason for analogy and rational explanation is at its highest in the history of these numerous attempts.

It will suffice for the purpose of this volume to mention only two, one by Newton and the other by Georges-Louis Lesage (1724-1803), a French-Swiss physicist and mathematician. The story of the attempt to give a material explanation to the force of gravitation is similar to that of the long struggle to endow the ether with substantial properties. One striking difference is to be observed, however. Out of the efforts of the investigators of the ether there finally emerged a magnificent theory of light in terms of electro-magnetic forces; from it we acquired the boon of wireless communication and a tool of great power in exploring the mysteries of energy. The attempts to endow gravitation with a material explanation have been sterile. We have acquired no music of the spheres; we have gained no insight into the relationships between useful quantities. Gravitation stands alone in the universe, a strange and perplexing entity. Nothing connects it with electricity, magnetism, or light. It is constant and unchanging, a sphinx,whose riddle cannot be answered by the delicate balances of the laboratory. Its extreme minuteness baffles experiment; it acts in one direction only and does not exhibit the phenomenon of repulsion common to the two apparently similar entities of electricity and magnetism.

We have already quoted the famous passage of a letter written by Newton to Rev. Richard Bentley in which he expresses the opinion that "it is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter, without mutual contact." This matter evidently recurred often to him as we see in allusions scattered through his writing, but twenty-five years after the letter to Bentley we find that he has reached a tentative explanation. In an appendix to the second edition of his *Opticks*, published in 1717, he reflects upon the nature of the all-pervading ether with which he thought space endowed, and states his postulates as follows:

"Is not this medium much rarer within the dense bodies of the sun, stars, planets, and comets, than in the empty celestial spaces between them? And in passing from them to greater distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great bodies toward one another, and of their parts towards bodies; every body endeavoring to go from the denser parts of the medium toward the rarer? . . . And though this increase of density may at great distances be exceedingly slow, yet if the elastic force of this medium be exceedingly great, it may suffice to impel bodies from the denser part of the medium toward the rarer, with all that power which we call gravity."

This statement of Newton is a curious exhibit of the desire of the mind for material explanation. We must remember that Newton's age marked the first genuine swing away from the occult mysticism and the nebulous speculation of the philosophy of the middle ages. The tenet of some philosophers has been that to have the appearance of wisdom one must be obscure. It reminds one of the story of "The Emperor's New Clothes" as related by Hans Christian Andersen. Two rogues disguised as tailors entered the monarch's domain and commenced the task of weaving new robes for the emperor. They caused the statement to be circulated that the new garments would appear invisible to anyone who was unfit to hold his office or who was incorrigibly stupid. Those who were called in to view the imaginary fabrics upon the looms exclaimed how beautiful their texture was and admired the details of workmanship. Finally the day came for the emperor to parade before his people in his new splendor; the imaginary garments were placed upon him, and he exhibited himself upon the streets. "What beautiful robes," cried the people, until a little child exclaimed, "Why the king hasn't any clothes on!" and the spell was broken.

One likes to turn to the other view of Newton, expressed at the conclusion of the third book of the *Principia,* where he says: "Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypothesis; for whatever is not deduced from the phenomena is to be called an hypothesis. . . . To us it is enough that gravity does really exist and act according to the laws which we have explained."

One of the most ingenious theories advanced to give a mechanical explanation of gravity was that due to Lesage who constructed his hypothesis in 1747 when he was only twenty-three years of age. He is said to have been so elated over the idea that he wrote to his father: "Eureka! Eureka! Never have I felt such satisfaction as at this moment, in which I have just succeeded in explaining completely, by the simple laws of rectilinear movement, the principle of *universal gravitation*."[22]

The theory advanced by Lesage was that space was filled with a huge swarm of "ultramundane corpuscles" of very minute size moving rapidly in all directions. An object placed in such a sea of particles would naturally remain at rest because as many corpuscles would strike it on one side as upon the other and there would be no pressure differential to create motion. It would be otherwise, however, with two bodies immersed in such a "gravitational fluid," because each would screen off some of the corpuscles from the other and the difference in pressure that would thus result would tend to drive the two objects together. Moreover, it is possible to show that the impelling force would vary inversely as the square of the distance between the two bodies.

This theory of Lesage appealed to the philosophers of the nineteenth century because of its simplicity and ingenuity. To be sure it substituted for an incomprehensible attraction, acting at a distance, a metaphysical swarm of ethereal corpuscles, but the picture seemed easier to grasp and accorded more closely with material models. Even Poincaré devotes several pages of his philosophy to a consideration of the consequences of the mechanical problem thus presented.[23]

The weakness of the theory was found to be precisely the weakness of the elastic solid theory of the ether. As soon as one tried to investigate the elasticity of the corpuscles or the material density of the medium which their swarm created, the old dilemmas reappear. If, for example, the particles were assumed to be perfectly elastic there would be no momentum given to the bodies immersed in them and hence no attraction. Furthermore, the law of inverse squares is not derivable exactly from the Lesage picture unless the corpuscles are completely inelastic, and this leads to difficulties with the astronomical observation that the planets move about the sun without encountering any resistance from the space medium. To escape this difficulty it is necessary to assume that these ultramundane corpuscles must have a velocity at least $24 \times 10^{17}$ times that of light, but if this be granted then the heat generated by the impact of the particles upon such an object as the earth would be enough to raise the temperature $10^{26}$ degrees a second. The heat necessary to maintain such a gradient may be calculated to be $10^{20}$ times the total output of the sun's radiation. Alas for the double eureka of the imaginative Lesage! [24]

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[22] In its completed form it appeared in the transactions of the Royal Academy of Berlin in 1782 under the title: "Lucrece Newtonien." For an historical account see W. B. Taylor: "Kinetic Theories of Gravitation," *Smithsonian Inst. Report* for 1876, pp. 205-282.

[23] *Foundations of Science*, pp. 517-522.

### 5. *Space, Time, and Gravitation.*

In this, as in the case of the ether, mechanical analogies and material concepts fail us. On the one hand we have the fact of attraction, and one would be mad to deny the evidence of the delicate balances of Cavendish; on the other we have the fact of the transport of energy, and we should be equally mad to deny the evidence of diffraction. But when we try to postulate a transcendental fluid and endow it with material properties of any kind we are faced with contradictions. One who in the enthusiasm of youth wishes to try the old problem again and to enter the lists with the lance of imagination, must be very cautious how he exults over the apparent demise of his metaphysical enemy. The only permanent conquest is likely to be found in the mystical symbols of a mathematical formula which is capable of transformation into conclusions in accord with laboratory experiment.

We turn finally to a consideration of a recent great explanation of gravitation. That it is the ultimate theory may be doubted. That it meets the criteria just laid down, first of formalization in terms of equations and second of reaching conclusions vindicated by experiment is a point not to be controverted. Einstein, following the traditions of Mach and under the stimulus of the success of his special theory of relativity, fell to musing upon the negative characteristics of the gravitational field. Suddenly he was struck by the following question: *Is not gravitation, itself, an attribute of space and time rather than an attribute of matter?* In other words, perhaps the negative character of these propositions with regard to gravitation may be explained by the assumption that we are experimenting with space and time rather than with matter itself.

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[24] See "Considerations on Gravitation" by H. A. Lorentz, *Proceedings of the Royal Academy of Amsterdam*, Vol. 2, 1900, pp. 559-574. The theory has recently been revived in a non-mathematical form with radiation pressure replacing the impact of the Lesagian particles in James Mackaye's: *The Dynamic Universe*, 1931.

This inspired reflection was destined to bear great fruit. According to the theory of Einstein it should be possible to shut off gravitation, not by the interposition of a screen of matter, but by changing the character of space. Isotropic space is itself changed by the presence of matter and from this interaction there follows the phenomenon of gravitation. Comparable with the classical story of Newton's apple is another relating to the development of the geometrical theory which we are describing. Einstein, according to the story, once saw a laborer fall from the roof of a building. Rushing up to the man, who was fortunately unhurt, Einstein asked him whether in his fall he had felt anything actually pulling upon him. The man replied in the negative and this answer corresponded exactly with the conclusions reached by Einstein in his musings.

The point may be clarified, perhaps, or at least illustrated, by a passage in the early pages of *Alice's Adventures in Wonderland* by Lewis Carroll. Thus we read:

"The rabbit-hole went straight on like a tunnel for some way, and then dipped suddenly down, so suddenly that Alice had not a moment to think about stopping herself before she found herself falling down what seemed to be a very deep well.

"Either the well was very deep, or she fell very slowly, for she had plenty of time as she went down to look about her, and to wonder what was going to happen next. First, she tried to look down and make out what she was coming to, but it was too dark to see anything: then she looked at the sides of the well, and noticed that they were filled with cupboards and book-shelves: here and there she saw maps and pictures hung upon pegs. She took down a jar from one of the shelves as she passed: it was labeled 'ORANGE MARMALADE,' but to her great disappointment it was empty: she did not like to drop the jar, for fear of killing somebody underneath, so managed to put it into one of the cupboards as she fell past it.

" 'Well!' thought Alice to herself. 'After such a fall as this, I shall think nothing of tumbling down-stairs! How brave they'll all think me at home! Why, I wouldn't say anything about it, even if I fell off the top of the house.' (Which was very likely true)."

While this passage is accepted as a childish fantasy in keeping with the other adventures of Alice, it is not entirely absurd when examined from the point of view of the new theory. According to Einstein, it is the easiest thing in the world to destroy a gravitational field entirely. Just start to fall. The gravitational field immediately disappears. Take your watch out of your pocket and place it in the air at your side. It will float there without support. The only practical difficulty in the situation is found if one should encounter some other material object such as the surface of the earth which is falling in the opposite direction. The explanation of the adventure related by Carroll appears to be that the hole into which Alice tumbled was itself in motion with an acceleration that partly destroyed the original field of gravitation.

To put the matter a little more scientifically let us consider an illustration used by Einstein himself. Suppose that we are riding in an elevator and have no knowledge whatsoever of the world outside. We set out to examine the laws of nature as we find them and we set up a pendulum or a gyroscope or some other device whose behaviour depends upon the gravitational field in which it is placed. If the elevator is at rest with respect to the earth everything will take place inside the elevator according to the same laws of matter that we find in our ordinary text books of mechanics. But suppose that the elevator is moving upward with a uniform acceleration. Not knowing anything about the universe outside we should be under the impression that we are living in a universe with a tremendous field of gravitation. If the elevator moved fast enough we might duplicate even the great gravitational field of the planet Jupiter.

On the other hand if the elevator moved with an acceleration of thirty-two feet per second, which is equal to that of the field of the earth, all gravitation would completely disappear. We should not need tables, because table-cloths could be spread out on the air and dinner served upon them without the slighest chance of an accident. There would be some difficulties, of course, such as the pouring of tea which would have to be transformed into the pumping of tea, since no force of gravitation would be there to perform the necessary act of pouring. There would be neither up nor down, and space would be exactly the same in all directions. The only forces to be overcome would be those due to the inertia of matter. Commodities such as sugar and flour would no longer be sold by the pound weight, but by the pound inertia. There is an interesting story in point in the tragedy of two miners who found a large quantity of gold near the Arctic circle. This they carefully weighed on a spring balance and shipped to San Francisco for mutual division. But when the gold was re-weighed at the California city a certain amount of it was found to have disappeared. The loss of something like three pounds in a thousand was due to the variation in gravity (the difference between the gravitational pull of the earth and the opposite centrifugal force created by the rotation of the earth). An unhappy tragedy attendant upon the discovery of this loss in the gold would have been avoided had the original measurements been made according to inertial mass. This could have been accomplished by the use of a beam balance instead of a spring balance in the original weighing.

It is also possible to invert gravitation completely. Suppose that the acceleration of the elevator be increased beyond thirty-two feet. Our table-cloth would now no longer float in the air but would fall upwards toward the ceiling. The whole situation would be that of Topsy Turvey land.

## 6. *Einstein's Theory of Gravitation.*

What then is Einstein's theory of gravitation? Gravitation, according to Einstein, is an intimate property of the space-time continuum, because it is clear that we can do away with it by a proper choice of the space-time with which we are to experiment. This may not always be desirable in practical affairs, because none of us would care to live very long in a falling elevator, but one can see from the illustration that it is possible to reverse gravitation and otherwise to change it by the device suggested.

Let us see what conclusion can be drawn from this point of view.

First, it explains why gravitation has for so many years given negative results to our experiments. Second, it explains the equivalence of inert mass and weight. For it is clear that a property of space-time could not be screened away by the interposal of matter, nor should it be affected by means of such a material agent as temperature.

These remarks may at first sight appear as metaphysical as those of the old sophists, but here is the marvel. Einstein actually derived an equation for space-time in the neighborhood of gravitating matter. The technical machinery by means of which he reached this interesting equation is very difficult mathematical reading and has led to the remark that only a dozen or so mathematicians are able to understand relativity. While the mathematics is difficult, the point of view and the conclusions are not necessarily so.

The actual words used by Einstein in explaining his novel concept, while not entirely free from technical nomenclature, will yield their gem of thought if they are carefully scrutinized. The following significant passages are quoted from Einstein's lectures delivered in May, 1921:[25]

"In the first place, it is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum) which acts itself, but which cannot be acted upon. This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in an unaccelerated motion relatively to space, but relatively to the center of all the other masses in the universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo. In order to develop this idea within the limits of the modern theory of action through a medium, the properties of the space-time continuum which determine inertia must be regarded as field properties of space, analogous to the electro-magnetic field. The concepts of classical mechanics afford no way of expressing this. For this reason Mach's attempt at a solution failed for the time being. . . . In the second place, classical mechanics indicates a limitation which directly demands an extension of the principle of relativity to spaces of reference which are not in uniform motion relatively to each other. The ratio of the masses of two bodies is defined in mechanics in two ways which differ from each other fundamentally; in the first place, as the reciprocal ratio of the acceleration which the same motional force imparts to them (inert mass), and in the second place, as the ratio of the forces which act upon them in the same gravitational field (gravitational mass). The equality of these two masses, so differently defined, is a fact which is confirmed by experiments of very high accuracy (experiments of Eötvös), and classical mechanics offers no explanation for this equality. It is, however, clear that science is fully justified in assigning such a numerical equality only after this numerical equality is reduced to an equality of the real nature of the two concepts.

"That this object may actually be attained by an extension of the principle of relativity, follows from the following consideration.

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[25] *The Meaning of Relativity*, 1923, Princeton University Press, pp. 62-65; third edition, 1950, pp. 55-58.

A little reflection will show that the theorem of the equality of the inert and the gravitational mass is equivalent to the theorem that the acceleration imparted to a body by a gravitational field is independent of the nature of the body. For Newton's equation of motion in a gravitational field, written out in full, is

(Inert mass) . (Acceleration) = (Intensity of the gravitational field) . (Gravitational mass).

It is only when there is numerical equality between the inert and gravitational mass that the acceleration is independent of the nature of the body. Let now $K$ be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to $K$, free from acceleration. We shall also refer these masses to a system of co-ordinates $K'$, uniformly accelerated with respect to $K$. Relatively to $K'$ all the masses have equal and parallel accelerations; with respect to $K'$ they behave just as if a gravitational field were present and $K'$ were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that $K'$ is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only $K$ is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, $K$ and $K'$, we call the 'principle of equivalence'; this principle is evidently intimately connected with the theorem of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to coordinate systems which are in non-uniform motion relative to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to $K$) or under the combined action of inertia and gravitation (with respect to $K'$). The possibility of explaining the numerical equality of inertia and gravitation by the unity of their nature gives to the general theory of relativity, according to my conviction, such a superiority over the conceptions of classical mechanics, that all the difficulties encountered in development must be considered as small in comparison.

"What justifies us in dispensing with the preference for inertial systems over all other coordinate systems, a preference that seems so securely established by experiment based upon the principle of inertia? The weakness of the principle of inertia lies in this, that it involves an argument in a circle: the mass moves without acceleration if it is sufficiently far from other bodies; we know that it is sufficiently far from other bodies only by the fact that it moves without acceleration. Are there, in general, any inertial systems for very extended portions of the space-time continuum, or, indeed, for the whole universe? We may look upon the principle of inertia as established to a high degree of approximation, for the space of our planetary system, provided that we neglect the perturbations due to the sun and planets. Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of the special theory of relativity . . . hold with remarkable accuracy. Such regions we shall call 'Galilean regions.' We shall proceed from the consideration of such regions as a special case of known properties."

We can summarize these conclusions of Einstein in the following way. It is first assumed that gravitation is an aspect of geometry and as such can be made to reveal the secret of the kind of space-time in which we dwell. One will recall from previous explanations in this book that there are three subjective geometries. One of these, due to Lobachevski and Bolyai, assumes that the sum of the angles of a triangle is less than 180 degrees; a second, due to Euclid, assumes that the angle sum is exactly equal to two right angles; the third, to which the name of Riemann is attached, postulates that the sum of the angles exceeds 180 degrees. What then is the geometry of space-time in which a gravitational force reveals itself?

If the gravitational force found always in the presence of material things is to be regarded as evidence of a warp in space-time, this should be capable of mathematical expression. This idea of a warp in space is found, curiously enough, in a short document by W. K. Clifford (1845-1879) published in 1876. This brilliant young mathematician postulated: "(1) That small portions of space *are* in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them. (2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave. (3) That this variation of the curvature of space is what really happens in that phenomenon which we call the *motion of matter,* whether ponderable or ethereal. (4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity."

Einstein, following the ordinary procedure of geometers, first set down a very general mathematical formula, which he defined as the metric of space-time. In this formula there occurred ten coefficients, which were to be chosen in such a manner that two fundamental criteria were to be satisfied. The first of these criteria was the very natural one *that the geometry, or in other words, the space-time metric, was to be independent of the mathematical coordinates used.* To put this other-wise, what does nature care for the language man uses to describe her laws? The second criterion merely assumed *that the coefficients of the space-time metric were to be so chosen that space-time was "flat" at distances remote from gravitating matter.* In one way this statement is equivalent to the special theory of relativity, which may be regarded as a statement of the invariance of "flat" space-time.

It is difficult, if not impossible, to describe the new theory more adequately without invoking the aid of mathematics. Einstein, employing the absolute differential calculus, an abstract mathematical theory developed from the fundamental work of G. F. B. Riemann (1826-1866) and E. B. Christoffel (1829-1900) by G. Ricci and T. Levi-Civita (1873-1942), solved his problem and found an explicit evaluation for his ten gravitational coefficients. The space-time metric of the objective universe was revealed to his view. The problem of gravitation was now merged in the problem of space-time. Some of the more speculative consequences of the new metric will be postponed to the next chapter, and we shall turn to a review of the three crucial experiments which vindicated the new theory.

### 7. *The Vindicating Experiments.*

The first of these experiments concerns the bending of light rays in the vicinity of a gravitation body. Newton had previously held the view that gravitation would influence the path of a ray of light, but for a very different reason. Light in his hypothesis was composed of a swarm of material particles and hence like any other body should suffer deviation in the neighborhood of a large material body like the sun. The deflection of light passing close to the limb of our sun would, on the basis of the corpuscular theory, amount to an angle equal to 0.87 of a second. But the space-time metric of Einstein gave a different answer. Space-time in the neighborhood of the sun, due to the gravitational field, would vary from the space-time of Euclid's geometry by an amount sufficient to deflect a ray of light by an angle twice that predicted by Newtonian theory or 1.74. This bending is not to be attributed directly to gravitational pull, you will understand, but is due to the *warped nature of space-time* in the vicinity of the gravitating body.

Is there experimental evidence for this belief? When Einstein made his spectacular announcement it immediately became the goal of several eclipse expeditions to verify or disprove the statement. When the shadow of the moon covers the surface of the sun, the stars in the immediate neighborhood of the shadow may be seen. Their light is caught upon a photographic plate and a comparsion can then be made between their apparent and real positions as computed for the time of the eclipse. Figure 1 shows what was actually found in the plate obtained by the Crocker Eclipse Expedition to Wallai, Western Australia, 1922. The dots represent stars and the dashes attached to them measure the displacements. It will be evident that on the average the stars near the limb of the sun suffer a greater displacement than those further away. A somewhat more vivid picture of the agreement between theory and experiment

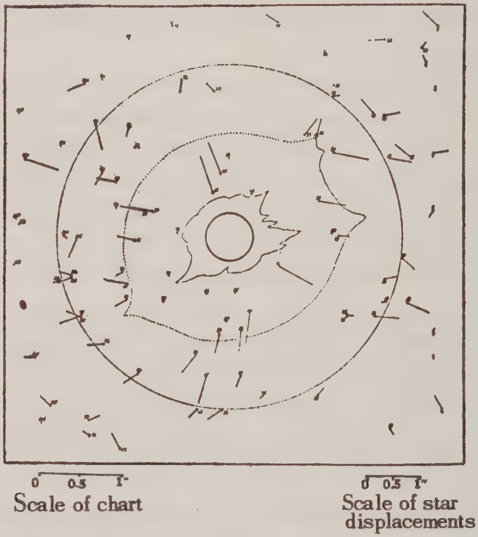

Figure 1. The actual displacement of stars observed during an eclipse of the sun at Wallai, Western Australia, 1922. The irregular figure about the sun is the outline of the corona. (From *Lick Observatory Bulletin*, No. 346.)

is found in the diagram in Figure 2, the broken line being the average value of the position of the stars. This is regarded generally as a very satisfactory agreement when all the sources of experimental error are taken into account.[26] This same effect was reported again in studies of the eclipse of 1952.

A second triumph for the theory of the space-time view of gravitation has been found in the explanation afforded of the one outstanding discrepancy between Newton's theory of gravitation and actual observation. Mercury, the tiny planet that lies so close to the sun that it can be seen with the naked eye only a few times a year, has never behaved according to Newton's law of gravitation. The perihelion of its elliptic orbit is changing in space at the approximate rate of 43 seconds of arc per century. This tiny discrepancy has long puzzled astronomers, and until it has been explained away no one will feel that the law of gravitation as stated by Newton is absolutely correct. As a matter of fact it may be the key that unlocks new mysteries of space and time. One should remark in passing, however, that the tiny variation which we have mentioned comes out of a tremendous distillation of mathematical equations. Urbain Jean Joseph Leverrier (1811-1877), the great French astronomer who shares with Adams the glory of having discovered Neptune, and

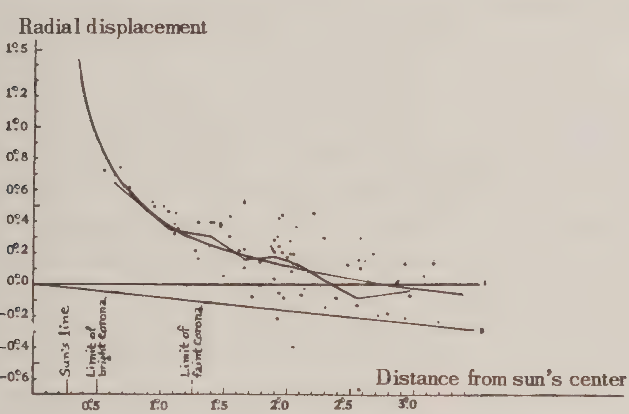

Figure 2. Diagram showing the displacement of the stars the light from which passes close to the sun. The smooth line is Einstein's prediction; the broken line is the average displacement. (From *Lick Observatory Bulletin*, No. 346.)

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[26] From the *Lick Observatory Bulletin*, No. 346.

Simon Newcomb (1835-1909), father of the American ephemeris, both agreed that, after the long equations determining the perturbations of the planets have been distilled and redistilled, there remains a small residual error which can be accounted for by introducing the perihelion shift already noted. That there may exist other neglected facts in this very difficult problem is a possibility not wholly to be discarded, and the transits of the tiny Mercury across the great disk of the sun are watched with interest by the astronomers.

Because of the great importance of the perihelion shift, K. P. Williams undertook in 1929 the arduous task of recomputing this value, making use of transits of Mercury, which were not available to either Leverrier or Newcomb. The former had published in 1859 the value 38".3 for this shift which Newcomb in his table for Mercury in 1898 had increased to 43".37. Taking unusual precautions, Williams published his result in 1939, which reduced Newcomb's value, the generally accepted one, to 40".56.[27] Einstein's theory requires the value 42".90.

The thing significant in the present instance is the fact that the theory of gravitation as announced by Einstein accurately accounts for this perihelion discrepancy. Space-time in the neighborhood of the huge gravitating sun has been sufficiently changed so that Mercury, the smallest of the planets, exhibits a slight variation in its orbit. One would not care to state that this single triumph is justification for belief in Einstein's theory, but there is another experiment of similar character which adds new luster to the victory.

In another chapter it will be pointed out that some hold to the belief that matter radiates heat, light, wireless waves, etc., because the tiny constituent electrons are constrained to move in elliptical orbits with dimensions depending upon the frequency of the radiation. The radiation is thoroughly characterized by the lines of its spectrum, so we should expect to find some evidence for or against the postulates of relativity in the velocity of the electrons. That this is a tenable hypothesis has been pointed out in the spectrum of hydrogen. Hydrogen is the simplest of the gases and, according to a theory developed later in the book, consists of a single electronic planet revolving at high speed around a single proton nucleus. Should not the high velocity of the electron make a difference in the space-time in its immediate neighborhood? According to A. Sommerfeld this is actually the case and the effect to be looked for is a precession of its perihelion exactly like that of the planet Mercury although much more rapid. If such an advance of perihelion actually exists it should make itself known by a fine division of the spectral lines of hydrogen known as *the fine-structure* of the lines. The fine structure thus predicted has been found experimentally. According to Sommerfeld this beautiful fact furnishes "ocular evidence not only of the actual occurrence of the elliptic orbits but also of the variability of the electronic mass."[28]

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[27] *The Transits of Mercury.* Indiana University Publications, Science Series, No. 9. See, in particular, the *Supplement*, independently published, p. 5.

Another triumph of even more spectacular character has come about through the third prediction of Einstein. "Time," said he, "changes in heavy gravitational fields so that the time on the sun is different from our time on the earth." A very good clock to use in testing this conclusion is the vibration of radiating atoms. These vibrations we know with very great precision through a study of the lines of the spectrum. How, then, shall we decide whether time is the same on the sun as on the earth? The answer is to compare the lines in the spectrum of an element on the sun with lines created by heating the same element in a laboratory on the earth. If the lines in the solar spectrum are shifted toward the red end we can interpret this difference to be due to the slower vibration of the atom in the sun. In other words clocks on the sun will thus be proved to be slower than on the earth.

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[28] From *Atomic Structure*, Eng. translation, 1923, Chap. 8. It must be confessed, however, that this relativity explanation has not proved competent to explain all the phenomena of fine-structure. To it there has been added the concept of an *electron spin* which connects the phenomena with electronic magnetism. The brilliant work of P. A. M. Dirac, however, has been able to unite the two theories of the origin of fine-structure by showing that the electron spin is a direct consequence of the relativity principle when the latter is applied to the equations of the wave mechanics (See Chapter 10). Employing these results Sommerfeld has been led to conclude that electron magnetism is itself relativistic in origin. "Instead of an incomprehensible accident we see here the manifestation of a deep-seated identity which must ultimately lead to a solution of the problem of the electron itself, which includes such questions as: Why do negative and positive electricity occur in elementary quanta, why is their mass unequal, what is their structure, why do they hold together?" *Wave Mechanics.* English edition, 1930, p. 118.

One must remember, however, that there is another interpretation given to the shifting of spectrum lines, which is known to astronomers as the *Doppler effect.* Just as the pitch of a train's whistle rises as the train approaches, and lowers when it recedes, so the spectrum lines move toward the violet as the star approaches and toward the red as it recedes from us. In the accompanying plate we see this Doppler shift clearly in the lower spectrum obtained from the light of Mu Orionis. The lines of the star's spectrum show a displacement corresponding to a recessive velocity of twelve miles per second. Hence, conclusions formed on the basis of a spectrum shift become a little doubtful. Shall the shift be attributed to recessive velocity or to the structure of time? This question has to be decided from other contributory evidence.

After prolonged experimentation on these exceedingly delicate differences between earth-time and sun-time, C. E. St. John of the Mount Wilson observatory finally committed himself as follows:[29]

"The conclusion is that three major causes are producing the differences between solar and terrestrial wave-lengths, and that it is possible to disentangle their effects. The causes appear to be the slowing up of the atomic clock in the sun to an amount predicted by the theory of generalized relativity, radial velocities of moderate cosmic magnitude and in probable directions, and differential scattering in the longer paths traversed through the solar atmosphere by light coming from the limb of the sun."

But here is the most surprising part of the story. Astronomers were not quite satisfied with the conclusions reached in the investigation of the difference between sun-time and earth-time; they looked about in the sky for another object where there might be unmistakable differences in space-time due to the gravitational difference. Fortunately there is a remarkable object in the heavens, the tiny white dwarf companion of Sirius, the Dog Star. This small object is only of the 8.5 magnitude, which means that it is beyond naked-eye visibility. It has been ascertained that this star, the white dwarf, has about two-fifths the mass of the brilliant Sirius and from an examination of its spectrum it is estimated that its surface temperature is greater than that of our own sun. In order to reconcile these facts thrust upon us by a hot, heavy star that is beyond naked-eye visibility, it has been necessary to assume that the density of this amazing object reaches the unbelievable value of at least 50,000 times that of water, approximately one ton per cubic inch.

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[29] "On Gravitation Displacement of Solar Lines," *Monthly Notices of Royal Astro. Soc.,* Vol. 84, 1923, pp. 93-96. See also *Proc. of National Academy of Sciences,* Vol. 12, 1926, pp. 65-68.

Astronomers at first found such densities difficult to believe, but the evidence increases that stars exist with even a much greater concentration of mass. G. P. Kuiper has reported the existence of a star with a density one thousand times as great as that of the Sirius dwarf.[30] The characteristics of these curious objects, some forty of which have been reported according to recent lists since the original discovery of the companion of Sirius, will be discussed in more detail in another chapter. But since the existence of such heavy stars is itself a matter of much interest, independent of the relationship of their masses to the gravitational theory of Einstein, it is worth while to review the arguments by which one reaches the conclusion as to their densities.[31]

Considering the companion of Sirius, we know from observation that the period of rotation of the visible star is 49.3 years and that the major axis of the path of the companion is 20 earth-radii. From these figures and the third law of Kepler, we can deduce without difficulty that the mass of the double star is approximately 3.5 times that of the sun. Moreover we can measure not only the path of the white dwarf companion, but also the path of both stars relative to the center of gravity of the system. This leads to the conclusion that Sirius has a mass 2.5 times that of the sun and that the mass of the dwarf, consequently, is equal to that of the sun.

In the face of this conclusion a surprising fact is now observed. At a distance of nine light-years away the white dwarf is a star of only 8.5 magnitude, which, in less technical language, means that it appears to be only 1/376*th* as bright as our sun. This is not so remarkable, of course, until we learn from a study of the dwarf's spectrum that it is considerably hotter than our sun, having, as a matter of fact, a surface temperature of about 8,000° centigrade as compared with 5,900° for our sun. The Stefan-Boltzmann law of radiation (See Chapter 10) states that the total radiation of energy per unit volume is proportional to the fourth power of the absolute temperature of the radiating body. Hence the surface brightness of the dwarf must be actually 3.23 times that of the sun.

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[30] See *Pub. Astro. Soc. of the Pacific*, Vol. 47, 1935, p. 307.

[31] See A. S. Eddington: *Monthly Notices of Royal Astro. Soc.*, Vol. 84, 1924, p. 308; also *A Debate on Relativity*, Chicago, 1927, pp. 102-104.

The facts, then, appear to be these: that the dwarf, while having the same mass as the sun and a surface brightness three and a quarter times as great appears to be a star only 1/376*th* as bright as our sun. The obvious conclusion that we can draw from these figures is that the dwarf has a surface only 1/1215 as large as that of the sun and hence must have a radius only 0.029 as large as the radius of the sun. Since the density of the sun is approximately 1.4 times that of water, and since the relative densities of the sun and the dwarf are proportional inversely to the cubes of their radii, a simple calculation shows that the dwarf must have a density in excess of 50,000 times that of water.

Returning now to the main problem, the testing of the third prediction of Einstein, we see that we have in the white dwarf with its amazing density an excellent object with which to investigate the nature of space-time in the vicinity of concentrated matter. According to theory, the shift of the spectral lines on the companion of Sirius should be about forty times the corresponding shift for our sun.[32] When the measurements were made under great technical difficulties at Mount Wilson, the actual shift was found to be slightly larger than the theoretical one. This has been regarded by some as the greatest achievement of the Einstein gravitational theory.

In commenting upon this result W. S. Adams, who was responsible for the details of the experiment, remarked:[33]

"Although such a degree of agreement [between predicted and observed values] can only be regarded as accidental for observations as difficult as these, the inherent accord of the measurement made by different methods, and in particular with the registering microphotometer, is thoroughly satisfactory. The results may be considered, therefore, as affording direct evidence from stellar spectra for the validity of the third test of the general relativity theory, and for the remarkable densities predicted by Eddington for the dwarf stars of early type of spectrum."

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[32] The shift is actually 0.3 of an Ångstrom unit as compared with 0.008 for the sun. An Ångstrom unit is $10^{-8}$ cm.

[33] "Relativity of Displacement of the Spectral Lines in the Companion of Sirius," *Proc. of Nat. Academy of Science,* Vol. 11, 1925, pp. 382-387.

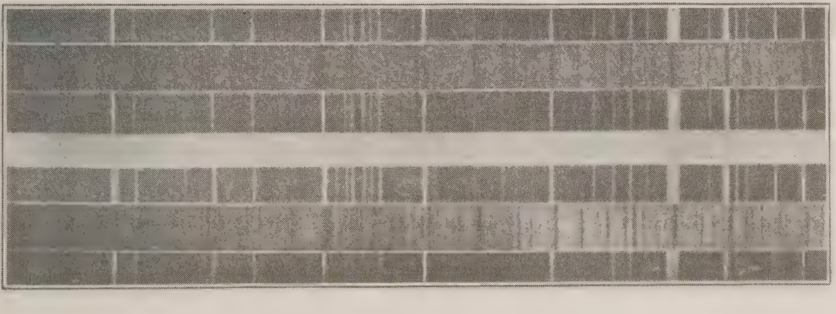

PLATE VI

The central spectrum in each series is that of the spectroscopic binary Mu Orionis. The dark lines are compared with the corresponding lines of titanium and show the Doppler shift toward the red end of the spectrum. This is interpreted as due to a recessive velocity. The lower spectrum indicates a velocity of twelve miles per second. (Yerkes Observatory).

### 8. *The Music of the Spheres.*

In the mystical pseudo-science of Aristotle we find attributed to Pythagoras the idea of the music of the spheres. "Some authorities," says Aristotle,[34] "maintain that large revolving bodies must cause sound, since bodies on our earth, which are smaller and move with less speed, do so. Inevitable, they say, there must be a terrific noise caused by the sun, moon, and other large heavenly bodies moving at such speed. Assuming this and assuming that their velocities and the intervals between them are harmonious, they declare that the revolving stars have a musical sound. They say that the reason why we do not hear this sound—a thing which seems unreasonable—is that it has been part of our experience from birth and is not distinguishable from silence. For sound and silence are only distinguishable with reference to each other. It is similar to the case of coppersmiths, who on account of their regular habit think nothing of the sounds made by their own tools. In the same way mankind generally does not notice the sound of the revolving heavenly bodies. As I have already said, there is much elegance and beauty in this theory, but it cannot be true." The slower and near bodies radiate deep notes and the swifter and further bodies give out the higher notes which blend into a celestial or cosmic octave.

Is there any truth to this mystic science, in spite of the adverse opinion of Aristole? In 1918 Einstein discussed the existence of gravitational waves[35] on the basis of his relativity theory and reached the interesting conclusion that "every change in the distribution of matter produces a gravitional effect which is propagated in space with the velocity of light." This idea is not a wholly modern one since the possibility of finite propagation of gravitation has been discussed by various mathematicians and astronomers in the past. We find Laplace investigating the velocity on the basis that gravitation is an evidence of fluid pressure. In order to reconcile the fact that the planets move without appreciable loss of energy due to resistance encountered from the space-filling entity, the eminent astronomer was led to the conclusion that the velocity of propagation of attraction could not be less than six million times that of light.

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[34] From *De Caelo,* II, 9. Translation made by R. H. Coon.

[35] See "Über Gravitationswellen," *Sitzungsberichte der Preuss. Akad. der Wissenschaften,* 1918, pp. 154-167; also H. Weyl: *Raum, Zeit, Materie.* Eng. trans., 1921, pp. 248-252.

F. Zöllner,[36] years later, making use of a formula originally due to H. Weber, suggested a modification of the simple law of Newton, in which terms were introduced containing a velocity factor. If this velocity were chosen equal to that of light the modification caused in the orbit of Mercury would be a precession of the perihelion equivalent to 14.52 seconds, which is, of course, significantly inferior to the one agreed upon by Leverrier and Newcomb. In order to attain the precession of 38 seconds, found by Leverrier and later increased to 43 seconds by Newcomb, a velocity of propagation equal to three-fifths that of light would be required. Similar modifications by Riemann, Gauss, Clausius, Seeliger, and others failed to identify in any natural way the velocity of the propagation of gravitation with the velocity of light so that the perihelion shift could be satisfactorily accounted for.[37]

Only P. Gerber in a remarkable paper published in 1898, seventeen years before the appearance of Einstein's gravitational theory, succeeded where the others failed.[38] Unfortunately his law of gravitation has never received the attention that it deserved, although it is mentioned as noticeable by E. Mach in his *Science of Mechanics*, who comments on the achievements with this remark: "This would speak in favor of the ether as a medium of gravitation." As a matter of historical record it should be pointed out that Gerber actually wrote down the formula given by Einstein for the perihelion shift of Mercury, using, however, his assumed velocity of gravitation where Einstein used the velocity of light. Postulating the existence of the shift as an astronomical reality, Gerber substituted its value in his formula to compute the velocity of gravitation, which he was able thus to identify with the velocity of light.

The new approach of Einstein has yielded the answer that appeals as a reasonable one and its derivation is a natural consequence of the law of gravitation which takes its structure from the character of the space-time metric. Verification experimentally has naturally been impossible to attain because of the infinitesimal differences involved. If all this is true, however, there is meaning, perhaps, to the music of the spheres. If we could only devise an Eötvös balance sensitive enough we might be able to catch these ethereal vibrations and turn them into sound as we now do with the waves of the radio. Is it too wild a dream to hope that someday we may hear the soft vibrations of the planet Mars or the orchestral symphony of the moons of Jupiter?

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[36] *Principien einer electrodynamischen Theorie der Materie*, Leipzig, 1876.

[37] For an account of these researches see F. Tisserand: *Traité de Mécanique Celeste*, Vol. 4, Paris, 1896, chap. 28.

[38] "Die raümliche und zeitliche Ausbreitung der Gravitation." *Zeitschrift für Math. und Phys.*, Vol. 43, 1898, pp. 93-104.

### 9. *What is Gravitation?*

And what are our conclusions about the mysterious action at a distance, with which we started in the first chapter? We have traced its history from the laws of Kepler, through the magnificent postulates of Newton, to the new revelation of Einstein. We have seen it start with the metaphysical assumption that objects are influenced by a force which acts through the vacuum of space; we have reviewed its struggle to gain objective reality by the postulate of an ethereal fluid composed of rapidly moving "ultramundane corpuscles"; we have traced the attempts of careful experimenters to discover some objective characteristic of gravitation other than that of attraction; we have finally witnessed the overwhelming of these objective pictures in the subjective equations of Einstein, in which space-time becomes the reality. Here at last we are able to gain new experimental evidence where all previous efforts had been in vain.

And what conclusion shall we draw from the history of this noble conquest? Will it not be similar to that derived from the struggle of science to endow the ether with rational existence? Both concepts lose their identity in the reality of space-time; all other objective existence is denied them. Their objectivity is replaced by sets of equations, which for the one, determine the paths of the planets with marvelous precision, predict the deflection of rays of light in the neighborhood of matter and the shifting of spectrum lines in heavy stars; and for the other, tell the story of the transfer of radiant energy through the emptiness of space, explain diffraction patterns, and predict the experimental behaviour of light.

Do these conclusions satisfy us or is there still that desire of the reason to see further into these perplexing things, to make them more tangible in the sense of making them more akin to sensuous experience? Are we talking metaphysics or are we now seeing a new vision of the meaning of reality?

# CHAPTER 7.

# IS NATURE FINITE?

"No man can have in his mind an image of infinite magnitude, nor conceive infinite swiftness, infinite time, infinite force, or infinite power. When we say anything is infinite, we signify only that we are not able to conceive the ends and bounds of things; having no conception of the thing, but of our own inability."

Thomas Hobbes in his essay *Of Man.*

"I say finitude is incomprehensible, the infinite in the universe *is* comprehensible. . . Can you suppose an end of matter or an end of space? The idea is incomprehensible. Even if you were to go millions and millions of miles the idea of coming to an end is incomprehensible. . . You can go from one to ten, and ten times ten and then to a thousand without taxing your understanding and then you can go on to a thousand million and a million million. You can all understand it."

Lord Kelvin in a lecture on *Light.*

## 1. *Voltaire and the Infinities of Newton.*

François-Marie Arouet (1694-1778), better known as Voltaire, when he had reached the age of one and thirty years became involved in a quarrel with the Chevalier de Rohan and was forced into exile in England to escape imprisonment. During this period (1726-1729) Voltaire learned much about the activities of that island and in particular became acquainted with the ideas of Newton. They took fire in his imagination, and when he returned to France he published a series of essays entitled *Letters on the English*, which served among other things to introduce the thoughts of the English scientist to the French people. In one of these (XVII) he makes the following comments upon the philosophy of the new mathematics:

"The labyrinth and abyss of infinity is also a new course Sir Isaac Newton has gone through, and we are obliged to him for the clue, by whose assistance we are enabled to trace its various windings.

"Descartes got the start of him also in this astonishing invention. He advanced with mighty steps in his geometry, and was arrived at the very border of infinity, but went no farther. Dr. Wallis, about the middle of the last century, was the first who reduced a fraction by a perpetual division to an infinite series.

"The Lord Brouncker employed this series to rectify the hyperbola.

"Mercator published a demonstration of this quadrature; at about which time Sir Isaac Newton, being then twenty-three years of age, had invented a general method, to perform on all geometrical curves what had just before been tried on the hyperbola.

"It is to this method of subjecting everywhere infinity in algebraical calculations, that the name is given of differential calculations or of fluxions and integral calculation. It is the art of numbering and measuring exactly a thing whose existence cannot be conceived.

"And, indeed, would you not imagine that a man laughed at you who should declare that there are lines infinitely great which form an angle infinitely little?

"That a right line, which is a right line so long as it is finite, by changing infinitely little its direction, becomes an infinite curve; and that a curve may become infinitely less than another curve?

"That there are infinite squares, infinite cubes, and infinities of infinities, all greater than one another, and the last but one of which is nothing in comparison with the last?

"All these things, which at first appear to be the utmost excess of frenzy, are in reality an effort of the subtlety and extent of the human mind, and the art of finding truths which till then had been unknown."

This naïve statement of Voltaire is historically important since it served to introduce the mathematics of Newton into France. Today it interests us because it reflects our still puzzled ideas concerning the notion of infinity.

Whence came this concept into our minds? Is it an *a priori* judgment of the rational intuition? Is it of necessity a part of reason to postulate the existence of the infinitely great and the infinitely little?

Before turning to a consideration of infinity in objective nature, let us examine the logical foundations of this belief.

### 2. *Origins of the Infinity Concept.*

The idea connoted by the word infinity is of very ancient origin. It is fundamental to the paradoxes of Zeno which we have discussed earlier in the book. It appears again in the *Elements* of Euclid when he proves that an irrational number like $\sqrt{2}$ cannot be exactly represented by a rational fraction, that is to say, a fraction of the form $m/n$, where $m$ and $n$ are integers. Archimedes' extension of the *method of exhaustion*, originated by Eudoxus, comes so close to the modern theory of limits that Archimedes was able to evaluate elementary integrals by means of an infinite process. In a somewhat naïve manner Lucretius introduced the concept of infinity into his discussion of the bounds of the universe.

The idea occasionally appears in the thoughts of children. One has, perhaps, tried to out-count a rival in some childish game. "One hundred," you say. "A million," counters your opponent. "A billion," you reply. "A trillion," he names. "A vigintillion," you exclaim and expect victory since that is the name of the largest number to which a name has been assigned in your arithmetic. There is a pause in the struggle. "One more than that," says your opponent. You are overwhelmed by the boldness of the idea, until a thought occurs and you turn the tables. "One more than any number that you can name," you cry. Surely the battle is at an end. For a long time your enemy reflects. Suddenly his face lights up with inspiration and he mutters through clenched teeth: "Infinity!" What judge would have the boldness to settle such a controversy!

When we are asked to explain the meaning of the term infinity we often write the sequence 1, 2, 3, 4, . . . and state that infinity is its limit. It will be readily apprehended that this point of view may lead to paradoxes and so it does. One of these is formulated in the statement that *there are as many even numbers as there are odd and even together*. The demonstration is simplicity itself. We write in one line the sequence of the integers given above and below it a second line containing the sequence of the even integers as follows:

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & \ldots \\ 2 & 4 & 6 & 8 & 10 & 12 & \ldots \end{array}$$

Since to each number in the first row there corresponds a number in the second row, we argue that there must be as many numbers in the second row as in the first. In making this comparison we have invoked the principle of *enumeration*. By this we mean that we have made a *one-to-one correspondence* between the class of all the integers and the class of the even integers. It is clear that by means of enumeration we can compare the size of classes of objects without the use of a system of numbers to count them. If each object in one class can be matched with each object of another class, we can say that the classes are equal; otherwise, one is larger than the other.

When we are discussing finite classes it is customary to say that the whole class is greater than any of its parts. But this is not true in infinite classes, as we have just demonstrated. In fact, an infinite class may be defined as any class which can be put into one-to-one correspondence with a part of itself. An interesting example of such a class is the sequence of images of an object placed between two parallel mirrors. Another of similar kind is the picture which contains a picture of itself. The second picture must again reproduce itself and so on *ad infinitum.*

Even more astonishing than the example of the integers given above is the proposition that *the class of all the rational points between 0 and 1 is equal to the class of the integers.*

This will be proved if we can show that the rational points can be enumerated, that is to say, written in a linear sequence. This is easily accomplished, since there will be a point at 1/2, another at 1/3, another at 2/3, etc. until all are represented. This sequence is customarily written as follows:

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$$

It is clear that every rational point will appear in this array, since every integer appears in the denominator and the numerators consist of all the integers prime to the integers below the bars of the fractions.

It is customary to speak of any sequence of this character as *countable,* or *denumerable,* since we can make a one-to-one correspondence between its terms and the terms of the sequence formed from the positive integers. This fact is made evident by merely writing the integers successively below the terms of the series of fractions.

The ideas which we have sketched in the preceding account of classes were developed during the nineteenth century by a number of mathematicians. The theory may be said to have culminated in the work of Georg Cantor (1845-1918), who was the first to introduce the concept of *transfinite numbers.* Since any finite class has a cardinal number, let us say 25 if that is the number of objects in the class, then infinite classes also should have their numbers. Since these exceed any finite number, the cardinal which represents them must be transfinite. Thus Cantor invented a category of such numbers designated by the Hebrew letter Aleph with subscripts 0, 1, 2, etc. to indicate the order of the class to which it belonged. Thus, the cardinal of denumerable classes was Aleph-zero.

If we examine this strange number carefully we shall see that it has some peculiar properties. For example, *Aleph-zero, when augmented by any finite integer or by itself, remains unchanged.* No finite number can have these properties. It is a matter of more than usual interest to observe that the physicist has among his fundamental constants one that possesses what we might call the Aleph characteristics. This is the velocity of light, which we designate with the letter $c$. For we have seen that light transmitted from an object moving with the velocity $v$ still has only the velocity $c$, that is to say, $c + v = c$. Moreover, if a match be struck, then the two opposite fronts of the spherical wave, though advancing in opposite directions with the speed of light, are themselves separating only with the speed of light, or in symbols, $c + c = c$. Thus the incomprehensible nature of the first postulate of Einstein is absorbed in the incomprehensibility of the concept of infinity.

Let us now return to a consideration of the problem of counting all the points on a line of unit length. Is the number still that of the rational points? To show that all the points are not included in the rational set, we shall perform a simple experiment. Although points, by the definition of Euclid, have no dimension whatever, let us enclose each point in the rational sequence given above by a small length. For example, let us cover the first point with a line which has the length of the wave of sodium light. Although this is a minute value, it is inconceivably greater than the point. Let us assign to the next point one half of this length, to the next point one fourth, etc. until all the points have been covered. Although one may reflect that the last points in the sequence will have assigned to them very small lengths, indeed, nevertheless each one is covered since the intervals which enclose them, however small they may be, are still not zero.

Let us now add together all these lengths assigned to the rational points, a problem that can be readily solved by a simple mathematical formula. What a strange fact emerges from the calculation! This sum is found to be equal to but twice the wave-length of sodium light, which, when compared with the total length of the line, is nothing at all. The remarkable conclusion is thus reached that the entire set of rational points can be subtracted from the line without exhausting in any way the totality of points which comprise the line.

The points which remain are the irrational numbers, such, for example, as $\frac{1}{2}\sqrt{2}$, $\pi - 3$, etc. These, together with the rational points form what is called the *linear continuum.* It may be suspected by the reader that the cardinal number of the continuum is not Aleph-zero since the removal of the denumerable infinity of the rational points did not exhaust it. But the proof of this proposition requires delicate reasoning which cannot be given here.

From all of this we may gather that the continuum is a mathematical construction with very unusual characteristics. From it one learns that points are not arranged along a line like beads upon a wire, because between any two of them, no matter how close they may be together, there is another infinity of points as great as the infinity that measures the continuum itself. It will be readily understood that the line of chalk which is often used to represent a line of points is itself a thing quite different from the continuum which it designates, since the molecules of the chalk are finite and between any two of them there is a finite number of other particles. In a word the continuum represents the concept of infinity in all its glory.[1]

### 3. *Infinity and the Problem of Logic.*

The logical problem associated with notions of infinity originated in the concept of an irrational number such as the square-root of two, which, as we have indicated above, cannot be represented by a rational fraction. L. Kronecker (1823-1891) initiated the modern controversy by denying the existence of irrational numbers since they could not be represented as ratios of the integers.

This "mathematical nihilism" of Kronecker stimulated the researches of J. W. R. Dedekind (1831-1916), who, in the first edition of his work: *The Nature and Meaning of Numbers*[2] published in 1887 wrote as follows:

"My answer to the problems propounded in the title of this paper is, then, briefly this: Numbers are free creations of the human mind; they serve as a means of apprehending more easily and more sharply the difference of things. It is only through the purely logical process of building up the science of numbers and by thus acquiring the continuous number-domain that we are prepared accurately to investigate our notions of space and time by bringing them into relation with this number-domain created in our mind. If we scrutinize closely what is done in counting an aggregate or number of things, we are led to consider the ability of the mind to relate things to things, or let a thing correspond to a thing, or to represent a thing by a thing, an ability without which no thinking is possible. Upon this unique and therefore indispensable foundation, as I have already affirmed in an announcement of this paper, must, in my judgment, the whole science of number be established."

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[1] The reader who wishes to pursue this subject further will find it set forth in a non-technical manner in T. Dantzig's *Number, the Language of Science*, New York, 1930. A very lucid technical exposition will be found in the classical monograph: *The Continuum*, by E. V. Huntington, Cambridge, Mass., 1917, 2nd ed., 1938.

[2] This paper and an earlier one are translated by W. W. Beman in a volume entitled: *Essays on the Theory of Numbers*, Chicago, 1901.

It was indeed upon this foundation that Dedekind erected his theory of the irrational number, which has become the generally accepted one. His celebrated generalization, which he phrased in what is called the *principle of continuity*, runs as follows:

"If all points of the straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions."

From this we see that Dedekind has created what is called a *cut* in the continuum of points, and this cut *is* the number. That one cannot represent the square-root of two by a rational fraction is of no consequence. This number exists because it severs the continuum.

It is with this background that we can more readily understand the approach to the problem of mathematical truth, or mathematical existence theorems, as it has been developed recently from two different points of view. The first of these has been advanced by L. E. J. Brouwer, Dutch philosopher and mathematician, the other by David Hilbert, one of the most penetrating mathematicians of our times.[3]

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[3] See A. Dresden: "Brouwer's Contribution to the Foundations of Mathematics," *Bulletin of the American Math. Soc.*, Vol. 30, 1924, pp. 31-40; "Some Philosophical Aspects of Mathematics," *ibid.*, Vol. 34, 1928, pp. 438-452; "Mathematical Certainty." *Scientia*, 1929, pp. 369-376. D. Hilbert and W. Ackermann: *Grundzüge der Theoretische Logik*, Berlin, 1928. For the general problem of mathematical existence the reader is referred to J. Pierpont: "Mathematical Rigor." *Bulletin of the American Math. Soc.*, Vol. 34, 1928, pp. 23-53; O. Becker: *Mathematische Existenz*, Halle, 1927; H. T. Davis: "A Survey of the Problem of Mathematical Truth" (Introduction to H. von Helmholtz: *Counting and Measuring*, trans. by Charlotte L. Bryan), New York, 1930.

The fundamental proposition in the *intuitional* philosophy of Brouwer is that a thing exists only after it has been explicitly exhibited; that a thing logically conceived of by human intelligence does not exist by virtue of this logic alone, especially when an infinite process of construction is involved. The most novel aspect of the intuitional philosophy is found in its denial of the universal application of the so-called "law of the excluded middle."

As is well known the foundations of Aristotelean logic proceed from three postulates:

1. The postulate of identity; namely, "$A$ is $A$."

2. The postulate of contradiction: "$A$ is $B$" and "$A$ is not $B$" cannot hold simultaneously.

3. The postulate of the excluded middle: of the two propositions "$A$ is $B$" and "$A$ is not $B$" one must always hold irrespective of what $A$ and $B$ may be.

There is no denial of the universal application of these principles in all finite processes of thought. The main contention of Brouwer and his school applies to the transfinite case: that when the proof of a proposition rests upon an infinite construction, such as for example the method of mathematical induction, then one cannot assert the universality of the third Aristotelean postulate.

This statement may be clarified by an example due to the Dutch mathematician. Let us think of the number $\pi = 3.14159 \ldots$ developed as a decimal fraction and let us denote by $d$ the $n$th integer after the decimal point. Now construct the number $N = (-1/2)^m$, where $m$ is the position of $d$ when $d$ equals 0 and the next nine values are the integers 1, 2, 3, 4, 5, 6, 7, 8, 9. We readily perceive that $N$ is a real number which equals zero if there is no such sequence in the decimal development of $\pi$, but which is either greater or less than zero if such a sequence actually exists and $m$ is, in the first case, an even or, in the second case, an odd number. Logically the number $N$ exists since we have given a construction for it, and it is in one of the three catagories according to Aristotelean canons. But this is denied by Brouwer since the tremendous calculations which would have to be made to settle the question probably bar forever an answer to the intelligence of man. The number $N$ exists by construction, but since this construction involves an infinite sequence of operations the law of the excluded middle is not valid.

This looks like a very sane and comfortable point of view. When we are confronted by a logical situation, which requires an infinite sequence of operations to determine it, we deny the validity of the law of the excluded middle. The introduction of infinite operations into thought except in certain kinds of attainable limits is thus the creation of a daring but mistaken intelligence.

Let us not bother about the continuum over which Achilles passed in the Zeno problem. The fact that he caught the tortoise is convincing proof of the answer. Points without dimensions filling up a continuum create a perplexing concept to which the dictum "either it is true" or "it is not true" does not apply.

Hilbert, however, focuses attention upon another aspect of the question and in what is known as the *formalist* theory shifts the point of view from the third postulate of Aristotle to the affirmation of a *transfinite axiom* which may be added to the axioms of the domain of finite ideas so that the system thus augmented remains free from contradictions.

A great freedom results from this enlargement of the domain of logical principles. In that uncertain realm beyond the finite we are privileged to make such postulates as we may desire provided only that they do not contradict the postulates of finite logic. The quarrel between the intuitionalists and the formalists is really over the proposition whether such a freedom from contradiction can be attained without the denial of the law of the excluded middle. The struggle is waged about certain paradoxes which have been so very troublesome that Bertrand Russell was led to his famous aphorism: "mathematics is the science in which we never know what we are talking about, nor whether what we say is true."[4]

It will not be profitable for us to proceed further in the investigation of these subtle questions. The primary purpose of this book is to investigate the nature of the principles upon which natural philosophy has erected its structure. We are therefore seeking those postulates which by empirical discovery in the laboratory we have found to be *isomorphic*, that is to say, *in correspondence*, with the facts of nature. This is a point that is very fundamental and has been investigated by R. D. Carmichael in an attempt to discover how man makes his discoveries.[5] Logic is a subjective creation; we can invent its infinity and its infinitesimal and from them construct massive edifices of thought. But does this fact convince us that there is necessarily the slightest connection between them and objective nature? This is the problem. The continuity of the ether and the atomicity of matter inevitably force the concept of infinity upon us. Is this concept a necessity of nature? "As in the days of Newton and Leibniz so now the notion of infinity is our greatest friend; it is also the greatest enemy of our peace of mind," says J. Pierpont. "We may compare it to a great water-way bearing the traffic of the world, a waterway however which from time to time breaks its bounds and spreads devastation along its banks."[6]

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[4] For further elucidation of this proposition the reader is referred to B. Russell: *Introduction to Mathematical Philosophy*, 1919.

[5] *The Logic of Discovery.* Chicago, 1930.

The purpose of this chapter is to investigate an interesting question, to which laboratory science is beginning to indicate an answer. This question may be stated as follows: Will the denial of an infinitely great element or an infinitely small element accord with discoveries in the objective world to which these subjective considerations must finally be applied?

### 4. *How Does An Electron Know How Large To Be?*

The logical difficulty associated with the infinitesimal in nature is clearly defined in the following reflections of Alice:

"'What a curious feeling!' said Alice. 'I must be shutting up like a telescope!'

" . . . she waited for a few minutes to see if she was going to shrink any further: she felt a little nervous about this; 'for it might end, you know,' said Alice to herself, 'in my going out altogether, like a candle. I wonder what I should be like then?' And she tried to fancy what the flame of a candle looks like after the candle is blown out, for she could not remember ever having seen such a thing."

The point at issue with Alice was: What does it feel like to be an infinitesimal? There is a curious proposition in this which is not wholly foolish. A. E. Eddington, the eminent English astronomer and exponent of the theory of relativity, asks the question: How does an electron know how large to be? As he picturesquely states:[7] "An electron could never decide how large it ought to be unless there existed some length independent of itself for it to compare itself with." The adventures of Gulliver among the pigmy Lilliputians and again among the giant Brobdingnagians force the matter to our attention. "Undoubtedly philosophers are in the right," says Gulliver, "when they tell us that nothing is great or little otherwise than by comparison." Micromegas, the giant traveller from a satellite of the great Sirius, according to the record of Voltaire, once established communication with some philosophers on the earth. Marvelling at the insignificance of their size and the greatness of their wisdom he remarked: "I am now more than ever convinced that we ought to judge nothing by its external magnitude. O God! who hast bestowed understanding upon such seemingly contemptible substances, Thou canst with equal ease produce that which is infinitely small, as that which is incredibly great; and if it be possible that among thy works there are beings still more diminutive than these, they may, nevertheless, be endued with understanding superior to the intelligence of those stupendous animals I have seen in heaven, a single foot of whom is larger than this whole globe on which I have alighted." The observation of Micromegas faces us with the question of subjective imagination versus observed magnitude.

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[6] *Mathematical Rigor.* (loc. cit.), p. 47.

[7] *Mathematical Theory of Relativity.* Cambridge, 1923, p. 154.

This brings us then to the point of our present theme. Is the universe finite? Is the relationship between space-time and matter of such a character that it is meaningless to talk of boundlessness in nature? If the answer to these questions be affirmative, we have broken definitely with the mathematicians. There is a delightful essay in point in the stimulating volume *The Human Worth of Rigorous Thinking* by C. J. Keyser who, almost alone in the mathematical world, dares to assert that infinity is undemonstrable.[8]

"And now to my final thesis I venture to invite the reader's special attention, and beg to be held with utmost strictness accountable for my words. The question is, whether it is possible, by means of the new concept, to demonstrate the existence of the infinite; whether, in other words, it can be proved that there are infinite systems. That such demonstration is possible is affirmed by Bolzano, by Dedekind, by Professor Royce, by Mr. Russell, and in fact by a large and swelling chorus of authoritative utterance, scarely relieved by a dissenting voice. After no little pondering of the matter, I have been forced, and that, too, I must own, against my hope and will, to the opposite conviction. Candour, then compels me to assert, as I have elsewhere briefly done, not only that the arguments which have been actually adduced are all of them vitiated by circularity, but that, in the very nature of conception and inference, by the most certain standards of logic itself, every potential argument, every possible attempt to prove the proposition, is foredoomed to failure, destined before its birth to take the fatal figure of the wheel."

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[8] "The Axiom of Infinity," p. 157.

One is always most certain when he is explaining something that no one else knows anything about. Perhaps this accounts for the many pages in philosophy devoted to infinity. There would undoubtedly be more were it not so difficult to attach concrete attributes to the idea. After the paradoxes have been exhibited the question is sterile, for there are no answers. Nowhere in physical nature do these concepts intrude. To be sure we sometimes speak of creating the Newtonian potential function by bringing matter together from infinite dispersion, but that is rather a figure of speech. A very great distance would do fully as well. W. Nernst, the chemist, reflecting upon the problem of what would happen to entropy if the reciprocal of the absolute temperature became infinite was led to his *third law of thermodynamics*, denying this possibility.[9] There is no need for infinity or the infinitesimal in physical nature since the subjective postulate affirming the contrary apparently can always be dispensed with.

This is a bold idea, but it does not seem to be unreasonable from the discoveries in modern physics. The one difficulty in the path is the fact that the chemistry of our minds has in some way made us Euclidean. We want to believe that the ordinary concept of parallel lines as taught in Euclidean geometry is the natural one; that the sum of the angles of a triangle equals 180 degrees; that space is infinite. We feel with Lucretius, as previously quoted, that "The existing universe is bounded in none of its dimensions; for then it must have an outside."

Of course it is possible for a thing to be finite yet boundless, which may afford a partial reply to the argument advanced by Lucretius. Consider, for example, the surface of a sphere. Enlarge it to such tremendous dimensions that its curvature cannot be detected by our most accurate scientific instruments; clearly the surface would be finite though boundless. Upon it there would be no end point from which the spear of Lucretius could be hurled past the uttermost boundary of the surface. We are not playing with words as might appear. By *finite* we mean that the area of the surface is limited, and by *boundless* that there is no point or line of points beyond which we shall find no more of the spherical surface.

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[9] *The New Heat Theorem*, London, 1926.

### 5. *Space-Time and Infinity.*

Let us now apply this idea to the four-dimensional manifold which Einstein has linked together into the concept of space-time. To some this invoking of the fourth dimension may seem mystical and unreal. But that such is the fact is far from being the case. By the most accurate scientific experimentation a true physical validity has been given to the underlying framework of events. This we now call space-time instead of the ether. Can there not be a finite quantity, volume, so to speak, of the four-dimensional framework, without a bound existing to it?

Reflect a little longer on the case of the surface of the sphere. Here we are dealing with a surface which has all the appearance of reality to us because it is immersed in a three-dimensional manifold which our human intellect thinks it comprehends. But how mysterious the notion of finite yet boundless would be to a being endowed with faculties that appreciated only two dimensions! Similarly the difficulty of carrying the projected analogy over into the four-dimensional manifold is inherent in our chemical structure which holds as rational only that which has length, breadth, and thickness, and as transcendental that which involves higher dimensions. To a being endowed with a five-dimensional intellect the problem of a four-dimensional surface requires no greater mental agility than the case of the surface requires of us. But there is a minor difficulty in the way. Who cares to postulate the existence of a five-dimensional intelligence!

In order not to appear too mystical we shall abjure such speculation and return to a safer point of view. Let us regard the equations of the Einstein law of gravitation as possessing objective existence and confine our attention to the conclusions which can be drawn by mathematical deductions from them. In this way we shall keep our feet solidly upon experimental data, while we reach for the secret of the clouds.

It will be recalled that Einstein was able to calculate, on very reasonable and essentially simple assumptions, the form of the space-time metric. This calculation involved the density of matter which acted upon space-time in the capacity of a distorting or twisting agent. This distortion made itself manifest in the three crucial experiments, one of which concerned the perihelion of Mercury, the second the behaviour of rays of light, and the third the shifting of star spectra. If these experimental observations seem confirmatory evidence of sufficient magnitude to make us adopt the Einstein philosophy, we may essay the bold adventure of exploring the structure of the four-dimensional manifold of reality.

At the very outset we are met by a profound question about which we must make a hypothesis. What causes the motion of the Foucault pendulum? Why is it that cyclones in the northern hemisphere turn in one direction and those in the southern hemisphere in the opposite direction? Why does the earth bulge at the equator and flatten at the poles? In other words, why is there an absolute rotation of the earth independent of the framework of the fixed stars?

To this puzzling question we can give at least two answers. The first of these assumes that inertia, which is at the bottom of the problem, is derived from the presence of all the other matter in the universe. The second leads to the de Sitter universe, which we shall consider later. In the first we postulate that a single particle of matter in an otherwise empty universe would have no inertial mass. To invoke Newton's laws of motion in such a vacuum would be wholly meaningless because we ask: With respect to what could such a particle change its direction of motion? To put it otherwise, if the sun and all the stars and the galaxies were removed to infinite distances from us so that their influence were completely annihilated, then the Foucault pendulum would cease to behave as it does. There would be no absolute rotation of the earth, the polar caps would not flatten nor the equator bulge. This suggestive thought, as we have stated in chapter two, goes back to Mach, who saw much that Einstein later formulated and who exerted a large influence upon the latter's views. At the conclusion of his Princeton lectures upon relativity Einstein makes the following exposition of his position:[10]

"The idea that Mach expressed, that inertia depends upon the mutual action of bodies, is contained, to a first approximation, in the equations of the theory of relativity; it follows from these equations that inertia depends, at least in part, upon mutual actions between masses. As it is an unsatisfactory assumption to make that inertia depends in part upon mutual actions, and in part upon an independent property of space, Mach's idea gains in probability. But this idea of Mach's corresponds only to a finite universe, bounded in space, and not to a quasi-Euclidean, infinite universe. From the standpoint of epistemology it is more satisfying to have the mechanical properties of space completely determined by matter, and this is the case only in a space-bounded universe. An infinite universe is possible only if the mean density of matter in the universe vanished. Although such an assumption is logically possible, it is less probable than the assumption that there is a finite mean density of matter in the universe."

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[10] *The Meaning of Relativity,* Princeton, 1923, p. 119.

Einstein's answer to the ancient problem of the extent of space and time is that the question of the limitation of the space-time metric of reality is inextricably united to the question of the limitation of matter. The essential postulates that are to be made are these:

1. *The fundamental metric which throws the cause of the gravitation of ponderable bodies back upon the character of space-time is the substratum of reality.*

2. *Inertia is a cosmic phenomenon which depends upon the totality of matter in the universe.*

It must be realized that in making postulates about things whose nature is little known one has great leeway. We should never be wholly unmindful of the sarcasm of Voltaire who advanced the proposition that Aristotle's discussion on the soul should be quoted in the Greek language, since "it is but reasonable we should quote what we do not comprehend in a language we do not understand."[11] The scrutiny we should apply to such hypotheses depends wholly upon such experimental evidence as may be brought to bear either directly upon the basic assertions or indirectly upon the conclusions to be derived from them.

In the case of the first hypothesis Einstein gained overwhelming prestige by his bold prediction of phenomena at the time undiscovered. That there would be adherents for such a theory was inevitable. One must settle the question of his own belief in the new science however, by weighing the probability that such predictions could be wholly fortuitous against his own intuition that such propositions can actually represent the facts of nature. Scientific feeling today seems to be in favor of adopting the new point of view while waiting patiently for the accumulation of new data.

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[11] *Micromegas,* chap. 6.

The second assumption of Einstein rests upon a far less secure basis. We have the one solitary fact of inertia—baffling, isolated, mysterious. The earth turns in space both on the evidence of the fixed stars and on the evidence of the pendulum swinging within the walls of the laboratory. What is its cause? Does it reside in the totality of matter in the universe or is it independent of other bodies? If all the material which fills proximate space were slowly withdrawn from our neighborhood would the Foucault pendulum gradually cease turning and finally come to rest as earth approaches isolation? Is this the source of the kinetic energy which we assume the earth to have? Are its billions of potential horse power derived wholly from the presence of other matter in the universe? Those who wish to make the positive reply with Mach and Einstein travel in a great company; those who wish to seek the explanation in another direction have as yet no large body of experimental facts to reconcile with any hypothesis that they might wish to make.

### 6. *Einstein's Cosmology.—The "Ghost Star" Theory.*

It is entertaining at least to explore the cosmology of Einstein which results from his postulates.[12]

First, the time ordinate of space-time is infinite in both directions and is not curved. We pass on from event to event along the world lines of experience and never return again to the point from which we started. Time no longer has the old meaning in Newtonian mechanics, but it shares with Newtonian thought an infinite past and an infinite future.

In the second place, however, the affair is different with respect to the space coordinates. Space has a curvature and a straight line prolonged far enough will ultimately return to the space-point from which it started. This idea has led to the poetical theory of "ghost stars," which has received considerable comment in popular accounts of Einstein's theory. Simply stated it is this. If space has a curvature then rays of light should finally return from their journey around the universe and should converge at the point from whence they started, to form ghostly images of the material parent stars. Thus we have a stellar universe peopled with the images of countless suns. The point of light that we call Polaris may be but the light from an empty spectre, the real star having long since lost itself among the constellations of the Milky Way.

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[12] Einstein's cosmology was first set forth in his paper: "Kosmologische Betrachtungen zur allgemeinen Relativitatstheorie," *Berlin Sitzungsberichte,* 1917, pp. 142-152.

Of course one may wonder where the image of our own sun is. It is probable that light in its long path around the universe would gradually be absorbed or scattered by matter diffused in space or it might perhaps be deflected by gravitational fields so that the focus would be blurred and dim. Someone has advanced the suggestion that the mysterious Gegenschein, which can be seen occasionally at sunset 180 degrees removed from the sun and in the place where the sun's ghost should appear, might be evidence of this long trip of the sun's light around the universe. It is much more probable, of course, that the Gegenschein is caused by reflected light from quantities of cosmic dust. Analysis made by the spectroscope shows that the light is partially polarized and identical with sunlight that has been reflected from very small particles or molecules of gas. However, the other possibility remains. Fantastic as these ideas may seem at first they are alive with the poetry of astronomy, and no apology need be made in bringing them to one's attention.

Some scientists have been curious to know what limitations are put upon this finite universe. The fascinating aspect of the theory is the enunciation of the belief that the amount of space-time depends upon the amount of matter. Create a single electron and you have created with it a little quantity of space-time; create two electrons and more of the substratum of reality has appeared; create the stars and the spiral nebulae and you have brought into existence the sum total of the space-time metric which reveals its existence and its actual extent by the behaviour of the Foucault pendulum through the inertia of matter. This concept has been put paradoxically in the statement: "The more matter there is, the more room there is for it to exist in."

One of the principal arguments first urged against the Einstein cosmology was that it posited the existence of vast quantities of matter which had not yet been discovered. This objection to the Einstein theory, however, has proved to be one of its most convincing arguments. The cosmological speculations of the father of relativity preceded the interpretation of the spirals as new galaxies of stars and hence the deficiency of world stuff which faced the theory in 1920 has acted as a stimulus to the discovery of undreamed treasures of cosmic matter in the ensuing decade. We shall enlarge upon these matters in the next sections.

### 7. *Astronomical Space.*

Let us turn from this speculation for a moment to review the story, never old and always stirring, of the extent of visible space. One will recall in Mark Twain's, *Captain Stormfield's Visit To Heaven*, the great difficulty encountered by the captain in explaining to the keeper of the celestial gates whence he had come. "Finally he came down," reports the captain, "and said he thought he had found that solar system, but it might be fly-specks. So he got a microscope and went back. It turned out better than he feared. He had rousted out our system, sure enough. He got me to describe our planet and its distance from the sun, and then he says to his chief—'Oh, I know the one he means, now sir. It is on the map. It is called the Wart.' " To most people this is all that one needs to know and perhaps can ever know about cosmology. However, the astronomers labor ceaselessly pushing back the boundaries of space. The great 100-inch reflecting telescope on the top of Mount Wilson in California has contributed greatly to this expanding knowledge. The new 200-inch instrument on Palomar Mountain north of San Diego will contribute even more.

Let us turn to a few figures of common knowledge and see what modern investigation has to say about the nature of matter in visible space. Let us start with the earth and its nearest neighbor, the moon. The earth has a diameter of approximately 8,000 miles and the moon a diameter approximately 3/11*ths* of this. The latter is on the average about 239,000 miles away from us. The earth and the moon belong to the solar system which consists of the sun, its planets, the planetoids, a little cosmic dust and a few comets. If the sun were represented by a circle an inch in diameter the earth could scarcely be seen at all by the naked eye and Jupiter, the largest, would appear as a tiny speck one tenth of an inch across. It is difficult to appreciate cosmic distances, and yet, in order to comprehend space and time, some numerical knowledge is necessary. It does not mean a great deal, to be sure, to know that the earth is ninety-three million miles from the sun, and that Pluto, the outer planet, is 3,671 million miles away, but relative distances are better understood if, as above, we represent the sun's diameter by an inch. Then the earth, in terms of these magnitudes, will be about nine feet from the sun and Pluto approximately 354 feet away.

As soon as we leave the solar system, space increases in a terrific manner. No longer is the distance from the earth to the sun or even the diameter of the solar system itself large enough to use as a measuring rod. We must employ light-years—the distance that light will travel in 365 days. In terms of this "yard stick" the distance from the earth to the sun is only a few seconds over eight minutes and the distance to Pluto is approximately five and one-half hours. Alpha Proxima, our nearest neighbor among the stars, is between three and four light years away. In the scale adopted above, an inch for the diameter of the sun, this would put our closest stellar companion between three hundred and four hundred miles distant. Astronomers have found the light-year an inconvenient unit to use and they more often employ the distance known as the *parsec*. The parsec by definition is the distance at which a star would have a parallax of one second of arc which is equivalent to 3.258 light-years or more than 19 trillion miles.

How vast the universe and how trivial these figures make us feel! Will increasing knowledge tend to reduce man to utter insignificance, a cosmic infinitesimal, or is there some meaning behind the fact that he is able to open up for himself these tremendous vistas? Perhaps this insignificance is only an apparent one.

### 8. *Is Matter Uniformly Dense Throughout Space?*

Before considering the possibility of the existence of a structure to the universe of stars, let us first reflect upon the following problem: Is there a uniform density of matter in the universe? That is to say, do the stars recede indefinitely from us and does each cubic parsec of space contain approximately the same number as every other cubic parsec?

The astronomers have been bold enough to give a negative answer to this question. For all of the blaze of glory in the clear evening sky, the uncountable multitude of stars, "which stand as thick as dew drops on the fields of heaven," there appears to be evidence that the density is not uniform.

Certainly it appears true that the density of the luminous matter in the sky is not the same in every part of visible space. The first argument that can be advanced is that there is not a proper increase in the number of the stars of higher numerical magnitude over those of lower numerical magnitude. In order to understand this argument it is necessary to comprehend the meaning of the word magnitude.

The astronomers, in order to compare the relative luminosity of celestial objects, introduced the idea of magnitude. As early as 150 B. C. Hipparchus, father of modern astronomy, classified the visible stars into six magnitudes. About twenty of the most brilliant, such as Sirius, Canopus, Vega, Arcturus, Rigel, etc., were put into the first magnitude class, while those so faint that they were on the margin of visibility were classified as of sixth magnitude. Stars of intermediate brightness were then placed in the remaining four magnitudes.

A more scientific classification of light intensity was introduced into the science in 1854 by the English astronomer N. Pogson (1829-1891), who observed that first magnitude stars on the average were about 100 times as bright as those of sixth magnitude. This made the ratio of the light from stars of adjacent magnitudes equal to 2.512, nearly 2½, or the fifth root of 100.

Since some stellar objects, as for example, Sirius, are brighter than the average of the 20 stars of first magnitude, it was necessary to introduce zero and negative magnitudes. Thus Sirius has a magnitude of -1.6, Venus a magnitude of -4.3, the full moon a magnitude of -12.5, and the sun itself a magnitude of -26.7. The *absolute magnitude* of a star, as contrasted with its *apparent magnitude,* namely the one which we have just described, is the apparent magnitude which it would have if it were observed at a distance of ten parsecs. This gives the astronomers a means of comparing the intrinsic brightness of stars. For example, the absolute magnitude of the sun is 4.85, while that of Sirius is 1.26, since the intrinsic brightness of the latter is about 87 times that of the sun.

With this explanation we can now return to the proposition that there is not a proper increase in the number of stars from magnitude to magnitude to support the hypothesis that matter is distributed uniformly throughout space. To make this statement clearer we must first assume that the stars on the average are of the same intrinsic brightness, that is to say, if they were all observed from a distance of ten parsecs they would on the average have the same apparent magnitude. This, of course, is not true, but the variations in actual luminosity among the stars is not too great, if we speak in terms of statistical averages, to vitiate the following argument made upon such an assumption. On this hypothesis then it may be shown mathematically that there would be four times as many stars of the second magnitude as of the first, four times as many of the third as of the second, etc. As a matter of experimental fact the actual ratio of increase is slightly less than three for the stars visible to the naked eye and sinks to slightly less than two for those of twentieth magnitude. Matter thins out with distance apparently, and while stars may exist to the utmost conceivable limits of space their distribution seems to be something like that of prime numbers, which gradually diminish in number as we get into numerical reaches far from the integer *one*.

Of course it may be argued on reasonable ground that this apparent scarcity of stars at remote distances from us is due to occulting matter or the absorption of light travelling over so long a path. The picture in the accompanying plate, one of the most magnificent in the gallery of astronomy, gives evidence of the existence of dark masses in the sky, great black nebulae perhaps, reaching with shadowy arms across the canopy of stars. It is equally certain, however, that these regions of obscuration are comparatively rare, and that there is not, in general, loss of light from distant sources, even though they may be as remote as 100,000,000 light years from us.

A very beautiful argument, the conclusions of which are in agreement with those just advanced upon the evidence of the luminosity of the stars, was first made by Lord Kelvin.[13] This argument starts with the hypothesis that the stars are uniformly distributed in space within the configuration of some geometrical figure and are moving under their mutual gravitational attractions.

Let us examine carefully the admissions which this involves. The first proposition is the one which we wish to controvert and the second comprises an assumption to which no reasonable objection can be raised, since it is merely that of universal gravitation. For simplicity let the geometrical figure of this cosmic gas be that of a sphere, and let us assume that the density of the stellar distribution is one star of mass equal to that of our sun within a volume equal to a sphere one parsec in radius. Under these hypotheses it can be shown that the period of revolution of a star on the outside of this gravitating cloud of stellar bodies would be 93,000,000 years, no matter what the radius of the sphere might be. It is not a serious mathematical problem to show that any star near the center of the sphere, in order to escape from this vast stellar cloud, must have a velocity 1.73 times that of the star revolving on the outside. For a sphere of radius 1,000 parsecs this velocity of escape must be 112 kilometers per second; for a sphere ten times this, 1,120 kilometers

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[13] "On Ether and Gravitational Matter Through Infinite Space," *Phil. Mag.*, Vol. 2, sec. 6, 1901, pp. 161-177, in particular p. 168 et seq.

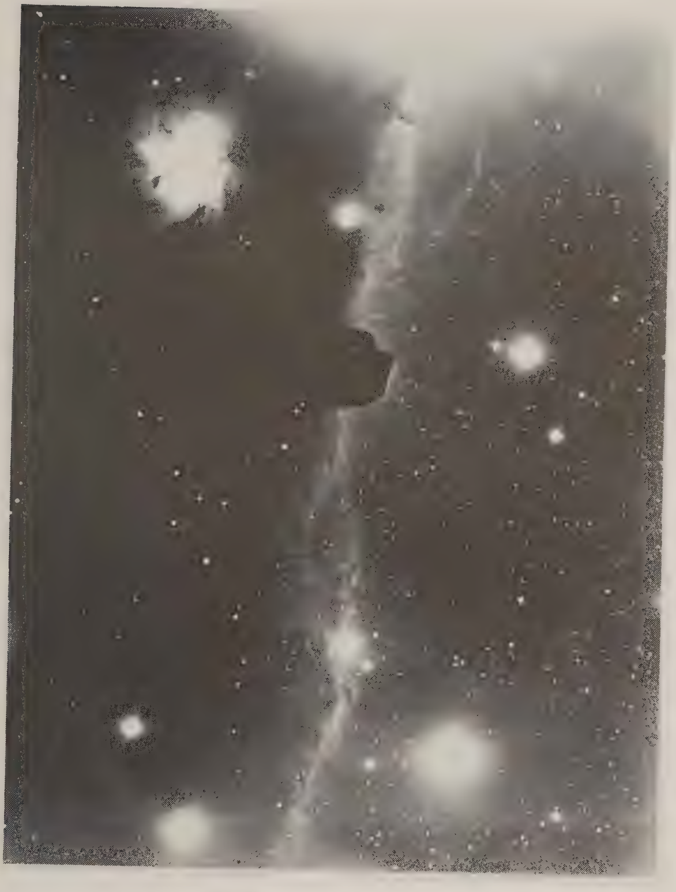

PLATE VII

The Cosmic Horse's Head. This photograph of the region south of zeta Orionis gives evidence of the existence of dark masses in the sky stretching with shadowy arms across the canopy of stars.

per second, etc. With increasing radius the escape velocity enormously increases, and for a sphere of nearly infinite dimensions it is beyond calculation.

Now the velocities of the stars in the regions visible from the earth are not remarkably great, being for the most part less than 50 kilometers per second, although there are a few notable exceptions, in which the velocity exceeds three hundred kilometers. It is thus clear that if the stars are distributed with the assumed density of one per cubic parsec in a sphere as large as 10,000 parsecs radius none of the stars whose proper motions are now known could ever escape from the gravitational influence of the stellar gas. Hence those stars which are at a greater distance must be assumed to be moving in paths which will never bring them close to our part of the universe. If on the contrary we assume that the stellar gas-ball has a radius inferior to 1,000 parsecs, the faster stars are escaping and will sometime disappear into the depths of space, exactly as the atmosphere of the moon is believed to have escaped from the weak gravitational field of that body. Neither of those conclusions seems to be a satisfactory hypothesis. We must either deny the original hypothesis of uniform density or modify our ideas about the universality of gravitation. The former seems the easier to disbelieve.

What a magnificent conclusion from such slender data. To be sure we have made the tacit assumption that the stars of the Milky Way are arranged in a globe, whereas the most likely figure is that of a thin disk; but the general argument and its conclusions are in no wise invalidated by this postulate. The mathematics is only slightly more complicated and dynamical considerations are involved in the possible rotation of the entire mass.

### 9. *The Galaxy and the Spirals.*

Let us return from such engrossing speculations to a more specific consideration of our galaxy, or as it is usually referred to, the Milky Way. It is now commonly held that all of the forty billion or more stars in the heavens belong to the galaxy. The reason why we see stars in all directions from us is that we ourselves are within the system. To judge from the distribution of the stars and the apparent increasing density which we find in the plane of the Milky Way as compared with the density toward the galactic poles, the galaxy is shaped like a lens, and its thickness is from one-sixth to one-tenth of its diameter. Estimates of its diameter vary from 60,000 to

300,000 light-years, the latter being the estimate of Harlow Shapley. This astronomer has located the center of our huge system about 52,000 light-years from the sun in the densely populated region near Sagittarius.[14] More recently there has been a tendency to reduce these estimates somewhat. The diameter is now believed to be of the order of 100,000 light-years, and the distance to the center not greater than 10,000 parsecs.[15] The mass of this vast system has been estimated to be of the order of 200 billion ($2 \times 10^{11}$) suns, but there is so much more space than matter that the average density is perhaps around one electron per cubic centimeter.

The objects in our galaxy may be arbitrarily classified into four different types. The first of these are the stars, of which probably more than a billion are within the reach of the great 100 inch reflecting telescope at Mount Wilson. As has been mentioned before, the velocities of most of these great suns are less than 50 kilometers per second with respect to our solar system. The second objects of interest are the globular star clusters. There are more than a hundred to be seen in the heavens. They contain from ten thousand to a hundred thousand stars, and some have the comparatively large relative velocity of 300 kilometers per second. A third class of objects, the subject of much investigation, are the diffuse nebulae. These are enormous tenuous cloud-like masses, which are moving with very low velocities. There are probably ten thousand such nebulae in the sky. Finally the planetary nebulae are small round objects which generally have a central star as nucleus. Their spectra seem to indicate a gaseous condition of at least a part of their material. There are more than one hundred fifty such objects in the heavens, and they are moving on the average with velocities slightly less than a hundred kilometers per second.

Outside of our galaxy lie the spiral nebulae. For this reason, and to distinguish them from the diffuse gas-clouds observed within our own system, astronomers commonly refer to them as extra-galactic nebulae. More than a million of these stellar objects are within the reach of our telescopes, and Hubble has estimated that a hundred times this number could be observed were it not for the obscuring material in the universe which hides vast numbers of them from us. This conclusion follows from the fact that the spirals are to be found in that part of sky where the stars are fewest; that is to say, they tend to accumulate in the neighborhood of the galactic poles, approximately the same number being observed on each side of the Milky Way. One of their most peculiar features is found in the fact that they appear to have on the average a truly enormous radial velocity (velocity in the line of sight), and this *velocity is recessive.* In these very distant objects the Doppler shift, which we discussed in some detail in the last chapter, is toward the red-end of the spectrum, which indicates that they are moving away from us. The truly spectacular aspect of this cosmic puzzle is found in the observation that recessive velocities of the order of 10,000, 15,000 and even 25,000 miles per second have been reported. Since the velocities of the stars and other bodies of our own galaxy almost never exceed 50 miles per second, and are generally much below this figure, our imagination is beggared by a contemplation of these tremendous masses of world-stuff hurtling with an incomprehensible speed into the mysterious depths of space. Without doubt such a fact must have some profound significance, since the theory of probability is here wholly violated if the motions of the celestial objects are to be regarded as random.

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[14] *Star Clusters,* New York, 1930, p. 177.

[15] See, for example, W. M. Smart: *Stellar Dynamics,* Cambridge, 1938, p. 389.

What is the nature of the spiral nebulae? Are they great "island universes," to use the picturesque expression of von Humboldt, like our own galaxy, each containing within its boundaries other billions of stars? Are they so far away that their diffuse, nebulous light is merely the hazy pattern that distance gives to the intense energy of an unresolved aggregation of countless stars?

The saga of the discovery of the true nature of the spiral nebulae is one of the most romantic stories of astronomy and is worthy of retelling here. It began with the conjectures of Thomas Wright (1711-1786), an almost forgotten astronomer of the eighteenth century, who published his ideas in a small volume entitled *An Original Theory or New Hypothesis of the Universe,* which appeared in London in 1750. In this work Wright first stated the proposition that our solar system belonged to the cloud of stars which comprised the Milky Way, and that this collection of cosmic matter formed an isolated system in space.[16] Much impressed by the theory of

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[16] R. G. Aitken has reconstructed the life of Wright in a short paper: "Thomas Wright of Durham and the Birth of a Great Idea," Leaflet No. 199, 1945, *Astronomical Society of the Pacific.*

Wright, Immanuel Kant (1724-1803) in his *General History of Nature and Theory of the Universe* (1755) enlarged upon the idea of the unity of the galactic system and said: "One can conjecture further that these higher world orders do not exist without reference to one another and thus through their mutual relationship create even vaster systems. In fact one sees that the elliptical figures of that type of nebulous star which Maupertuis has cited, have a very close resemblance to the plan of the Milky Way. This seems to open a wide field for discovery, to which observation must supply the key."[17] A similar conjecture regarding the possibility of other galaxies was expressed six years later by J. H. Lambert (1728-1777), the mathematician, in his *Cosmological Letters*, which appeared in 1761.

Sir William Herschel (1738-1822), with his new telescopes, was the first to apply extensive and systematic observations to the nebulae and thus begin the construction of the key required by Kant. In 1785 he wrote as follows: "I shall now proceed to show that the stupendous sidereal system we inhabit, this extensive stratum and its secondary branch, consisting of many millions of stars, is, in all probability, a detached nebula."[18] In the same paper he remarked further: "Among the great number of nebulae which I have now already seen, amounting to more than 900, there are many which in all probability are equally extensive with that which we inhabit; and yet they are all separated from each other by very considerable intervals." Herschel himself attempted a sketch of the outline of our galactic system and estimated its extent. About this sketch he remarked: "From this figure, however, which I hope is not a very inaccurate one, we may see that our nebula, as we observed before, is of the third form; that is: A very extensive, branching, compound congeries of many millions of stars." Estimating the size of the galaxy in terms of our distance to Sirius (8.6 light-years by modern measurement), Herschel proposed an average width of 900 units and a breadth of 225 units, or in round numbers dimensions of 8,000 by 2,000 light-years.

Many years later, perhaps unfortunately for the subsequent development of astronomy, Herschel changed his opinion about the nature of the spiral nebulae and revoked his theory that they were vast collections of stars. Thus in 1811 he wrote as follows:[19]

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[17] See Kant's *Werke*, Berlin, 1902, Vol. 1, pp. 255-256.

[18] "On the Construction of the Heavens," *Phil. Trans. of the Royal Soc.*, Vol. 75, 1785, pp. 217-266. Also *Collected Papers*, London, 1912, Vol. 1, p. 245.

"...We may also have surmised nebulae to be no other than clusters of stars disguised by their very great distance, but a long experience and better acquaintance with the nature of nebulae, will not allow a general admission of such a principle, although undoubtedly a cluster of stars may assume a nebulous appearance when it is too remote for us to discern the stars of which it is composed.

"Impressed with the idea that nebulae properly speaking were clusters of stars, I used to call the nebulosity of which some were composed, when it was of a certain appearance, *resolvable*; but when I perceived that additional light, so far from resolving these nebulae into stars, seemed to prove that their nebulosity was not different from what I had called milky, this conception was set aside as erroneous."

But Herschel's first opinion was not completely rejected by others in the ensuing years. To it was attached the name the "island universe" theory of the spiral nebulae. The origin of this name, as we have said above, may be traced to Alexander von Humboldt (1769-1859) who, although scarcely to be classified among the astronomers of his period, began his ambitious survey of the physical universe, *Kosmos* (1845-1862), with a description of astronomical objects. Thus he says:[20]

"If we compare the regions of the universe with one of the island-studded seas of our planet, we may imagine matter to be distributed in groups, either as unresolved nebulae of different ages, condensed around one or more nuclei, or as already agglomerated into clusters of stars, or isolated spheroidal bodies. The cluster of stars, to which our cosmical island belongs, forms a lens-shaped, flattened stratum, detached on every side, whose major axis is estimated at seven or eight hundred, and its minor one at a hundred and fifty times the distance of Sirius."

By the beginning of the twentieth century the island-universe theory of the spirals had almost completely disappeared. One writer of that period (1906) voiced the general opinion of the astronomical world when he wrote about them: "Their spectra indicate that they are perhaps largely in a solid or liquid condition. On the other hand, their transparency indicates their tenuity. Hence they seem to be perhaps vast swarms of incandescent or liquid material surrounded by gaseous material."

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[19] "Astronomical Observations Relating to the Construction of the Heavens...," *Phil. Trans. of the Royal Soc.*, 1811, pp. 269-336. Also *Collected Papers*, London, 1912, Vol. 2, pp. 459-497.

[20] Vol. 1, Chap. 1.

In 1921 a celebrated discussion of the "Scale of the Universe" was held between Harlow Shapley and Heber D. Curtis (1872-1942) in which both the extent of our own galaxy and the nature of the spiral nebulae were the primary issues.[21] With respect to the latter question Shapley reaffirmed his earlier views: "It seems to me that the evidence, other than the admittedly critical tests depending on the size of the galaxy, is opposed to the view that the spirals are galaxies of stars comparable with our own. In fact, there appears as yet no reason for modifying the tentative hypothesis that the spirals are not composed of typical stars at all, but are truly nebulous objects." Curtis, on the other hand, after surveying the evidence for and against the island-universe theory, stated his conclusion thus: "I hold, therefore, to the belief . . . that the spirals are not intra-galactic objects but island universes, like our own galaxy, and that the spirals, as external galaxies, indicate to us a greater universe into which we may penetrate to distances of ten million to a hundred million light-years."

The controversy thus stood at the century-old impasse in 1921. In the mean time, however, a new and remarkable theory had turned the speculative eye of the astronomer toward the spiral nebulae from another point of view. As we have stated earlier, Einstein in 1917 had published a paper in which the cosmological consequences of his general theory of relativity were set forth. Associating the size of space with the average density of matter which it contained Einstein required more matter than was then visible to the astronomers. It is astonishing that the proponents of the island-universe theory did not seize avidly upon this remarkable new argument which had been injected so strangely into the controversy. But such was not the case, doubtless because those who scanned the theory believed that it needed more proof than the galactic character of the spirals which it demanded. Writing in 1924, L. Silberstein surveyed the mass-requirements of the Einstein universe and concluded that at least $10^{10}$ such galaxies as our own would be needed to account for the known dimensions of the universe. These galaxies, he rather cautiously suggested, might be found in the island universes as represented by the spiral nebulae. He concluded with the suggestive remark that the new theory "affords at the same time the best reason for sending the opponents to look after the indispensable masses farther and farther into unknown regions of space."[22]

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[21] *Bulletin of National Research Council,* Vol. 2, 1921, pp. 171-217.

But dramatic discoveries were even then in the making. The classical work of Henrietta S. Leavitt (d. 1921) on the Cepheid variable stars (See Chapter 11) had shown as early as 1912 that *the absolute magnitude, or intrinsic brightness, of these stars was a function of the length of the period of their variation.* Shapley in 1917, computing the average distance of several Cepheids, was able to construct a curve by means of which the absolute magnitude of such stars could be estimated from the period of their variable light-curve. Here, then, was an astonishing new tool by means of which distances could be probed that were inconceivably greater than those accessible by the ordinary parallax measurements. For by comparing the absolute magnitude of a Cepheid, as determined by its period, with its apparent magnitude, as directly observed, its distance could be accurately estimated. And hence the distance to any celestial object, no matter how remote, is similarly determinate provided it contains a single observable Cepheid.

One of the most magnificent objects in the astronomer's gallery is the great Andromeda spiral, known in star catalogues by the less romantic name of Messier 31, or simply M 31. Visible to the unaided eye and reported as early as the tenth century by Al Sufi, this great spiral has been the subject of study and speculation for many centuries. In 1924 E. P. Hubble determined the period of one Cepheid in its outer portion and from the observed magnitude and the absolute-magnitude diagram of Shapley estimated that the nebula was at the incredible distance of 900,000 light-years from us. Although this figure has been reduced by later observations to about 720,000 light-years, its magnitude had settled the age-old controversy. The nebula is extra-galactic and in the words of Herschel it is "a very extensive, branching compound congeries of many millions of stars."[23]

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[22] *The Theory of Relativity*, 2nd ed., 1924, p. 481.

[23] For an authoritative account of these discoveries see E. Hubble: "Extra-galactic Nebulae", *Astrophysical Journal*, Vol. 64, 1926, pp. 321-369.

The original measurements of Hubble were soon followed by others and the overwhelming evidence convinced the astronomical world of the correctness of the island-universe theory of the spirals. The size and mass of these great objects has also steadily increased as the extent of the peripheral stars has been enlarged by long and careful observation. These stars, because of their immense distances from us, are at the very edge of visibility in the largest telescopes. Writing in 1943, Shapley says: "We find that the length of the Andromeda nebula is 35,000 light-years and the width about 8,700 light-years." Including the "haze of stars that surrounds the main body of the system," he remarks, "the area on the sky now recognized as covered by the Andromeda nebula has in consequence been increased ten times to about fourteen square degrees, equal to approximately seventy full moons or one third the area of the bowl of the Big Dipper. In appearance, as in fact, it is a gigantic galaxy, if one looks deep enough."[24]

The spiral character of the nebulae seems to indicate that these vast systems have an angular rotation. Difficulties of observation and the large recessive shift of the nebular spectra preclude at this time too definite a judgment on this point. However, within our own island universe, there exists strong evidence for such systematic motion. Thus the pioneer investigations of J. C. Kapteyn (1851-1922) established statistically the existence of definite streams of stars within the galaxy and subsequent investigations have tended to confirm the original pronouncements.[25]

The conclusion most readily derived from this brief survey of the romantic story of the spiral nebulae is that we dwell within a universe of unimaginable magnitude. The spirals have given us new assemblages of stars, and they in turn may prove to be but atoms in a vaster system which we have not yet apprehended. To this total collection of cosmic matter, the stars, the clusters, the galaxies and all the other objects of the universe, the name of the *metagalaxy* has been given. In such a picture man himself may be compared with the infinitesimal intelligence which may perhaps reside upon an electron in the planetary system of an oxygen atom within the center of a flat wafer-like stone on the surface of the earth. What would this diminutive being be able to conclude from the confines of his prison? Neighboring rocks would be spiral nebulae perhaps; the earth an assemblage of stars beyond the bounds of comprehension, the stars of our cosmology—well, who has an imagination that cares to go so far? Man lives between two worlds—a macro-universe of apparently measureless extent, and a micro-universe so small that its dimensions stagger our imagination. In one of them distance expands and increases like the wildest of De Quincey's dreams, but time slows up and velocities become comparatively small; in the other distances shrink and shrivel up, while time and velocities are marvelously accelerated. This forces upon us the old problem: What is the factor that determines size? How do we know how large to be? Why are we creatures between the macrocosmos and the microcosmos? Do these universes extend indefinitely in both directions from us? Is the statement of W. D. MacMillan that "the sequence of physical units is infinite both ways" the categorical imperative of rational belief?

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[24] H. Shapley: *Galaxies,* Philadelphia, 1943, vi + 229 p. In particular, see pp. 126-127.

[25] See W. W. Campbell: *Stellar Motions,* New Haven, 1913, Chap. 6; W. M. Smart: *Stellar Dynamics,* Cambridge, 1938.

### 10. *De Sitter's Cosmology—The "Wall of Time" Theory.*

We turn to another concept of the universe which is in some striking ways superior to that of Einstein. This concept is derived from the work of the great Dutch astronomer, W. de Sitter (1872-1934) who differed from Einstein and Mach in their assumption that inertia is an inherent phenomenon of the totality of matter and that the Foucault pendulum would cease to rotate if all the other matter in the universe were removed. De Sitter pointed out that the Einstein postulate gives too great reality to matter, and in order to have substantiation, must posit the existence of vast quantities of world matter. His theory perhaps presents too much the aspect of going back to material models and makes the reality of inertia too substantial. In a way de Sitter's position is a curious example of the desire, found increasingly in modern science, to swing as far away as possible from the material concepts; to regard with distrust explanations that have their roots too firmly fixed in solid matter or in analogies drawn from sensuous models. Maxwell's answer to the puzzle of the ether rests in a set of equations; modern atoms are real only in the symbols that represent their activities; gravitation vanishes in a metric of space-time. Are we not reactionary when we try to explain the inertia of matter in terms of matter, even though this be thought of merely as the sum of all the particles throughout space?

The naïve point of view of de Sitter is this. Let us look at infinity and try to find out the fundamental character of the metric of space-time from speculations based upon our own intuitive feelings with regard to the four-dimensional geometry of that remote region. What a glorious freedom for the new physics! What a declaration of independence is found in the following dictum of the eminent astronomer.[26] "How they [the gravitational potentials] are in those portions of space and time of which our observations have not yet penetrated, we do not know, and how they are at infinity [of space or of time] we shall never know. All assumptions regarding the value of them [the gravitational potentials] at infinity are therefore extrapolations which we are free to choose in accordance with theoretical or philosophical requirements."

Galileo would have us choose these gravitational potentials at infinity so that space-time would be flat or Euclidean; Einstein made his selection so that all were zero except the one involving time, which placed a logical blemish upon his theory because the set of values thus assumed was not wholly invariant for mathematical transformations. De Sitter, turning his back upon the material consideration that led Einstein to his cosmology, made his "philosophical requirement" the invariance of the potentials, and calmly set them all equal to zero in that remote and inaccessible region of infinity. The consequences of the de Sitter postulate agree spatially with those of Einstein but differ for the variable of time. The radius of the universe, spatially speaking, is the same as that of the Einstein, but the time-concept differs in this interesting conclusion that a ray of light could never make the complete circuit and return again to form a ghostly image of its former self. Thus the fanciful "ghost star" theory must be abandoned by those who go with de Sitter, but it is replaced by the equally poetical one which we may call the postulate of the "wall of time."

The picture that de Sitter wished to convey to us is that of a great wall of time barring the path of anything such as a ray of light which tries to make the circuit. As we look deep into space, at the spiral nebulae perhaps, time appears to slow up until at very remote distances from us it is "always six o'clock," to quote the mad Hatter. We have used the phrase "appears to slow up" which is the de Sitter concept. If we should actually travel out to this remote part of space we should find that the hands of our watches still moved with their ordinary velocity, but if we looked back at the earth that we had left, time would seem to have slowed up on that planet. In other words time, in regions remote from our own, flows less swiftly from our point of view, so that a ray of light, as it recedes from us, would seem to lose velocity, until in the neighborhood of the great wall of time it would cease to move altogether.

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[26] "On Einstein's Theory of Gravitation and its Astronomical Consequences." *Monthly Notices of the Royal Astronomical Association,* vol. 78, p. 3, November, 1917.

Is there experimental evidence for this strange concept? Most assuredly so and evidence of a remarkable kind. Every luminous body like the sun or like a star carries its own clock in the form of atomic vibrations. These vibrations we can measure in terms of spectral lines and compare them with the same vibrations on the earth. In other words we can measure a star's time with earth time by the most perfect mechanisms in the world.

If the spectral lines of a star shift toward the red end of the spectrum when compared with our own, this shift can be due to at least two causes as we have previously explained.

First, the star may be receding from us. This is known as the Doppler principle and is analogous to the change in pitch of a locomotive whistle as the engine moves away from us. Second, it may measure a genuine difference between star-time and earth-time. This difference, you will recall, was actually measured in the case of the heavy white dwarf which we knew was not receding from us with any great velocity, since it was the companion of Sirius.

We have already commented in the preceding section upon the strange fact that the spiral nebulae, on the average, are apparently receding from us with almost incredible velocities, some of these attaining the fantastic values of from ten to twenty-five thousand miles per second.[27] These recessive velocities, or the red-shifts which indicate them, have long been recognized as presenting astronomy with one of its most difficult problems. But with the recent measurements of the distances of the spiral nebulae a new aspect of the puzzle has presented itself. *The recessive velocities of the spirals increase linearly with their distances from us.* This fact is so well grounded in observation that it may be reduced to the simple law that the speed of recession of distant objects is approximately equal to 100 miles per million light-years. Thus the nebula in the faint cluster of the constellation Gemini, which is receding with a velocity of 15,000 miles per second, may be estimated to be 150,000,000 light-years from us.

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[27] The determination of these recessive velocities, or more properly the red-shifts in the lines of the spectra, is largely the work of Milton Humason of the Mount Wilson observatory.

De Sitter believed that this curious discovery of modern astronomy furnishes experimental evidence for his concept of space-time. Time, in the remote regions where the spirals exists, is slowing down a bit and the atomic watches of these remote masses are revealing this in the Doppler shift. The spirals are not all receding from us with those huge velocities, as we have thought. They are merely nearer the wall of time and their atomic clocks in consequence move more slowly.

### 11. *The Expanding Universe.*

The spectacular verification of the consequences of the de Sitter cosmology which foretold the mysterious recessive law of the spiral nebulae, and the equally spectacular discovery of the vast quantities of matter required by the Einstein cosmology stirred the imagination of the scientific world. Since the fundamental postulates of the two cosmologies were incompatible, the question was advanced as to which corresponded to the real universe. For although both are derived from solutions of the gravitational equations of the general theory of relativity, and in fact are the only two possible cases corresponding to a static universe in which the radius of space is constant, there is no way to choose between them except by making arbitrary assumptions as to the nature of the gravitational postulates at infinity.

An examination of the differences between the two cosmologies reveals also other difficulties. Thus the universe of Einstein relates the size of space to the quantity of matter which it contains, while the universe of de Sitter ignores the presence of matter altogether and assumes that its density is zero. Moreover, Einstein's cosmology makes fundamental use of the proposition of Mach that the inertial framework of the universe is a property of the totality of matter which it contains, whereas de Sitter asserts that the mass of the universe has nothing to do with the mystery of inertia. For the one the Foucault pendulum, which we have described in Chapter 3, would reveal no rotation of the earth in an empty universe, while for the other the annihilation of stars and galaxies would have no effect upon the experiment.

Into this conceptual impasse an interesting and suggestive hypothesis was introduced in 1922 by A. Friedmann, who assumed that the real universe might be a transition state between the material universe of Einstein and the empty one of de Sitter.[28] That is to say, instead of the two static models between which we are asked to choose, there might be another in which the radius of the universe is a function of time itself. This fruitful idea was found to be consistent with the gravitational equations of the general theory of relativity, and the new expanding universe emerged as an attractive solution of the problem. In Friedmann's formulation, the radius of the universe appeared as a periodic function of the time with the present phase one in which the expansion prevails. Assuming that the mass of the universe was of the order of $5 \times 10^{21}$ suns, Friedmann computed that the period from the limit of expansion to the limit of contraction and back again was of the order of $10^{10}$ years.

The next advance in the theory was made by Abbé G. Lemaître, who, following certain suggestions proposed by K. Lanczos in 1922,[29] published in 1927 a paper describing "A Homogeneous Universe of Constant Mass and Increasing Radius".[30] Although the ideas presented were apparently conceived independently of those of Friedmann, the essential content of the earlier paper was rediscovered. Lemaître, however, availed himself of a certain freedom of choice among the possible variable universes and discussed in detail that particular expanding model which began with an initial minimum radius (the Einstein solution) and is now in transition toward the model of de Sitter. His point of view is illustrated by the following quotation:

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[28] "Über die Krümmung des Raumes", *Zeitschrift für Physik,* Vol. 10, 1922, pp. 377-386.

[29] "Bemerkung zur de Sitterschen Welt", *Phys. Zeitschrift,* Vol. 23, 1922, pp. 539.

[30] "Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques." *Annales Société sc. Bruxelles,* Vol. 47, 1927, (A), pp. 49-59.

"The paradox [of the two static universes] is elucidated when one takes account of the fact that the solution of de Sitter does not correspond to all the necessities of the problem. Space is indeed homogeneous, of constant positive curvature; space-time is also homogeneous, all points of the universe are perfectly equivalent; but the division of space-time into space and time no longer possesses homogeneity. The chosen coordinates introduce a center to which nothing corresponds in reality; an immovable point at the center of space describes a geodesic of the universe, an immovable point other than at the center will not describe a geodesic of the universe. The choice of coordinates thus breaks the homogeneity which exists in the assumptions of the problem, and from this come the paradoxical results which appear at the 'horizon' of the center. When one introduces coordinates and a corresponding division of space and time which respect the homogeneity of the universe, one finds that the field is no longer static; one obtains a universe of the same form as that of Einstein, but a universe in which the radius of space, instead of remaining invariable, varies with time according to a particular law."

At the end of his classical paper on the nature of the spiral nebulae published in 1926, E. Hubble made use of Einstein's formula connecting the size of space with the average density of matter to estimate the present radius of space. Recognizing the possibility of an error of enormous magnitude in such an extrapolation, Hubble none the less gave the value of $2.7 \times 10^{10}$ parsecs as an estimate of this radius. To put this figure into slightly more intelligible terms, it would take light perhaps 550 billion years to make the circuit of space.[31]

Lemaître, accepting the estimate of Hubble for the size of our present universe, attempted to compute the radius which existed in the original universe of Einstein before the now visible expansion began. This he found to be of the order of $2.7 \times 10^8$ parsecs, or one hundred times as small as its present estimate.

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[31] Hubble gave this estimate in his paper: "Extra-galactic Nebulae," *Astrophysical Journal*, Vol. 64, 1926, pp. 321-369; in particular, p. 369. He made use of Einstein's formula: $R^2 = c^2/4\pi\rho G$, where $R$ is the radius, $c$ the velocity of light, $G$ the Newtonian gravitational constant, and $\rho$ the average density of matter in space. For this density Hubble assumed the value of $1.5 \times 10^{-31}$ grams per cubic centimeter. This he later changed to $5 \times 10^{-31}$ grams per cubic centimeter, which yields as a new approximation to the radius the value $1.5 \times 10^{10}$ parsecs. See, for example, H. P. Robertson: "Relativistic Cosmology", *Reviews of Modern Physics*, Vol. 5, 1933, pp. 62-90; in particular, p. 70.

At the end of his paper Lemaître, assuming with de Sitter that there exists a distance such that the light from bodies beyond this limit is thrown into the invisible region of the infra red, estimated that this *horizon of knowledge* is approximately ten times the range of the great telescope of Mount Wilson. Already our penetration into space is so great that the shift toward the red end of the spectrum corresponds to a Doppler velocity of 15,000 miles per second.

Stirred by the speculations of Friedmann and Lemaître, A. S. Eddington investigated the stability of the Einstein universe. This universe he discovered was in equilibrium, but the equilibrium was unstable. Any disturbance, however slight, was enough to tip it over the edge and start its course of expansion or contraction.[32] "The Newtonian attraction and the cosmical repulsion are in exact balance," he said. "Suppose that a slight disturbance momentarily upsets the balance; let us say that the Newtonian attraction is slightly weakened. Then repulsion has the upper hand, and a slow expansion begins. The expansion increases the average distance apart of the material bodies so that their attraction one to another is lessened. This widens the difference between attraction and repulsion, and the expansion becomes faster. Thus the balance becomes more and more upset until the universe becomes irrevocably launched on its course of expansion. Similarly if the first slight disturbance were a strengthening of the Newtonian attraction, this would cause a small contraction. The material systems would be brought nearer together and their mutual attraction further increased. The contracting tendency thus becomes more and more reinforced. Einstein's universe is delicately poised so that the slightest disturbance will cause it to topple into a state of ever-increasing expansion or of ever-increasing contraction."

An instructive point of view was introduced into the problem by E. A. Milne in 1935,[33] who returned to the first principle of the special theory of relativity for an explanation of the mystery of the recessive velocities of the spiral nebulae. He began with what he calls Einstein's cosmological principle, which he describes as follows: "We shall therefore say that two particle-observers *A* and *B* are equivalent when the totality of observations A can make on *B* can be described by *A* in the same way as the totality of observations which *B* can make on *A* can be described by *B*. When this condition is satisfied, we shall write $A = B$. A system of particles is then said to satisfy Einstein's cosmological principle when if *A* and *B* are two members of the system such that $A = B$, then *A*'s description of the whole system is identical with *B's* description of the whole system." As a direct consequence of this principle Milne is led to investigate: "... the phenomenon that if two observers in uniform relative motion send out an expanding light wave, each will forever see himself as the center of the spherical wave, in his own reckoning. In our system each uniformly moving particle-observer of the system sees himself for all time as the center of the system, and sees the system arranged nonhomogeneously but with spherical symmetry round himself."

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[32] For a popular account of these matters see Eddington's volume, *The Expanding Universe*, New York, and Cambridge, 1933, viii + 182 p.

[33] *Relativity, Gravitation, and World-structure*, Oxford, 1935, viii + 365 p.

Starting with this postulate and avoiding the assumptions of the general theory of relativity, Milne is able to explain the expanding universe. Introducing a point in time and space where all the world-lines of the observers originally concurred, the place of origin of the universe, or the epoch of creation, he is able to derive numerous properties of the spiral nebulae. Thus measuring time from the epoch of creation, he finds that the radius of the expanding universe is $ct$, where $c$ is the velocity of light. The distance of any nebula is less than this value and at distances just less than the radius the nebulae are almost invisible. The velocities of recession range up to the velocity of light, but in the neighborhood of this velocity the number of nebulae increases without bound. In other words, although the radius of space is finite, the amount of matter which it contains is infinite. About this curious proposition Milne makes the following statement: "The volume occupied by the universe is therefore probably finite although the total population is infinite. In that case the nebulae, infinite in number, are representable as an open set of points, crowded together at any given epoch towards the boundary, which would, however, be inaccessible and moving with the speed of light. The nebulae themselves, near the boundary, would be subject to an extreme Lorentz contraction, in the view of any observer, and would indeed in the limit appear squashed flat in the direction of their motion. There is therefore abundance of 'room' for them, infinitely dense though they be in distribution near the inaccessible expanding boundary."

It is clear that in a situation of this kind where the one observable fact is the recessive velocities of the spiral nebulae, imagination is no longer subject to the usual restraints imposed by scientific measurement. Numerous universes were engendered in the ensuing frenzy of cosmic speculation. In some of these models the total energy of the universe is conserved, as in the case of the Friedmann universe; in others, as with Lemaître, matter is finite and invariable; while in still others, as in the universe of Milne, space is finite though expanding, but the amount of matter which it contains is infinite. In some of the postulates, as we have already pointed out, there appears an exceptional point in the scale of time, the origin of creation, which thus imposes upon time itself a finite value. Such universes, however, encounter difficulty from the astronomers, since the period of time required for the universe to have attained its present form appears to be of the order of from 10 to 100 billion years, whereas the evolution of the stellar system as it has been constructed seems to require a time-scale in excess of the cosmological estimates.

A comprehensive survey of all these possibilities has been given by H. P. Robertson, who concludes with a statement of his belief that the ultimate truth lies within the formulation of the theory of relativity. Thus he says: "In giving this survey of cosmologies we are convinced that the underlying theory forms an integral part of the theory of relativity, and that although the choice of a particular model may for the present be influenced by the predilections of the individual, we can hope that the future will reveal additional evidence to test its validity and to lead us to a satisfying solution."[34]

## 12. *Is Nature Finite?*

What a vast distance we have now come from the philosophy of Kant! The subjective, *a priori* intuitions of the philosopher now merge into a single substratum of reality, the space-time of the astronomers. "The business of geometry," said J. J. Sylvester, the mathematician, "is with the evolution of the properties of space, or of bodies viewed as existing in space."[35] The laboratory of the sky evidently can contribute profoundly to this evolution.

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[34] "Relativistic Cosmology," *Reviews of Modern Physics*, Vol. 5, 1933, pp. 62-90; in particular, p. 83. A more popular account of these theories will be found in Robertson's lecture: "The Expanding Universe," *Science*, Vol. 76, 1932, pp. 221-226. An excellent account of the recent speculations in this problem is given by G. C. Omer in "A Nonhomogeneous Cosmological Model," *Astrophysical Journal*, Vol. 109, 1949, pp. 164-176.

We return finally to the main theme of our chapter: Is nature finite? It is a strange and startling fact to those scanning the night sky, probing the depths of what has hitherto among the poets and the philosophers been the symbol of the infinite, to find that science has begun to suspect that there is a bound to those phenomena with which material things are concerned. It is astonishing even to the scientists themselves to reflect that some of the hardier members of their fraternity will dare to set a bound upon time itself and speak of the epoch of creation. But science must have also its moments of fantasy and, as we have learned before, where we speculate about the transfinite universe we may make such postulates as our imagination wills.

But there have been other moments in the history of science where startling limits were set for the world of matter. It was one of the amazing discoveries of the nineteenth century that temperature had a lower bound. The location of this zero point in the absolute scale of temperature at minus 460 degrees Fahrenheit (-273 degrees centigrade) below the freezing point of water was an extrapolation as daring, when it was made, as some of those advanced by modern scientists. This material bound represents the state of matter when all its thermal energy has been pumped away. What matter, thus emptied of its activating entity, may then become we do not know, for this mysterious region is not yet fully explored, nor is it our concern at this time. The interesting fact is that temperature is bounded at one end. Is it also bounded at the other? There have been certain speculations in this direction likewise, and one tentative estimate sets the limit in the neighborhood of $1.1 \times 10^{13}$ degrees.[36] This fantastic temperature is what one might expect to find if all the molecules of a gas approached the velocity of light. But even the center of the hottest stars are far below this limit, since recent theories restrict these temperatures to something of the order of $2 \times 10^7$ degrees.[37]

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[35] *Collected Mathematical Papers*, Vol. 2, p. 5.

[36] See for example S. Suzuki: "The Existence of the Upper Limit of Temperature and a New Radiation Formula." *Proceedings of the Physico-Mathematical Society of Japan*, Vol. 8, 3rd ser. (1928), pp. 175-179.

We have already commented at sufficient length upon the maximum relative velocity of material quantities, that is to say, the speed of light, and upon the significance of this maximum in the new geometry of space. We shall merely recall it here as other evidence of the existence of natural limits.

In the preceding sections we have related the arguments of the cosmologists about the structure of space itself and have discovered that there is a strong tendency to regard the space-time universe as finite though boundless. What this really means, if its meaning can be divorced from the mathematical symbols of the metric, is that the geometry of space is not in agreement with the geometry of Euclid. It is rather the geometry of Riemann, which is founded upon fully as logical concepts as the other, however novel it may appear at first to our intuitions. The nature of this geometry, one will recall, is exhibited by the postulate that through a point exterior to a straight line no other line can be drawn so as to be parallel to the first. That is to say all straight lines, or more properly geodesics, since the meaning of the word straight has slipped a bit, will concur if sufficiently prolonged. By a simple deduction the sum of the angles of a triangle in such a space is greater than two right angles. Riemann's space is spherical in character and hence is finite, however difficult such concepts may be within the realm of our normal intuitions. The extent of this finite space and its astronomical implications we have already sufficiently explored in the preceding section. It is enough for our present purpose to indicate again the nature of this fascinating speculation and its evolution through the merging of mathematical geometries with the quest of the elusive ether and the conquest of the heavens. A more intriguing story than this is not to be found in the annals of man.

We turn finally to another speculation of similar character though of inverse magnitude and inquire whether or not it is possible that inferior limits are also to be found for natural dimensions. If there be an upper limit to space-time is it not possible that this value bears a fundamental relation to the smallest unit of space? In other words, as Eddington picturesquely asks, "How does an electron know how large to be?"

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[37] S. Chandrasekhar: *An Introduction to the Study of Stellar Structure*, Chicago, 1939, p. 232.

Eddington himself essays to answer this question and makes the following striking remarks:[38]

> There is strong ground then for anticipating that the solution of the unknown equations [of the electron] will be radius of electron in any direction = numerical constant × radius of curvature of space-time in that direction.

This concept of nature leads at once to the full atomistic philosophy. The problem of Zeno no longer exists, since we deny the existence of the continuum wherein the paradox resides. If there be a smallest object then there must also exist a finite number of them between any two other objects, and any partitioning of the interval below this smallest unit is as impossible as the attainment of a velocity exceeding that of light.

One cannot help but speculate as to the role of number in this vast but finite universe. What happens to the sequence 1, 2, 3, 4, . . ., which we considered earlier in this chapter, and the denumerable infinities of quantities which were put into one-to-one correspondence with this famous set? The great tool of the mathematician has been his infinity. But when he turns from the pure realm of logical speculation and applies the noble structure of mathematics which he has created to the interpretation of the natural world, then there exists the suspicion that the mathematical infinity may be unnecessarily large. In all the applications, we may cautiously aver, a very great though finite number would do just as well. How large this number should be is of no importance, although it should be great enough to count and measure all the quantities of the physical world. Perhaps it might be equal to the total number of particles in the universe, or lest we be accused of lacking a generous spirit, it might be set at not less than all the permutations of these particles.

There have been recent attempts to estimate the number of elementary particles, electrons and protons, in the universe. Now that the radius of space has been associated with the quantity of matter which space contains, any estimate of the size of the one will lead to an estimate of the size of the other. If we assume with the more daring astronomers that the total amount of matter in the universe is of the order of $5 \times 10^{21}$ suns, and if the mass of the sun is $2 \times 10^{33}$ grams, and if electrons and protons are equally paired in nature either as separate particles or in neutron combination, then since their combined mass is of the order of $1.62 \times 10^{-26}$ grams, it is merely a matter of division to find that the total number of these elementary particles is of the order of $10^{79}$.

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[38] *The Mathematical Theory of Relativity*, Cambridge University Press (1923), p. 154.

As far back as we can see in the history of thought there has existed a tendency toward number-magic. The Pythagoreans sought to relate the properties of integers to the fundamental mysteries of their cult. The Neoplatonists in the decadent days of the Alexandrian Museum sought for meaning in the number 6, which is called a perfect number, since it is equal to the sum of its factors, 1, 2, and 3. Even Archimedes in one of his most ingenious speculations attempted to estimate the number of grains of sand in the universe. It is a matter of perhaps more than historical curiosity to note that Archimedes, defining 10,000 grains of sand as equal to one poppy seed, proceeded to estimate that the number of such grains in the universe was of the order of $10^{63}$.[39] By such speculations as these are the dreams of the ancient world related to the fancies of the new!

Do we wonder then at more recent attempts to discover in nature some fundamental number which relates the very great to the very little? Eddington, who dared more than others to extrapolate far beyond the limits of scientific data, typifies this yearning of the human spirit to discover some deep-seated aspect of the natural world in pure number itself when he writes:[40]

". . . . among the constants of nature there is one which is a very large pure number; this is typified by the ratio of the radius of an electron to its gravitational mass — $3 \times 10^{42}$. It is difficult to account for the occurrence of a pure number (of order greatly different from unity) in the scheme of things; but this difficulty would be removed if we could connect it with the number of particles in the world—a number presumably decided by pure accident. There is an attractiveness in the idea that the total number of the particles may play a part in determining the constants of the laws of nature; we can more readily admit that the laws of the actual world are specialized by the accidental circumstances of a particular number of particles occurring in it, than that they are specialized by the same number occurring as a mysterious ratio in the fine-grained structure of the continuum."

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[39] *The Sand Reckoner*, written perhaps around 250 B. C.

[40] *The Mathematical Theory of Relativity*, loc. cit., p. 167. Eddington enlarges on this thesis in later works. See, for example, *The Expanding Universe*, New York and Cambridge, 1933, Chap. 4.

Our conclusion must be like that of all who tilt with the concept of infinity. We can assert nothing with certainty. However, it becomes increasingly apparent from the writings of the wise that modern scientists, the mathematicians, the astronomers, and the physicists, are not wholly loathe to assume the following postulates:

1. *A thing does not exist by virtue of its logical conception alone. There may be doubt where the canons of Aristotle's logic involve the use of infinite operations; or if one prefers to regard the problem in another way, we are limited in the realm of transfinite consideration only by the postulate of consistency with finite specialization.* (Modern mathematical rigor).

2. *The physical universe in its material elements is finite.* (Modern physics).

By these postulates we may perhaps reorder the structure of the universe in such a way that the mysteries of the continuum of Zeno and Cantor disappear in an atomic concept of all properties associated with matter.

# CHAPTER 8.

## THE ATOM CONCEPT.

> ". . . all experience is an arch wherethro'
> Gleams that untravell'd world, whose margin fades
> Forever and forever when I move!"
>
> Alfred Tennyson in "Ulysses".

### 1. *Continuity and the Atom.*

Like most of the great theories of science, the origin of the atomic concept of nature is found in ancient documents. The first chapter opened in the fourth century B. C. when Democritus of Abdera, following the ideas of his teacher Leucippus and perhaps of even earlier writers, enunciated the belief that matter is not indefinitely divisible, but that all substances are formed of indivisible particles, or atoms. These primary constituents of the universe are eternal and unchangeable; they are separated from one another by void, and a combination of them constitutes what we call matter. They are not all the same, says Democritus, for those of water must be round and smooth so that they can the more readily glide over one another, while those of iron are hard and rough and hook together to form this rugged element.[1]

We have discussed previously some of the characteristics of the atomistic views of Democritus, but it is time to reflect more philosophically upon this strange doctrine. By the assumption just stated we give up the notion of continuity, which is certainly one of our most treasured intuitional concepts.

The atomists would have us believe that there is a smallest physical unit, from which all larger objects are constructed; the position of the continuists is wittily put in the well known verse of Dean Swift:[2]

"So, Nat'ralists observe, a Flea
Hath smaller Fleas that on him prey.
And these have smaller Fleas to bite 'em,
And so proceed *ad infinitum.*"

All modern mathematics with its infinitesimals and its infinity, its point-sets and its continuum, is based upon the assumption of continuity. To be sure some strange results have emerged from this assumption. One of the most remarkable of these is the curve of the German mathematician, Karl Weierstrass (1815-1897), which has the extraordinary property of being everywhere continuous but nowhere possessing a tangent. The existence of such a curve makes us pause and scrutinize again the concept of continuity, which Poincaré characterizes as "a belief which it would be difficult to justify by apodeictic reasoning, but without which all science would be impossible."[3] Complete atomistic philosophy applied to the logic of mathematics would surely be fatal to certain branches of it. On the other hand the postulation of complete continuity in natural phenomena would appear to be equally disastrous since even to account for so common a thing as the high compressibility of gas would impose a superhuman agility upon the imagination of the continuists.

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[1] See E. Zeller: *Pre-Socratic Philosophy*, London, 1881, Vol. 2, P. 207.

[2] From "On Poetry: A Rhapsody."

The most celebrated atomist of antiquity was the Latin poet Lucretius, whose philosophy of space, infinity, and the atom has already been commented upon earlier in the book. *De Rerum Natura,* in which the poet states his reflections upon the nature of the physical universe, has some remarkable passages dealing with the atomic theory of matter. The following quotation is wholly typical of the depth of his ideas, which are interesting and instructive in showing how far an intellect, unaided by laboratory instruments, can see into the difficult problem:[4]

"Again things which look to us hard and dense must consist of particles more hooked together, and be held in union because welded all through with branch-like elements. In this case first of all diamond stones stand in foremost line inured to despise blows, and stout blocks of basalt and the strength of hard iron and brass bolts which scream out as they hold fast to their staples. Those things which are liquid and of fluid body ought to consist more of smooth and round elements; for the several drops have no mutual cohesion and their onward course too has a ready flow downwards. All things lastly which you see disperse themselves in an instant, as smoke mists and flames, if they do not consist entirely of smooth and round, must yet not be held fast by closely tangled elements, so that they may be able to pierce the body and enter it with biting power, yet not stick together: thus you may easily know, that whatever we see the senses have been able to allay, consists not of tangled but of pointed elements. Do not however hold it to be wonderful that some things which are fluid you see to be likewise bitter, for instance the sea's moisture: because it is fluid it consists of smooth and round particles, and many rough bodies mixed up with these produce pains; and yet they must not be hooked so as to hold together; you are to know that though rough, they are yet spherical, so that while they roll freely on, they may at the same time hurt the senses. And that you may more readily believe that with smooth are mixed rough first-beginnings from which Neptune's body is made bitter, there is a way of separating these, and of seeing how the fresh water, when it is often filtered through the earth, flows by itself into a trench and sweetens; for it leaves above the first-beginnings of the nauseous saltness, inasmuch as the rough particles can more readily stay behind in the earth."

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[3] *Foundations of Science,* p. 173.

[4] From Munro's translation: Book 2, lines 444-477.

The significance to be attached to this crude picture is the evidence which it affords of the necessity of postulating primary units whose combinations make up the structure of various states of matter. Those who believe in the continuity of universal substance tend to draw their picture in terms of a fluid; those who postulate atoms ask how, otherwise, the structure of a gas with its enormous powers of condensation and rarefaction, can be rationally explained. It is a curious fact that those who deal with phenomena by means of differential equations, the nature of which is grounded in considerations of continuity, speak of the tubes of electromagnetic forces as moving through a continuous fluid medium filling all space and permeating all material bodies; only Mendeléeff, the chemist, speaks of the ethereal atom. The point at issue is, indeed, one of vital importance. It is a battle as old as human thought. The issue today is nearly as obscure as it was in the early dawn when Democritus and Lucretius began to translate crude experience into hypotheses. Poincaré in his article on "The Quantum Hypothesis," written at a moment when this theory was just beginning to stir the world of science, makes the following significant observation:[5]

"Is discontinuity destined to reign over the physical universe, and will its triumph be final? Or will it finally be recognized that this discontinuity is only apparent, and a disguise for a series of continuous processes. The first observer of a collision thought he was witnessing a discontinuous process, but we know today that what he saw was the result of changes which, although very rapid, were continuous. Any attempt at present to give a judgment on these questions would be a waste of paper and ink."

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[5] *Dernières pensées,* Chap. 6, Sect. 8, Paris, 1913, p. 192.

The weight of the authority of Aristotle was thrown in favor of the continuity concept; Newton, on the other hand held the contrary view. In discussing the structure of solid bodies he expressed his belief[6] "that the smallest particles of matter may cohere by the strongest attractions, and compose bigger particles of weaker virtue; and many of these may cohere and compose bigger particles whose virtue is still weaker; and so on for divers successions, until the progression ends in the biggest particles, on which the operations in chemistry, and the colors of natural bodies, depend, and which, by adhering, compose bodies of a sensible magnitude. If the body is compact, and bends or yields inward to pression, without any sliding of its parts, it is hard and elastic, returning to its figure with a force rising from the mutual attraction of its parts. If the parts slide upon one another, the body is malleable or soft. If they slip easily, and are of a fit size to be agitated by heat, and the heat is big enough to keep them in agitation, the body is fluid; and if it be apt to stick to things, it is humid; and the drops of every fluid affect a round figure, by the mutual attraction of their parts, as the globe of the earth and sea affects a round figure, by the mutual attraction of its parts by gravity."

### 2. *An Atomic Picture of Matter.*

Before we enter into a discussion of the scientific methods which have been applied to the nature of matter, let us speculate for a moment upon the structure of a piece of stone. Let us suppose that our eyes are assisted by a lens of such high resolving power that we can look beyond the surface of the solid matter. What a land of magic we now behold! For that which our crude sense of touch thought was solid is now found to be incredibly porous, mostly void, but with a vast number of tiny particles in tremendous agitation moving through it. The principle difference between the stone and the air which surrounds it, we now see, is that the particles of the one are mainly bound to relatively stable positions in the void, while those of the other may wander freely through the space, subject only to frequent collisions with their fellows. The appearance of solidity in the stone is merely a statistical illusion due to the mobility of the particles, which permits them to be many places at almost the same time.

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[6] See Sir David Brewster: *Life of Sir Isaac Newton,* chap. 17, 2nd ed., p. 270.

To those to whom this picture seems bizarre the consideration of a single elastic ball in a large room which contains no other object is recommended. If the ball rests upon the floor the room will appear to be almost entirely empty. But let the ball acquire some speed and bounce several times per second off of the walls. The emptiness of the room to the visitor is now no longer so evident as it was. But let the ball attain the velocity of light and he will affirm that the space is entirely filled with flying particles so ubiquitous is the ball from the incredible number of reflections which it has from the walls. Time and velocity, one thus observes, are not inconsiderable items in the illusion of solidity.[7]

Are we then to regard atoms, where we use the word in its generic sense to mean the indivisible unit, as small, hard balls moving with ceaseless activity from place to place and producing to our crude sense of touch the illusion of solidity? If this is, indeed, our picture and it probably conforms to the mental image of the average man, then we might ask why we could not conceive of such an atom divided into two parts? The answer to this question is simple. We could so think of the divided atom, but the two parts, in consequence of our hypothesis, would lose their identity as things. The point that we are making is due originally to Clerk Maxwell whose view is an attractively modern one. He says in his *Theory of Heat* (page 285):

"We do not assert that there is an absolute limit to the divisibility of matter: what we assert is, that after we have divided a body into a certain finite number of constituent parts called molecules, then any further division of these molecules will deprive them of the properties which give rise to the phenomena observed in the substance."

There is a vast amount of room for division between the smallest atom ever conceived and the largest mathematical point. We are not imposing insuperable logical difficulties when we ask the question: Divide the atom and what then? The answer of Maxwell asserts that we shall mean merely that a divided atom loses its material identity and whatever else its constituent parts may be they are not to be regarded as material entities.

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[7] P. W. Bridgman has discussed this problem for a single molecule traveling in a horizontal line. *Proc. of the American Academy of Arts and Sciences*, Vol. 49, No. 1, p. 105 et seq.

### 3. *The Hypothesis of Avogadro.*

Historically, so far as modern science is concerned, the atomic theory started with the work of John Dalton (1766-1844), who conceived of atoms as small, indivisible constituents of matter which combined in various proportions to make the substances of the world. In his theory the atoms of any given substance are identical, but atoms of different elements are different and have different weights. For example if a gram of water is divided electrolytically into its components it will be found to consist of eight times as much oxygen, by weight, as hydrogen. Hence, according to Dalton, the water particle is made up of two atoms, one of hydrogen and the other of oxygen, the latter being eight times as heavy as the former.

When further insight into chemical processes was gained, however, it was found that a better hypothesis could be substituted for the purely atomic one. Amadeo Avogadro (1776-1856) in 1811 advanced the proposition that equal volumes of different gases contain equal numbers of *molecules*, that is to say, groups of atoms in combination. This law must be recognized to be merely a convenient agreement, much like the astronomical agreement to regard the sun as the center of the solar system rather than the earth. Equal *volumes* of gases instead of equal *weights* were made the basis for defining the relative size of atoms. One can readily appreciate the convenience of this hypothesis. For example, it is found that equal volumes of hydrogen and chlorine will completely combine to form hydrochloric acid. Hence we say that hydrochloric acid is composed of one atom of hydrogen and one of chlorine, the latter atom being thirty-five times as heavy as the former. Similary one part of oxygen, speaking volumetrically, will combine with two parts of hydrogen to form water, so we assume that the molecules of this common fluid consist of two atoms of hydrogen in combination with one of oxygen, the latter being sixteen times as heavy instead of eight, as in the Dalton theory.

We should mention in this connection the suggestion made in 1815 by William Prout (1785-1850) that the "first stuff" postulated by the ancient philosophers is realized in hydrogen.[8] His theory, truer than anyone then thought, was essentially the assumption of a basic atom from which by processes of alchemy all the other forms of matter could be constructed.

### 4. *The Atom of Boscovich.*

"It is remarkable that involuntarily we always read as superior beings," says Emerson in his essay on "History." With what pity, then, we look upon the masters of the past as they struggled with these weighty matters. How far down seems the foot of the mountain to one who has toiled laboriously up the slope. We must pause a moment to survey the guesses of our predecessors, as those who follow us will survey our own attempts to probe to the ultimate in nature.

The first of those belonging to the modern age of science who tried to give a material picture of the atom was Ruggiero Giuseppe Boscovich (1711-1787), an Italian natural philosopher, born at Ragusa in Dalmatia. His concept began with the traditional one of the indivisible particle, merging into the monadism of Leibniz, since he failed to endow his atom with finite size. This primary substance was able to move continuously in a line and could occupy only one place at a given time. It possessed mass and could be changed from its straight-line propagation only by the impressing of force. The chief characteristic of the Boscovich atom, however, was its varying spheres of action. For a given distance, a small fraction of an inch, perhaps, atoms were attracted to one another by a force varying inversely as the square of the distance between them. But within a second sphere of influence the attractive force was changed to one of repulsion. More than one such sphere might probably be conceived of, but the final domain of activity was necessarily one in which repulsive forces held. This led to the second fundamental assumption of the Boscovich atom, which asserted that no two atoms could ever coincide since the ultimate repulsive forces would always increase as the two approached one another, until any conceivable force could be exceeded. Clerk Maxwell makes the following comment upon this hypothesis:[9]

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[8] In an anonymous paper in the *Annals of Philosophy*, 1815.

[9] From Maxwell's article on the "Atom." *Encyclopedia Britannica*, 9th ed.

"But this seems an unwarrantable concession to the vulgar opinion that two bodies cannot coexist in the same place. This opinion is deduced from our experience of the behaviour of bodies of sensible size, but we have no experimental evidence that two atoms may not sometimes coincide. For instance, if oxygen and hydrogen combine to form water, we have no experimental evidence that the molecule of oxygen is not in the very same place with the two molecules of hydrogen. Many persons cannot get rid of the opinion that all matter is extended in length, breadth and depth."

Although the concept of the Boscovich atom continued to interest a number of scientists, including Lord Kelvin, for many years, it may finally be regarded as furnishing merely another model for the historic gallery. But only upon these magnificent theories of the past can we hope to build the ever-more-certain theories of the future. The atom of Boscovich has one feature of peculiar interest to our speculation: it substitutes for the substantive particle of Democritus and Lucretius a pure theory of action at a distance. The atom was of infinitesimal dimension, wholly replacing the property of finite size by the more elusive property of repulsion and attraction. This was a fine idea and we turn from it with certain regrets.

### 5. *The Vortex-ring Theory of the Atom.*

Another famous concept of the atom was due to Lord Kelvin, and it has already been mentioned briefly in another place under the name of the vortex-ring theory. The basic idea was a very simple one, being merely the generalization of smoke rings. In 1858 Helmholtz published a short paper entitled, "On the integrals of the hydrodynamic equations which express vortex-motion." This remarkable paper was translated for the *Philosophical Magazine* in 1867, and thence came to the attention of Lord Kelvin, then William Thomson. This scientist, with his passion for seeing physical interpretations in everything, suddenly had the inspiration that here was a model for the atom. Vortex-rings in the ether might be the solution of the ancient problem. His ideas are vividly told in the following quotation from a letter written by him to Helmholtz on the event of his conception of this atom.[10]

"Take one side (or the lid) off a box (any old packing-box will serve) and cut a large hole in the opposite side. Stop the open side

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[10] Thompson's *Life of Lord Kelvin* (loc. cit.), Vol. 1, pp. 513-514.

AB loosely with a piece of cloth, and strike the middle of the cloth with your hand. If you leave anything smoking in the box, you will see a magnificent ring shot out by every blow. A piece of burning phosphorous gives very good smoke for the purpose; but I think nitric acid with pieces of zinc thrown into it, in the bottom of the box, and cloth wet with ammonia, or a large open dish of ammonia beside it, will answer better. The nitrite of ammonia makes fine white clouds in the air, which, I think, will be less pungent and disagreeable than the smoke from phosphorous. We sometimes can make one ring shoot through another, illustrating perfectly your description; when one ring passes near another, each is much disturbed, and is seen to be in a state of violent vibration for a few seconds, till it settles again into its circular form. The accuracy of the circular form of the whole ring, and the fineness and roundness of the section, are beautifully seen. If you try it, you will easily make rings of a foot in diameter and an inch or so in section, and be able to follow them and see the constituent rotary motion. The vibrations make a beautiful subject for mathematical work. The solution for the longitudinal vibration of a straight vortex column comes out easily enough. The absolute permanence of the rotation, and the unchangeable relation you have proved between it and the portion of the fluid once acquiring such motion in a perfect fluid, shows that if there is a perfect fluid all through space, constituting the substance of all matter, a vortex-ring would be as permanent as the solid hard atoms assumed by Lucretius and his followers (and predecessors) to account for the permanent properties of bodies (as gold, lead, etc.) and the difference of their characters. Thus, if two vortex-rings were once created in a perfect fluid, passing through one another like links of a chain, they never could come into collision, or break one another; they would form an indestructible atom; every variety of combinations might exist. Thus a long chain of vortex-rings, or three rings, each runnning through each of the others, would give each very characteristic reactions upon other such kinetic atoms. I am, as yet, a good deal puzzled as to what two vortex-rings through one another would do (how each would move, and how its shape would be influenced by the other). By experiment I find that a single vortex-ring is immediately broken up and destroyed in air by enclosing it in a ring made of one's fingers and cutting it through. But a single finger held before it as it approaches very often does not cut it and break it up, but merely causes an indentation as it passes the obstacle, and a few vibrations after it is clear."

An interesting question to be considered in connection with the vortex-theory is this: If the atoms are indestructible, once they are set up in the ether, how were they created in the first place? But perhaps this is too metaphysical to be considered a legitimate question. The vortex theory finally met destruction in the same way as all other theories which rely in any essential manner upon material properties of the ether. The theory, on the basis of mathematical consideration, required a finite density for the ether, and, as we have shown in previous chapters, this assumption is untenable.

### 6. *The Ether-Squirt Atom.*

We must not forget that the main objective of this volume is to exhibit the speculative character of some of the fundamental assumptions upon which the structure of modern physics is built. That this speculative aspect of reality is not a sudden reversal of scientific attitude, nor a violent overthrow of previous belief, is another thesis just as important as the first. Great philosophical currents join with a stream of thought built upon careful laboratory experience to show us that that which was physical is now metaphysical and that the metaphysical problems are now the problems of reality.

An interesting example of this merging of the two ideas is to be found in a paper written by Karl Pearson (1857-1936) under the alluring title: "Ether squirts, being an attempt to specialize the form of ether motion which forms an atom in a theory propounded in former papers." This paper was not published, as its title might indicate, in the magazine section of a Sunday newspaper, but appeared in 1891 in the conservative *American Journal of Mathematics*.[11] It runs to fifty pages bristling with equations. That Pearson was quite aware of the bizarre nature of his new ideas is readily apprehended from the fact that he quotes as an apologia the following statement which he attributes to Fitz Gerald:

"It has become the fashion to indulge in quaint cosmical theories and to dilate upon them before learned societies and in learned journals. I would suggest, as one who has been bogged in this quagmire, that a successor in this chair might well devote himself to a review of the cosmical theories propounded within the last few years. The opportunities for piquant criticism would be splendid."

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[11] Vol. 13, pp. 309-362.

The theory of Pearson starts with the assumption "that an atom or the ultimate element of ponderable matter is an ether squirt"; the definition of squirt is due to W. K. Clifford, who says:[12] "Suppose that the lines of flow are straight lines diverging from a fixed point, so that the fluid is everywhere streaming from this point; that there is no spin anywhere, and no expansion except at the fixed point." Such a streaming Clifford designates as a *squirt.*

To one unfamiliar with the history of the problem of the atom as it developed late in the nineteenth century, the ideas presented by Pearson might seem to represent a kind of scientific atavism, a return to the fanciful theories of an earlier and more ignorant age. But this, indeed, was far from the case. Searching through the literature of dynamics and elasticity, the scientists of that day were seeking for some plausible model which might provide a formulation which would satisfy all the requirements of the rapidly accumulating new knowledge in chemistry and physics. One by one the earlier models had been rejected. Little remained in the classical concepts upon which such a theory might be based. But Pearson found in the definition of the squirt a new possibility which, in its inception, was certainly little stranger than the highly successful postulate of the moving tubes of force upon which Clerk Maxwell erected the structure of his electro-magnetic ether.

Obvious difficulties which underlay the vortex-atom made a radical change in point of view necessary. Pearson conceived of the atom as being in some way a change in the structure of the ethereal fluid. One way to account for such a change, or rather one way of looking at this change in mathematical language, was to assume that it was created by a streaming of something into or out of the point in space where the atom resided. But this raised a troublesome question. What was streaming and whence did it come? Pearson's answer to the first was that the streaming was the ethereal fluid, and to the second that it didn't concern a physical theory to make such explanation. "From whence the ether flows and why its flow resists variations are problems which, as they fall outside the range of physics, I leave to the metaphysicians to settle."[13] Later, however, in his now classical *Grammar of Science,* he does offer the explanation that the ether squirt may be a flow of ether into our three-dimensional manifold of existence from a four-dimensional space transcending our sensations. He concludes his speculations with the interesting remark:[14] "Should a time ever come, which may, perhaps be doubted, when a happy conception as to the structure of the prime-atom is discovered to be a *perceptual* fact, then if such a conception involves the existence of four-dimensioned space, our friends will have done yeoman service in preparing a way for a scientific theory of the supersensuous—*out through the doorway of matter*!"

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[12] W. K. Clifford: *Elements of Dynamics,* London, 1878, p. 212.

[13] *Amer. Journal of Mathematics* (loc. cit.), p. 309.

The ether-squirt theory of the atom, if one is willing to admit the metaphysical structure upon which it rests, avoids many of the difficulties of other ethereal concepts. By defining the mass of the atom to be the mean rate at which ether is squirting into space at the point in question, one can derive in a natural manner the law of inverse-square attraction. Other phenomena, such as chemical affinity, porperties of light and electromagnetism, cohesion, etc., are amenable to treatment by the strange theory.

The possibility of *ether-sinks* also occurs as soon as we admit ether-sources, and these would behave like atoms of negative mass. Since no such phenomena had been observed in nature at that time, Pearson concluded that the repulsion exerted by one upon the other had finally driven the ether-sink atom out of the universe of our perceptions. This theory, unpalatable to Pearson's contemporaries, would not be regarded today perhaps with the same suspicion. For we find a strange parallel in the theory of the positron, the ephemeral positive electron which we shall describe later, as proposed by P. A. M. Dirac more than a third of a century after the ether-sinks had been forgotten. For Dirac discovered *negative-energy* solutions for the wave equation of the electron. Pondering the significance of such an anomalous matter, he reached the conclusion: "An unoccupied negative-energy state will now appear as something with a positive energy, since to make it disappear, i. e. to fill it up, we should have to add to it an electron with negative energy. We assume that *these unoccupied negative-energy states are the positrons.*"[15] Such is the magic and mystery with which the romance of the atom is clothed in every age.

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[14] *The Grammar of Science,* Second ed., London, 1900, p. 270.

[15] *The Principles of Quantum Mechanics,* 2nd ed., Oxford, 1935, p. 271.

### 7. *Crookes' Fourth State of Matter.*

Abandoning now these pre-radioactive atoms, we turn at last to the modern picture. We are now no longer primarily interested in trying to explain gravitational attraction by our atomic models, but we have turned to the great question of radiation. Let us put the matter as picturesquely as we can: Consider a ball of iron at ordinary room temperature. Let us apply a source of heat to it. At first the material sphere will remain dark, but from it there is nevertheless emanating a peculiar thing. Our hand, held near it, becomes warm; invisible radiations are conveying energy to our flesh across the medium separating us from the iron. Apply the heat a little more and the phenomenon becomes visible. The iron begins to glow, at first a dark red, merging into scarlet, and finally into a white glowing ball difficult to look upon. There is between us and the iron a radiant bond just as real and just as mysterious as the bond of gravitation. What is its explanation?

Another phenomenon, which we must recall in order to set the stage for the modern exploration of the structure of the atom, is that of X-ray radiation. This remarkable radiation was discovered in 1895 by William Konrad Röntgen (1845-1923) while experimenting with a Crookes' tube.

Many years previous several investigators, notably Julius Plücker (1801-1868), J. G. Hittorf (1824-1903), and William Crookes (1832-1919), had commenced to study the discharge of electricity through tubes in various stages of evacuation. When the poles of an electrical generator are connected to two metal electrodes fused into the ends of a glass tube several inches in length, and the air is gradually withdrawn by a vacuum pump, a series of interesting phenomena is observed. At first the electricity has great difficulty in getting through the tube, due to the large resistance offered by the air, but as the pressure is gradually reduced irregular streamers of pink light appear, which finally broaden until almost the entire tube is filled with a pink glow. While this transformation is taking place a luminous tuft of violet light appears at the cathode, or negative electrode, and as the pressure is further diminished, this grows until it entirely surrounds the metal target. Between the two glows appears a darker region known as the *Faraday dark-space.* With greater rarefaction another vivid glow appears, this time at the anode, or positive electrode, and the pink light breaks up into thin fluctuating disks or striae. As the pressure is continually reduced these laminar glows thicken, the Faraday dark-space enlarges, the cathode glow increases in brightness and volume, and a beautiful green fluorescence appears on the walls of the tube. As the tube is further exhausted the cathode glow breaks away and moves further into the tube, while a second violet glow takes its place. Between these two violet lights appears a dark region known as the *Crookes,* or *cathode dark-space.* Still reducing the pressure this dark-space gradually increases, the negative glow slowly fades until all trace of luminosity has disappeared, and finally the dark-space fills the entire tube. As this phenomenon is taking place the tube begins to shine in the region of the cathode and finally throughout with a brilliant green fluorescence characteristic of X-ray vacua. If the pressure (now less than one fiftieth of a millimeter) is further reduced the resistance of the tube increases until the electricity will no longer pass through the tube. (See Figure 1).

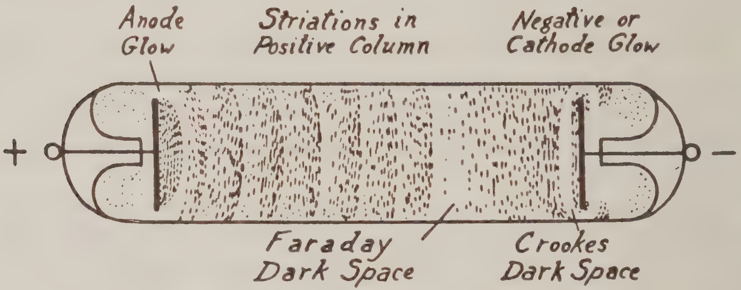

Figure 1. Phenomena observed in a Crookes' tube.

Here, surely, was mystery enough to keep a generation of scientists at work! Attention was soon centered upon the last strange fluorescence of the tube, and this was attributed to rays originating from the cathode, now known as cathode rays. At first violent disagreement arose as to their nature, but when it was found that they were capable of deflection by magnetic forces those who would characterize them as rays similar to light were soon vanquished. Crookes himself, in a moment of inspired intuition suggested that they might represent a fourth state of matter. At the end of his classical article published in the *Philosophical Transactions* for 1879 he makes the following observation:[16]

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[16] The Bakerian Lecture: "On the Illumination of Lines of Molecular Pressure, and the Trajectory of Molecules," *Trans. Royal Phil. Soc. of London,* Vol. 170, 1879, pp. 135-164. In particular pp. 163-164.

"The modern idea of the gaseous state of matter is based upon the supposition that a given space of, say a cubic centimeter, contains millions of millions of molecules in rapid motion in all directions, each having millions of encounters in a second. In such a case the length of the mean free path of the molecules is excessively small as compared with the dimensions of the vessel, and properties are observed which constitute the ordinary gaseous state of matter, and which depend upon constant collisions. But by great rarefaction the free path may be made so long that the hits in a given time are negligible in comparison to the misses, in which case the average molecule is allowed to obey its own motions or laws without interference; and if the mean free path is comparable to the dimensions of the vessel, the properties which constitute gaseity are reduced to a minimum, and the matter becomes exalted to an ultra-gaseous or molecular state, in which the very decided, but hitherto masked properties now under investigation come into play.

"The phenomena in these exhausted tubes reveal to physical science a new world—a world where matter may exist in a fourth state, where the corpuscular theory of light may be true, and where light does not always move in straight lines, but where we can never enter, and with which we must be content to observe and experiment from the outside."

What is the secret of cathode radiation? The answer to this question is that we have been describing the birth of the *electron,* a particle far smaller in size than the atom and moving with a tremendous velocity. The old concept of Lucretius has been reduced a step further and the atom has been subdivided into units incredibly smaller than the indivisible units of the older theory.

### 8. *X-Rays and Radio-activity.*

It seems rather curious that the investigations of Crookes did not lead earlier to the discovery of X-rays, but the facts are that these radiations were not described until 1895, as has already been related. X-rays are formed when the electrons of the Crookes' tube impinge upon a metal target, or, as a matter of fact, upon any matter. The discovery of Röntgen was made quite by accident, as many such discoveries are. He had been working on the problem of invisible light rays and during his investigation turned on a low pressure tube which was concealed under a covering of heavy black paper. To his surprise he noticed that a fluorescent screen some distance away from the apparatus commenced to shine. Then by interposing matter between it and the tube so as to cast shadows upon the screen, he was able to trace the invisible radiation to the Crookes tube. Another dramatic moment was at hand in the annals of man's quest of the unknown. X-rays had been discovered.

Investigation by many men who were instantly attracted to this discovery revealed, after much ingenious and penetrating labor, the nature of the new radiation. The cathode rays were tiny particles possessing mass and a charge of negative electricity; X-rays were radiations similar to light although of very much smaller wave length and with a corresponding ability to penetrate opaque matter.

The next great discovery in these closing years of the nineteenth century was that of radio-activity by Antoine Henri Becquerel (1852-1908) in 1896. Like his predecessor in the discovery of activities in nature which would penetrate opaque objects Becquerel was investigating the possible existence of invisible radiations. His experiments proceeded from phosphorescent substances, which he placed upon a photographic plate wrapped in black paper. By rare good fortune the phosphorescent substance chosen for the investigation was a preparation of uranium, and when the plate was developed it was found to have been darkened. Further study of this interesting phenomenon soon revealed the fact that phosphorescence had nothing to do with the matter, but that uranium itself possessed a hitherto unsuspected property of emanating something that would penetrate opaque substances.

Soon after this there appeared the epoch-making work of Pierre (1859-1906) and Marie Curie (1867-1934) who discovered radium. This remarkable element, similar in many ways to uranium and other radio-active substance, though possessing these powers in much higher degree, emits three types of radiation which are called by the first three letters of the Greek alphabet, Alpha, Beta, and Gamma rays. The first two rays differ in a very fundamental way from the third, since they exhibit the characteristics of corpuscles or particles while the last is an undulatory phenomenon. The alpha rays are much more sluggish than the beta particles, traveling with velocities of a few thousand miles per second, while the latter may be made to approach the ultimate velocity itself. The alpha particles are also more massive and carry a positive charge of electricity. Their mass can be measured by deflections created in their path by a magnetic field and is found to be equal to the mass of the helium nucleus.

Other tests that can be applied confirm the identification of these two. Beta rays, on the other hand, are as light as they are swift. They can also be weighed by deflections caused in their path by a magnetic field, and this mass is found to identify them with the electron, when proper correction is made for the change in mass predicted by the relativity theory. The gamma radiation, on the other hand, is a true wave motion, similar in character to that of light, but very much more penetrating. Gamma rays are not corpuscular in character as many experiments show. Magnetic fields, for example, will not affect their propagation, nor will they leave a positive or negative charge upon matter on which they fall. Their velocity, subject to experimental error, is that of light itself.

We must pause a moment and reflect upon this remarkable proposition. Here science stumbles by accident upon a phenomenon in which three types of things are involved. The first two are corpuscular in character. They possess some of the properties of matter; they have finite mass and velocities smaller than that of light; they can be deflected by magnetic fields, they possess charges of electricity, and otherwise behave like quantities for which we have definite objective feeling. But look at the mysterious gamma ray. Here is a new property of the ether; this ray behaves like a vibrating entity similar to light but with incredible powers of penetrating matter. It resembles the X-ray, with the one difference that it has a smaller wave length. Oh curious word! How can we think of wave length when there is no substance of material quality for it to undulate in? For the electron we have intuitive feeling; for the mysterious radiation in the evanescent ether we have none. Here suddenly a new world of rays and minute quantities is opened to the speculation of man before he has had time to coordinate his ideas about the sub-stratum of reality in which these phenomena are taking place. This discussion is interpolated here so that the sequel will be more vivid.

### 9. *Radium and the New Alchemy.*

The mystery of radium and uranium and of the forty or more other natural radio-active substances found since the great discovery is explained by saying that we are witnessing in these remarkable substances the destruction of atoms. The dream of alchemy, "that art without art, which has its beginning in falsehood, its middle in toil, and its end in poverty," has at last become a reality.

The decay of radio-active atoms is in many cases a very slow process. Uranium, for example, suffers transmutation of half of its atoms in about 4,560 million years. Since the process of decay is exponential in character, it is customary to describe the disintegration in terms of what is called the *half-life* period of the substance. Thus it would require another 4,560 million years for half of the remaining uranium atoms to disintegrate, and another equal period for the disintegration of the half of the half, etc. This period of decay of the atoms appears to be an intrinsic property of the substance, since nothing can be done to increase or diminish it. The rate is independent of temperature, pressure, or other external conditions that have been imposed.

But as uranium thus slowly melts away, it is not destroyed, merely transmuted into other substances, which in turn disintegrate until a stable element is reached. The first product of this transmutation process has a short half-life of twenty-four days, and yields in turn a substance of the most ephemeral character, since half of its atoms decay in approximately 69 seconds. The second product of uranium, however, is much more stable since its half-life is estimated to be of the order of 300 thousand years. Among all the elements, only thorium has a slower rate of disintegration than uranium, its half-life being equal to thirteen billion years.

The intricate relationships between these heavy elements have been traced by Ernest Rutherford, the English physicist, Frederick Soddy, his colleague in much of this research, and many others, step by step from the massive uranium, through radium with its half-life of 1590 years, through radium's inert emanation (niton) of half-life equal to five days, to the final product, lead. What a wonderful picture of the creation of the elements is this! But the old question persists, just as it does in the problem of energy. Where did uranium come from? Why in the lapse of countless geologic ages has the decay not persisted until all the radio-active elements have broken down into their final structure?

By a series of ingenious experiments Rutherford and Soddy formulated two fundamental laws of this modern alchemy. (1) A substance derived from another by the emission of an alpha particle will have chemical properties similar to those of an element two places to the left of the parent element in the periodic table of the elements. The atomic weight of the derived substance will be found to have been reduced by four. (2) A substance derived from another by the emission of a beta particle will have chemical properties similar to those of an element one place to the left of the parent element, but the atomic weight will be unchanged.

Here, then, at last was the first glimmering of the solution of a problem that had intrigued the human race from its most remote past. Guided by dreams of creating gold from baser metals, countless efforts have been made to find the key to the mystery of the difference between the elements. Even in early days there was fear of this black art for we read in the history of the Roman Emperor Diocletian that after he had conquered Egypt in 297 he "treated the Egyptians harshly and cruelly and having sought out the books written by their forefathers on the chemistry of gold and silver, burned them lest they, emboldened by riches, should in the future revolt against the Romans."[17]

But finally, near the beginning of the twentieth century, Rutherford and his associates found part of the key to the mystery. A relentless search for some effective method of controlled disintegration, or perhaps synthesis, of the atom was continued by their successors. More than a third of a century was to pass, however, before the full significance of these early experiments burst upon the human race like a clap of thunder. This story we shall continue in another chapter.

It is sufficient for our present purpose that radium and cathode radiation proved beyond any doubt that there is an atomic nature both to matter and to electricity. Even as late as 1897 Lord Kelvin was still discussing the possibility of a fluid structure of electricity, but these new discoveries were already pointing out the proof of the existence of an atomic nature of electricity as well as of matter.

### 10. *Sub-Atomic Exploration.*

With the birth of the electron and the verification that a fourth state of matter actually existed within the Crookes tube, the age of sub-atomic exploration was at hand. The name "electron" is due to G. Johnstone Stoney who in 1891 used it as a designation for the "natural unit of electricity."[18] This term is frequently specialized to denote the actual charge on the particles of the Crookes tube, and the word "corpuscle" is reserved for the material quantity by which the charge is carried. In this volume we shall adopt the definition of

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[17] Suidas (c. 10th century) on *Diocletian.*

[18] *Scientific Trans. of the Royal Soc.* of Dublin (II), Vol. 4, 1891, p. 563.

Lorentz, who characterizes electrons as "extremely small particles, charged with electricity, which are present in immense numbers in all ponderable bodies, and by whose distribution and motions we endeavor to explain all electric and optical phenomena that are not confined to the free ether."[19]

At once some important questions arise which might be enumerated as follows: (1) How large is the charge of electricity carried by the electron? (2) What is the mass of the electron? (3) How fast does it move? (4) Can a volume be assigned to it? (5) Are there sub-electrons, that is to say particles smaller than the particles of the Crookes tube?

Very many ingenious methods have been devised to answer these important questions, but it will be impossible to review them adequately here, since they are technical in nature. A general idea of the attack made, however, is both instructive and interesting and can be set forth as follows:

We have already stated that the fundamental "corpuscular" character of the electrons was demonstrated by the deflections which they suffered while passing through magnetic and electric fields of force. It is easily deduced from the equations of mechanics, which, though they originally applied to the large ponderable masses dealt with in the ordinary affairs of the world, may be assumed to hold also for the tiny electron, that a relationship between the velocity of the electrons and the ratio of charge ($e$) and mass ($m$) can be derived from the size of the deflections caused by a known magnetic field. But the question cannot be so easily answered as to which is the value of the ratio $e/m$ and which the velocity.

The velocity of the particles in a Crookes tube was determined independently in 1897 by Sir J. J. Thomson[20] (1856-1940) and E. Wiechert.[21] The latter by means of a tube containing two magnetizing coils was able to control the deflections of the cathode beam falling upon a fluorescent screen, in such a way as to be able to measure the velocity of the particles. These he found attained the not inconsiderable speed of $5 \times 10^9$ centimeters per second, or one-sixth that of light. Hence, knowing the velocity, the ratio $e/m$ could then be easily calculated. Sir J. J. Thomson in his method deflected the electrons by means of a magnetic field into a hollow vessel where their aggregate electrical charge could be measured by an electrometer and where their total kinetic energy was determined by means of a delicate thermo-electric couple in terms of units of heat. From these two values, together with the strength of the deflecting field, and the radius of the path taken by the electrons, he was able to compute the average value of the velocities of the particles and hence the desired ratio.

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[19] *Theory of Electrons* (loc. cit.), p. 8.

[20] *Phil. Mag.* (5), Vol. 44, 1897, pp. 293-316.

[21] *Verb. der phys.-ökon.. Ges. zu Königsberg*, 1897.

The ratio of $e$ to $m$ was thus found to be a large number. This indicated either that $e$ was a comparatively large unit, or that the mass of an electron was exceedingly minute. If $e$ was of the same order of magnitude as the electrical charge on the hydrogen ion in solutions, the mass of the electron was only 1/1800th of the mass of the hydrogen atom. This latter possibility has the exciting consequence that if it be the correct interpretation, we shall have subdivided the atom and can peer into depths never before seen by man. The atom will then have structure and be composed of much smaller units than had previously been postulated. This possibility merely means, of course, that what was once considered the smallest divisible unit will have yielded to a smaller one, the sequence now being, molecules, atoms, and electrons. The smallest unit has merely changed its name.

### 11. *Weighing the Electron.*

Speculation next centered around the problem of how the value of the electric charge and the mass of the electron could be independently determined. Human ingenuity reaches one of its highest levels in the beautiful chain of experiments which culminated in the very accurate determination for which R. A. Millikan was awarded the Nobel prize in 1923. The fundamental idea goes back to J. S. Townsend, C. T. R. Wilson and others who saw in the formation of clouds the possibility of weighing the electron.[22] The basic assumption is that water vapor must have nuclei around which to condense if clouds are to be formed and these nuclei are ions.

Any one who has had the privilege of sitting upon a high peak and watching clouds form below him will always remember the event. The wind, laden with moisture, sweeps up the valley and strikes against the cold wall of the mountain. Puff! Out of nothing suddenly appears the cloud. Where a moment before the sun shone clearly through, there is now a weaving mist which scuds rapidly away to make room for the next. And what is the nature of this phenomenon? One will read in books on meteorology that the moisture laden wind sweeps up the valley and the water is condensed by contact with the cold cliff. But there is more than that in this interesting phenomenon. Suddenly a brilliant glare flashes across the valley, and one is aware that there are dynamic forces in the clouds. The water-vapor was not merely condensed by the cold air of the mountain, but carried within itself a charge of electricity. Each droplet contained as its core at least one elementary charge of this mysterious element, which first terrified the human race as an evidence of celestial wrath and now is harnessed to do the myriad tasks of mankind.

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[22] *Proc. of the Cambridge Phil. Soc.*, Vol. 9, 1897, pp. 244 and 345.

To put the matter somewhat more scientifically the following assumptions were made:

First, that each drop of condensed water vapor had one charge of electricity as its nucleus, so that the total number of *ions*, or charge bearers, was equal to the total number of drops.

Second, that the average weight of the water droplets making the cloud could be determined by watching their rate of fall and computing their radii from a theoretical law derived by G. G. Stokes.[23]

Admitting these two propositions the matter was theoretically a simple one. Townsend in his experiment determined the electrical charge per cubic centimeter of the cloud by means of a quadrant electrometer, weighed the total amount of water composing the fog and divided it by the number of droplets to get the total number of ions per cubic centimeter. The total electrical charge divided by the number of ions was the charge per ion.

While this is the crude outline of the experiment both of Townsend and of those who followed him, it is obvious that many sources of error are present. First the assumption of the equivalence between ions and drops may be doubted. Stokes' law had not been experimentally tested; the drops were not necessarily uniform in size; the obvious assumption that evaporation would take place had been neglected; convection currents certainly would play a part in the movement of the cloud particles.

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[23] *Cambridge Phil. Trans.*, Vol. 9, 1850.

The subsequent efforts of Sir J. J. Thomson, C. T. R. Wilson, H. A. Wilson, and R. A. Millikan, who made the fundamental contribution to physical science of computing accurate values for $e$ and $m$, were devoted wholly to overcoming the difficulties cited above.[24] The latter, whose values are generally regarded as being the most accurate, developed a very fine experiment by means of which he could actually study the movements of an individual drop with its nuclear charge which he desired to measure. The following quotation shows the elegance which characterized the investigation of the "star" upon whose motions so much of the modern work in atomic physics was to rest.[25]

"My original plan for eliminating the evaporation error was to obtain, if possible, an electric field strong enough exactly to balance the force of gravity upon the cloud and then by means of a sliding contact to vary the strength of this field so as to hold the cloud balanced throughout its entire life. In this way it was thought that the whole evaporation-history of the cloud might be recorded, and that suitable allowances might then be made in the observations on the rate of fall to eliminate entirely the error due to evaporation. It was not found possible to balance the cloud, as had been originally planned, but it was found possible to do something much better: namely, to hold individual charged drops suspended by the field for periods varying from 30 to 60 seconds. . . . The drops which it was found possible to balance by an electrical field always carried multiple charges, and the difficulty experienced in balancing such drops was less than had been anticipated.

"The procedure is simply to form a cloud and throw on the field immediately thereafter. The drops which have charges of the same sign as that of the upper plate or too weak charges of the opposite sign rapidly fall, while those which are charged with too many multiples of the sign opposite to that of the upper plate are jerked up against gravity to this plate. The result is that after a lapse of 7 or 8 seconds the field of view has become quite clear save for a relatively small number of drops which have just the right ratio of charge to mass to be held suspended by the electric field. These appear as perfectly distinct bright points. (The apparatus is illuminated by light from both sides.) I have on several occasions obtained but one single such "star" in the whole field and held it there for nearly a minute."

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[24] For an admirable account of the matters treated in this chapter the reader is referred to Millikan's book: *The Electron*, Chicago, 1917. A new edition under the title: *Electrons* (+ and - ), *Protons, Neutrons, and Cosmic Rays*, appeared in 1935, x + 492 p.

[25] "A new Modification of the Cloud Method of Determining the Elementary Charge and the most Probable Value of that Charge," *Phil. Mag.* (6), vol. 19 (1910), p. 209. See also: *The Electron*, p. 57.

Is there not poetry in this measurement of these "stars" and the weighing of that infinitesimal particle so many magnitudes below our visibility? There is drama in the movements of the tiny speck, the adjusting of the field to hold it in suspension, the weighing of that infinitely small particle, from whose myraid swarms come the phenomena of ponderable bodies. The value of the mass is wholly beyond our comprehension because of its minuteness, but is of the order of $9 \times 10^{-28}$ of a gram.

What mental picture can we form of this tiny quantity from which the structure of things is made? Is it spherical like a marble after the concepts of the Lucretian atom? Does it have size, or is it merely a strain in the metaphysical ether? For want of a better hypothesis, perhaps, one tentative agreement has been to regard the electron as a spherical entity, the mass of which is wholly derived from its electrical charge. This model is due to Sir J. J. Thomson who demonstrated that the apparent inertial mass of a sphere in motion is slightly increased if it be charged, and who showed how to calculate the total mass of such a body. Applying this formula to the electron and assuming that the entire mass is electrical, one finds that the approximate radius of the electron is $2 \times 10^{-13}$ centimeters; or in slightly more understandable language, if ten trillion such bodies were placed in a row one just touching the other their total length would not exceed an inch and a half.

We have already indicated above the fact that the electron, since its discovery in the Crookes tube, has been found throughout nature. It attains one of its most majestic places in the high-speed beta rays of radium. The progress of physical science in recent years has been greatly aided by the invention of new methods for giving very high velocities to electrons. Since the mass of the electron is very small, relatively simple apparatus will accelerate them readily. But a new difficulty intrudes. As the velocity increases so also does the mass. For radioactive substances the beta ray attains velocities in the neighborhood of 99 per cent of the velocity of light; some of the electrons even reach velocities of the order of 99.6 per cent that of light. But for such speeds the masses of the particles have been increased by factors of approximately seven in the first case and eleven in the second.

Numerous ingenious devices have been invented to give high velocities to electrons. One of the first of these was a tube invented by W. D. Coolidge upon which a high accelerating voltage could be imposed.[26] More recently an apparatus called the induction accelerator, or the *betatron*, has been highly successful in producing high-speed electrons. This device, the invention of D. W. Kerst, induces an alternating magnetic field in the orbit of the electron.[27] This creates a high electro-magnetic force in the orbit and the electron is thus continually accelerated. One of the advantages of this device is found in the fact that its operation is not affected by the increasing mass of the electron. Very high velocities have been attained by the betatron, the limit being perhaps in the neighborhood of 0.9998 that of light, where the mass has been increased by a factor of approximately 79.

## 12. *The Proton.*

So far in our story we have said nothing about the *proton*, or positive particle, which is found to be the constituent entity in the alpha rays of radium. The existence of such fundamental particles was first recognized as early as 1886 when E. Goldstein perforated the cathode, or negative electrode, of a Crookes tube and observed streams of luminous gas back of the perforated terminal.[28] These streams were called *canal rays*, but it was soon recognized that the luminosity of the gas was caused by a stream of positively charged particles, which flowed from the anode of the tube. It is this stream, atoms of the gas which have become ionized by the loss of one or more electrons in the discharge tube, which constitutes the canal radiation. These particles are now generally referred to as positive rays.

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[26] "Production of High-Voltage Kathode Rays Outside the Generating Tube", *Journal of the Franklin Institute*, Vol. 202, 1926, pp. 693-721. See also: *Science*, Vol. 62, 1925, pp. 441-442.

[27] *Physical Review*, Vol. 61, 1942, p. 93.

[28] *Berliner Preuss. Akad. Wiss*, Vol. 39, 1886, p. 691.

Just as in the case of the electrons, it was discovered that the positive particles could be deflected by electric and magnetic fields. Hence their velocities and the ratio of their charge ($e$) to their mass ($m$), that is to say, the fraction $e/m$, could be accurately determined. Both the velocities and the ratio were found to be much smaller than in the case of the electrons, which indicated that the mass of the constituent particles was greater than that of the electron. These quantities varied for different gases, but were found to reach a minimum when the gas was hydrogen.

By a series of ingenious experiments it was discovered that the smallest positive charge of electricity which could be produced was *equal numerically to the charge of the electron.* But the particle which carried this elementary charge has a mass approximately 1840 times as great as that of the electron and consequently equal to the mass of the hydrogen atom. To this elementary particle was given the name *proton,* from the Greek word for first.

The alpha particles of radium, examined from the point of view of the new protons, were found to have a mass equal approximately to four times that of the hydrogen atom and a positive charge numerically twice as great as that of the electron. This identified them with the nucleus of helium, that is to say, the atom of helium which had been stripped of its two electrons. But a mystery is here apparent which, although first recognized in the early part of the century, was not to be resolved for many years. Does the nucleus consist entirely of protons, and if that is the case, why should the charge be two units instead of four? There must exist some neutralizing agent within the nucleus which reduces the total number of electrical units by two so that the atom itself is without a charge.

The first evidence as to the size of the proton was obtained from the classical experiments of Rutherford in 1911 on the scattering of alpha particles by thin films.[29] It was found that when the particles passed through a gold foil, a certain small number of the order of say five in a hundred thousand were deflected into paths which made large angles with the path of propagation. Some few actually reversed their direction altogether. It was observed also that the number of particles thus scattered was proportional to the thickness of the foil, provided this thickness was sufficiently small.

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[29] *Philosophical Magazine,* Vol. 21, 6th ser., 1911, p. 669.

By statistical deduction from the experimental data it was inferred, first, that the scattering was caused by the near approach of the alpha particles to atomic nuclei and, second, that these nuclei were very minute in size. From the evidence of direct hits, that is to say, the case where the alpha particles were actually reversed in their direction, the conclusion was reached that the diameters of the nuclei could not be greater than some small multiple of $10^{-12}$ centimeters. Thus, in the case of the helium nucleus, it has been estimated that the best model is an elastic oblate spheroid with major and minor axes of the order of $16 \times 10^{-13}$ and $8 \times 10^{-13}$ respectively. And what about the proton itself? Although the difficulties of statistical inference still make the problem a hazardous extrapolation, it is now generally believed that the size of the individual protons may be much smaller, perhaps of the order of one two-thousandth that of the ubiquitous electrons.

And what of the atom itself? If we assume the model which was suggsested in 1913 by Niels Bohr, a model which we shall discuss in much greater detail later, the atom is a miniature solar system. The central nucleus is a massive, positively charged core around which revolve in elliptical orbits the rapidly moving electrons sufficient in number to neutralize the positive charge of the nucleus. But both the nuclear core and the electrons are insignificant in size when compared with the space which surrounds them. An elementary calculation will show this. For the diameter of the atom is estimated to be of the order of $10^{-8}$ centimeters, while the diameters of the electron and the nucleus have been shown above to be of the order of $10^{-12}$ centimeters. Hence the volume of the space within the atom compared with the volume of the particles which it contains is of the order of $10^{12}$, or, in other words, the volume of the space is approximately a trillion times as great as the volume occupied by the material objects. Is there little wonder, then, that the elusive ether could slip through the interstices of a material universe so porous!

### 13. *Do Macrocosmic Laws Hold in the Microcosm?*

If we are willing to believe the evidence that has been accumulated in the laboratory concerning these exceedingly small particles moving with their enormous velocities, the structure of the microcosmos may not be wholly discordant with the universe of man's activities. We have made assumptions that the laws of mechanics applicable to the large ponderable bodies apply also in the movements of these tiny particles; we have assigned an inertia to their movements identical with the mysterious inertia of our greater systems. This raises the speculation whether, if Mach and Einstein be correct, these tiny bodies are also inextricably connected with all the other matter in the macrocosm. We have regarded them as spheres of ponderable stuff with minute charges of electricity wholly conceivable and in accord with rational belief. We have merely replaced the crude atom of Lucretius by smaller particles with measurable properties and attributes consonant with experiment.

The picture does not surprise us. We look with certain feeling of confidence upon Bohr's image of the atom, a little solar system with a protonic nucleus and the electrons moving in elliptical orbits about it. Perhaps, we say, the clue to the structure of matter is to be found in astronomy; we apply to one the same dynamical equations used in investigating the other; the laws of Kepler for the planets are the laws for the hydrogen atom. That superb unity in natural law, long dreamed of by those who followed the trail of Newton and his successors, seems about to be revealed to us. The microcosm and the macrocosm are identical, and physical science can do nothing better than to accumulate more and more evidence to show this cosmic harmony. It is a beautiful picture; but, alas, the story is not yet told!

### 14. *The Photo-Electric Effect.*

We turn next to a remarkable fact, which is essentially the basis of the strange dilemma with which modern physics is now confronted. This fact is technically known as the *photo-electric* effect and can be described easier than it can be explained.

When light is allowed to fall upon a metal plate we discover that electrons are given off and the quantity increases with the intensity of the incident beam. One feels in this statement the approach of trouble. With these words we connect the electron with the great mystery of radiation. We have already prepared the way by citing the phenomena of the hot material body throwing off its streams of radiant energy. All about us in every conceivable activity of nature we find evidence of a union between matter and radiation. There is an undiscovered bond between the activities of the electrons in ponderable bodies and the elusive entity which conveys the evidence of their activity. It is the ancient metaphysical problem of matter and its relation to the substratum of reality. Just as we were wholly baffled in our attempts to give a rational explanation to the motion of ponderable bodies through the medium which conveyed radiation, so we should expect a similar difficulty when we try to trace the relationship between the motion of the electrons and the radiation that results from their activity.

But what is so mysterious in the photo-electric effect? That electrons should be cast out of a metal plate by the bombardment (technically spoken of as absorption) of light is not unreasonable. But here is the strange fact. The stream of photo-electrons consists of electrons whose velocities never exceed a value determined by the frequency of the incident light. Increase the intensity of the radiation and more electrons, but not more rapidly moving ones, are cast out of the metal. The greater the energy the greater the number, but not the greater their kinetic energy. Further investigation of the phenomena led to the conclusion that, allowing for a certain amount of energy necessary to detach the electron from the metal plate, the kinetic energy of the electrons was proportional to the frequency of the incident light, in which the factor of proportionality was a universal number called Planck's constant and equal in c. g. s. units to $6.55 \times 10^{-27}$ erg-seconds.

Does the peculiarity of the phenomenon now make itself apparent? If not, reflect upon the waves of the sea. Suppose one were standing upon the beach on a stormy day watching the water roll the pebbles upon the sand. And suppose the wind increased in violence and the waves came in with higher crests and greater volume. Would one not consider it a strange phenomenon if the pebbles which one had been watching were thrown no higher upon the beach, but merely that more of them were affected by the waves? And would it not seem stranger still if investigation showed that waves of a certain frequency, no matter how large their size, always moved their pebbles a certain distance and no farther? This analogy which is frequently used in explaining the new difficulties of physics is picturesque and accurate. The waves of light falling upon the metal plate may increase in intensity, but this intensity serves only to eject more electrons from the surface and not to give them greater energy.

### 15. *The Ether and Natural Law.*

With this brief description of an epoch-making phenomenon we shall bring the present chapter to a close. We started out auspiciously to study the atomic structure of matter, and beyond our highest hopes, were able to demonstrate the existence of primary units, which seemed to combine the mystery of matter with the mystery of electricity. The last fortress of the continuists, so far as objective nature was concerned, was taken. Nature was atomic: the fluid concept of electricity was to be replaced by a corpuscular theory, in which swarms of tiny particles bearing their charges of negative or positive electricity were to yield to the analysis of the mathematical theorems of the kinetic theory of gases.

But a remarkable aspect of the problem has been revealed. The ether, that strange necessity of thought, inaccessible to sensuous experience, baffling and mysterious has intruded upon the picture. Not content with overthrowing our *a priori* concepts of space, time, and gravitation, the ether has again encroached upon the cherished principles of Newtonian mechanics. One can not forbear to quote the "Ancient Mariner:"

"When looking westward, I beheld
  A something in the sky.
At first it seemed a little speck,
  And then it seemed a mist,
It moved and moved, and took at last
  A certain shape I wist."

As we have traced step by step the history of philosophical speculation in its relation to the problems of the world of matter this strange spectre has been with us. Its wraith-like form intertwines with every theory. We start out hopefully with a material model and with postulates firmly grounded in sensuous experience and upon rational intuitions. At some turn in the road the spectre waits. Our theories lose their reality, and the metaphysical finally replaces the physical in a set of equations which describe the phenomena, but leave the puzzle unresolved. We have not yet gone far enough in the present romance to be able to draw this conclusion with conviction, but the quantum theory and its resulting wave mechanics will prove to be an even more remarkable substantiation of our theme than the perplexities of the space-time manifold, the history of which we have completed in previous chapters.

# CHAPTER 9.

## THE LAWS OF CHANCE.

"All events, even those which by their insignificance appear not to obey the great laws of nature, exist or live by a sequence as necessary as the revolutions of the stars. Through ignorance of the laws which unite them to the entire system of the universe, we have made the final causes depend upon fact, or chance, according as they occur and succeed each other regularly, or without apparent order."

P. S. de Laplace in *The Analytical Theory of Probabilities.*

. . . "Chaos umpire sits,
And by decision more embroils the fray
By which he reigns: next him, high arbiter,
Chance governs all."

John Milton in *Paradise Lost,* Book II.

### 1. *Probability and the Discontinuity of Nature.*

In the last chapter we traced the history of the atomic concept of matter from its beginning in the vague outline of Democritus and Lucretius to the brilliant experiments which demonstrated the existence of the tiny corpuscles of the Crookes tube. We saw science postulate an atomic structure for electricity and matter, giving shape, size, inertia, and electric charge to these basic units of objective nature. The model was simple and satisfactory. It was easy to visualize and strained credulity only to the extent of imposing upon thought a discontinuous structure for the universe instead of a continuous one. Moreover this strain upon belief is apparent rather than real, for the latter view, if it be thoughtfully considered, will appear fully as difficult to conceive as the former. One we derive from considerations of a fluid, the other from the compressibility of a gas.

But after these satisfactory conclusions had been attained a grave difficulty appeared. Matter radiates the troublesome undulations whose history we have traced earlier in connection with theories of the ether. And these radiations are ubiquitous. Wheresoever matter is, there we find them, and where matter is not, they are also present. Radiant energy is the most common phenomenon of nature; in every cubic centimeter of space it is to be found. Its density varies from its tremendous value in the interior of the stars to its infinitesimal value in the dark cold reaches of interstellar space. It is the most important phenomenon in nature. Matter is either in equilibrium with it or seeks perpetually to attain this equilibrium. The mystery of its source and the mystery of its laws are the puzzles of modern physical science.

It may seem strange to some that a chapter upon the laws of chance should be interpolated between chapters on the atomic concept and the metaphysics of the quantum theory, but the mystery of radiation forces the issue upon us. What mathematical model shall we employ in its description? Poincaré's comment on the theory of quanta radiation is highly significant: "It is useless to remark how laws of physics will no longer be susceptible of being expressed by means of differential equations."[1] We have already witnessed in previous chapters the struggle of those who attempted to fit the ether into a system of differential equations and we have seen in the theory of relativity that the invariance of these equations for transformations in a non-Euclidean geometry was one of the principal concepts of Einstein's theory. Shall the new approach to the mysterious relationship between radiation and matter be taken through these continuous creations of subjective thought?

The answer is that the theory of probability furnishes a more satisfactory mathematical structure upon which to construct an approach to atomic concepts. Its foundations are rooted in the "law of large numbers" which implies by its very name the postulate of discrete quantities. In such an analysis the description of the activities of aggregates of particles is surely to be found. If nature can be studied through rational models here is the open road.

### 2. *The Nature of Natural Law.*

The mathematician is often accused of being impractical, a dreamer who spins intricate patterns out of the web of his imagination, who builds theories that have little in common with the practical affairs of life. The story is told of Euclid that a youth who had come to him for instruction, after having learned the first proposition, asked: "But what am I to get out of this?" and Euclid for reply commanded a slave to give him a coin, since he needs must have a gain for all that he learned.[2]

But there is one subject in mathematics which possesses the elegance of those abstract theories that delight the artistic spirit and yet, at the same time, has vital contact with the practical affairs of life. This is the theory of the laws of chance, a theory at once so elusive that one does not find complete agreement among mathematicians as to the full validity of its laws and yet a theory so sound that millions of dollars have been invested in insurance companies built upon its foundations.

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[1] *Comptes Rendus*, Vol. 153, 1911, p. 1103.

[2] *Stobaeus*, Edition Wachsmuth, 1884, Ecl. II.

"Yet how dare we speak of the laws of chance? Is not chance the antithesis of all law?" asks Bertrand at the beginning of his *Calcul des probabilités.* In commenting upon this question Poincaré says: "But this conception (of phenomena either obeying harmonious law or else rebellious to all law) is not ours today. We have become absolute determinists, and even those who want to reserve the right of human free will, undoubtedly will let determinism reign undividedly in the inorganic world at least. Every phenomenon, however minute, has a cause; and a mind infinitely powerful, infinitely well informed about the laws of nature, could have foreseen it from the beginning of the centuries. If such a mind existed, we could not play with it any game of chance; we should always lose. . . . Chance is only the measure of our ignorance. Fortuitous phenomena are, by definition, those whose laws we do not know."[3]

The calculus of probabilities should be regarded as an attempt to reduce to numbers the uncertainties of life and thus to afford a way of aiding judgment. The universe about us presents a maze of perplexities. We are ignorant of the past and uncertain of the future, but it is by no means true that this uncertainty and ignorance are uniform and homogeneous. We are not absolutely certain that the sun will rise tomorrow, but we should attach far more value to a guess about it than we should to a guess about our ability to throw a head in a single toss of one coin.

But from where do we derive our greater certainty? Suppose that we saw a sunrise for the first time and we should inquire: What is the chance that this phenomenon will occur again? In the game of matching pennies we observe that the chance of throwing heads is equal to the chance of throwing tails, or numerically stated it is 1/2. Suppose that we attach this probability, because of ignorance, to the phenomenon of the sunrise.

Further we might deduce from our experience of tossing coins that the chance of throwing heads ten times in succession is 1 to 1024; but we observe that the sun rises ten, twenty, a hundred times in succession. Hence it would appear highly unlikely that our original assumption of a probability of 1/2 was right. Moreover, if we continue to assign larger and larger fractional values, always leaving something for our uncertainty, to the probability of the rising of the sun and continue to collect data on the phenomenon, we shall eventually find that all our original assumptions are in error because no cases of failure appear.

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[3] *Science and Method*, chapter 4. *Foundations of Science*, p. 395.

Thus we are finally led to assign as the probability of the rising of the sun a number which differs from unity, or certainty, by a value indefinitely small. When such a probability exists it may be said to define a law.

By using properly the experience of many men we are acquiring much greater ability to predict the future than did our ancestors with all their magicians and soothsayers. It was once the fashion to consult the "*sortes* of Virgil" in order to learn the probable success of a venture, but today it is much more the custom to consult a table of statistics.

However, our progress is slow and the pitfalls in the way of chance are many. If one of us had the power of infallible prediction, it would not be long before this one would be ruler of the world. To him there would be no calculus of probabilities because all things would be certain. Failing, however, to possess this remarkable gift we must find some yardstick that can measure in a degree the extent of our ignorance and uncertainty. We flip a coin. Will it fall heads or tails? We buy a share of stock. Will it rise or fall? As in O. Henry's story of "The Roads of Destiny" we come to a parting of the ways. Do success and happiness lie along the middle road or along those to the right or left?

There is deep philosophical speculation in these ideas. Are we to regard the events of the universe as wholly fortuitous, guided by blind accident? Is the freedom of the will of man an illusion and are the acts of choice which we think that we perform daily merely predetermined acts for which we are but the vehicles of expression?

Laplace, to whose great treatise on probability we can trace most of the stimulus acquired by modern students of the laws of chance, has stated in a philosophical essay the following rational view which we shall tentatively adopt:[4]

"Present events are connected with preceding ones by a tie based upon the evident principle that a thing cannot occur without a cause which produces it. The axiom, known by the name of the

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[4] Page ii of the "Introduction" to his *Théorie Analytique des Probabilitiés*, 3d ed., Paris, 1820. See English translation of the "Introduction" by F. W. Truscott and F. L. Emory, 1902, p. 3.

*principle of sufficient reason,* extends even to actions which are considered indifferent; the freest will is unable without a determinative motive to give them birth; if we assume two positions with exactly similar circumstances and find that the will is active in the one and inactive in the other, we say that its choice is an effect without a cause. It is then, says Leibniz, the blind chance of the Epicureans. The contrary opinion is an illusion of the mind, which, losing sight of the evasive reasons of the choice of the will in indifferent things, believes that choice is determined of itself and without motives.

"We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formulas the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes. The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discoveries in mechanics and geometry, added to that of universal gravitation, have enabled it to comprehend in the same analytical expressions the past and future states of the system of the world. Applying the same method to some other objects of its knowledge, it has succeeded in referring to general laws observed phenomena and in foreseeing those which given circumstances ought to produce. All these efforts in the search for truth tend to lead it back continually to the vast intelligence which we have mentioned, but from which it will always remain infinitely removed. This tendency, peculiar to the human race, is that which renders it superior to animals; and their progress in this respect distinguishes nations and ages and constitutes their true glory."

### 3. *The Law of Large Numbers.*

The definition of mathematical probability is usually given as follows: *The probability that, among several equally likely events, a given event will happen is the ratio of the number of favorable cases to the total number of possible cases.*

There is much tacitly implied in this brief statement, however. In elucidation let us consider a simple example. We throw a die. Any one of the six faces may appear, and one is no more likely than another. Hence we say that the probability of any particular face appearing, for example five, is one-sixth. But this must be properly understood. Does this mean that once and only once the five spot will appear in every six throws of the die? Not at all. It means that on the average, if many throws are made, the five will be found to have appeared approximately one-sixth of the time. This proposition is known in the theory of probability as the *law of large numbers* and is the rock upon which the structure rests. It forces us to distinguish between *a priori* and *a posteriori* probability. The former, by definition, refers to the probability derived by deductive processes from reason alone, prior to experience. The latter applies to the probability derived from a study of empirical data. We reflect upon the tossing of a coin and conclude *a priori* from pure intuition that the chance of throwing heads in a single toss is 1/2; to test this conclusion by experiment, that is *a posteriori,* would require evidence derived from a large number of tosses of an actual coin.

Some definitions of probability include the law of large numbers as an integral part, as for example the statement of the Danish astronomer T. N. Thiele (1838-1910) that probability is the name "for the limiting value of the relative frequency of an event, when the number of observations (trials) under which the event happens approaches infinity as a limit."[5]

Let us illustrate this by determining through actual empirical trial the probability that a coin will fall heads. Toss the coin once and compute the *a posteriori* probability of the event on the basis of this single experiment. It will be either certainty (one) or impossibility (zero). But suppose that the experiment be continued to ten thousand tosses. It can be shown that the probability is so large that the difference between the calculated answer and the *a priori* one of one-half will be less than one one-thousandth. If we continue our tosses to a million, two million, three million, etc., the difference will be found continually to diminish until ultimately it may be made smaller than any limit we wish to set.

But whence do we derive this certainty? Does it come from the experience of countless penny tossers or does it have a more fundamental derivation from subjective intuition? It may be proved on intuitional postulates with some rigor and a minimum of assumption that if the *a priori* probability actually exists then the difference between it and the one obtained by experiment may be made as small as we choose. But who would care to state that the *a priori* probability always exists? Suppose we test the validity of the assumption that heads will fall half the time in a series of tosses with a single coin. A million actual trials will probably serve to show merely that one side of the coin is slightly heavier than the other or that some other systematic error exists.

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[5] Quoted by A. Fisher: *Mathematical Theory of Probabilities,* New York, 1923, p. 84.

On the other hand there is remarkable objective validity in the law of large numbers and unsuspected problems may be solved by its application. The following celebrated example was first given by George Louis LeClerk, Comte de Buffon (1707-1788) and can be tried with profit by any one who wishes to assure himself that the law is fundamentally applicable to the material universe.[6]

Take a stick of length $L$, say two inches, and from a short height drop it upon a table upon which is drawn a grid of lines a distance $D$, greater than $L$, say four inches apart. Then the stick will either cross one of these lines or it will not. Suppose that this experiment be tried $N$ times, say 1000, and the observation be made that the stick crosses one of the lines $M$ times. Is it not a beautiful conclusion that the value of $\pi = 3.1416...$, the ratio of a circumference to its diameter, can be computed with some degree of accuracy from these numbers, namely, $\pi = 2\ N\ L / M\ D$? Using the lengths 2 inches and 4 inches respectively for $L$ and $D$, we reduce the formula to $\pi = N/M$, that is to say, $\pi$ is approximated by the reciprocal of the probability, $M/N$. At the Chicago World's Fair in 1933 there was exhibited an apparatus which performed the experiment automatically. A box with glass sides and with floor and ceiling made of iron strips separated by thin lines of an insulating material was rotated slowly. At each rotation a needle was dropped through a maze of wires upon the floor of the box. If it crossed one of the lines an electric counter recorded this fact. Dividing the number of rotations of the box by the recorded number of crossings, the visitor was able to determine the approximate value of $\pi$ which had been attained by the apparatus.

The error in $\pi$ obviously depends upon the care taken with the experiment, the ratio of $L$ to $D$, and the number of trials recorded. Augustus De Morgan (1806-1871) had one of his pupils make 600 trails and thus calculated $\pi=3.137$.[7] In 1864 Captain O. C. Fox made 1120 trials using some precautions to get actual random falls for his stick and obtained $\pi = 3.1419$.[8] To the reflective mind it must be a perpetual wonder that this important number, the center about which revolves a conspicuous part of the mathematical universe, should have its secrets bared by the dropping of sticks upon a table top.

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[6] This example appeared first in Buffon's "Essai d' Arithmétique Morale," 1777, Vol. 4 to Supplement à *l'Histoire Naturelle*, although it was written about 1760.

Another example of similar character and one important in the history of the theory of chance is the probability of writing down at random a pair of numbers prime to one another. Thus suppose that $N$ pairs of numbers such as, for example 23 and 96, are written down at random, and account taken of the number, $M$, that are prime to one another. It is a curious fact that $\pi$ may be approximately calculated from the formula: $\pi^2 = 6N/M$.[9]

Those who remember the painful toil necessary to compute the ratio of the circumference to its diameter by the methods of elementary geometry will find in this an interesting reflection. Straight lines defined as rays of light, as we have seen in another place, might lead the paths of geometrical instruction to the laboratory; the law of large numbers assures us that abstract ratios in this same subject may be computed by empirical trials with sticks of wood or pairs of relatively prime numbers. Archimedes, instead of employing subjective methods to calculate the upper and lower bounds within which the value of $\pi$ must lie, might have attained the same limits and with almost the same accuracy by dropping sticks of wood for a sufficient number of times.

It is truly surprising how far the elementary definition of probability will lead. It is in this fact, perhaps, that the charm of this subject lies, namely that one can go so far upon so small a postulate. Many problems of geometrical nature are amenable to the calculus of chance, although it may seem strange that close connection should exist between branches of mathematics so divergent. Consider for example an interesting episode in Poe's fanciful story of "The Pit and the Pendulum." The hero of the tale is imprisoned in a dungeon of the Spanish Inquisition. He recovers from his first period of unconsciousness to find himself in complete darkness, unable to see the walls of the prison or anything about him. After some meditation he decides to make a circuit of the dungeon, and when this is accomplished he estimates that it has a perimeter of approximately fifty yards. This task completed, he decides to cross the prison from wall to wall, but in the darkness trips and falls upon his face, to find that he is lying upon the edge of a circular pit opened there to accomplish his destruction. An interesting problem in probability is suggested by this tale. Suppose that the circumference of the pit had been $C$ yards in length, what was the probability that the prisoner would have fallen into the pit had he attempted to cross the dungeon in any random direction? It is a surprising result that this mathematical probability, under the single assumption that both pit and dungeon were convex in shape, is merely the ratio of the length of the circumference of the pit to the perimeter of the dungeon, namely $C/50$. Thus if the pit had had a radius of four yards, that is to say, a circumference of approximately 25 yards, the prisoner would have had an even chance for his life.

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[7] *Budget of Paradoxes*, London, 1872, pp. 171-172.

[8] "On an Experimental Determination of $\pi$," by Asaph Hall. *Messenger of Mathematics*, Vol. 2, 1873, pp. 113-114.

[9] This result is due to the Russian mathematician, P. S. Chebyscheff.

Look about you and see the innumerable places where these probabilities occur. Count the letters on an English page and determine the probability of the occurrence of the letter $e$. One will remember how important this knowledge was in deciphering the mystery of Poe's "Gold Bug." We have already indicated the use of chance in considering cosmic puzzles such as the density of matter in the universe; that a law of some kind was working in the distribution of the asteroids is implied by the discovery of the existence of radial gaps in their orbits, the probability of which, on the assumption of a purely fortuitous distribution, is 1 to 137,438,953,472, according to the figures of Daniel Kirkwood (1814-1895), the astronomer.[10] Death, whose certainty is assured but whose time is subject to the whim of fate, holds for every age an empirical probability upon which the colossal financial structure of life insurance is erected.

The power of the theory of chance lies in its calm disregard of the causes of the events subject to its analysis, provided only that these causes be sufficiently numerous. We toss a penny and note that its trajectory is affected by many circumstances. Its ultimate position on the table top is determined by numerous things besides the law of gravitation; its initial position in the hand that throws it, the direction and force with which it starts, the air through which it falls, the elasticity and smoothness of the table top, upon which it drops, combine to form a complexity of causes that would defy the most skillful analyst to untangle. The same point is beautifully illustrated in the numbers in a table of logarithms. It would be folly, for example, to wager with a friend that he could not tell whether the first digit in the logarithm of a number named by you was greater or less than five. It would even be unwise to make the same wager with regard to the second digit for the law of its formation is not essentially complex; but it is perfectly safe to make an even wager upon the last number in a seven place table because for it the causes which affect its magnitude have become so numerous that its size is entirely fortuitous. This statement must be properly understood because it is evident that we can at any time calculate the digit in question by the known laws of logarithms. But to one uninstructed in the method the appearance of one number rather than another is a fortuitous event. Poincaré calls this "a first degree of ignorance" in contrast to the second degree of ignorance represented by a complex mechanical situation such as we find in the tossing of a coin. The smoothing process exerted by the complex series of causes in the calculation is shown in the fact that there are as many even numbers as odd in this seventh column of the table; there are as many sevens as there are twos. And if this be the case for a set of quantities subject to mathematical determination how much more we might expect it for those complex events which make up the world-lines of our existence.

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[10] "Distribution of Asteroids," *Sidereal Messenger*, Vol. 4, 1885, p. 259.

### 4. *The Laws of Chance.*

But the events with which we ordinarily deal are not usually isolated ones. They follow one another and there are numerous independencies of which account must be taken. To the definition of probability, two laws must be adjoined before the foundation of the structure is laid. The first of these has to do with compound probability and may be stated thus:

*The probability that two independent events will happen simultaneously or in succession is the product of the probability of the two events taken singly.*

For example, the probability of getting a head in one throw of a single coin is 1/2, and the probability of getting two heads in two throws of one coin or one throw of two coins will be (1/2) X (1/2) or 1/4.

This law must be carefully understood because a too hasty application of it may lead to error. For example, the probability of getting a head in one throw of a single coin is 1/2 and the probability of getting a tail is also 1/2, but the probability of getting a head and a tail in one throw of two coins is not 1/4. We might argue as d'Alembert did. There will be three cases:

*A*. Two heads; *B*. Two tails; *C*. One head; one tail.

We thus deduce that the probability is 1/3, which is again an answer that experience will not justify. The error lies in the fact that *A*, *B*, and *C* are not all equally probable. Thus in case *C* one coin might come down heads and the other tails or vice versa, while in *A* and *B* there is but one possibility. Hence the correct answer would be 1/2. However, if we throw one coin twice and specify in advance the order, thus one head and one tail, the probability becomes the same as in cases *A* and *B* or 1/4.

The second law of chance has to do with mutually exclusive events and may be stated as follows:

*The probability that a given event in a series of mutually exclusive events will happen and all others fail is called partial probability. The probability that any event whatever of the series will happen and the others fail is called total probability and is equal to the sum of the partial probabilities.*

To illustrate, suppose *A*, *B*, and *C* match pennies, odd man to win. What is the probability that either *A* or *B* wins? Since only one can win, the winnings by *A*, *B*, or *C* are mutually exclusive events. But *A*'s chance is 1/3 and *B*'s chance is 1/3, so the chance of either *A* or *B* winning is 2/3.

Another illustration is contained in the following. Suppose a difficult problem is to be solved and three men whose abilities are known are set to the task. Suppose the estimate of *A*'s chance of working the problem is placed at 1/2, *B*'s chance at 1/3, and *C*'s chance at 1/4. What is the probability that the problem will be solved?

It will be seen that the following are the mutually exclusive events which lead to the solution of the problem, namely *A*, *B*, or *C* solves the problem and the others fail; *A* and *B*, *A* and *C*, or *B* and *C* solve the problem and the other fails; or all three succeed. Consider one of these possibilities, for example the case where *A* and *B* solve the problem and *C* fails. The individual cases are mutually independent so that the probability of the event in question will be $(1/2) \times (1/3) \times (1-1/4) = 1/8$.

In a similar way the other probabilities may be computed, and since they all represent mutually exclusive events, in that the occurrence of one precludes the occurrence of any other, their sum, 3/4, will represent the total probability that the problem will be solved. Another way in which the same answer might have been obtained was by computing the probability that all three would fail to solve the problem, namely, $(1/2) \times (2/3) \times (3/4) = 1/4$, and subtracting this value from unity. Since it is certain that the problem will either be solved or not solved, these two events form a mutually exclusive system.

Many delightful problems arise out of the combination of these laws, but we cannot tarry to examine them. Paradoxes are to be expected, but the root of the trouble is usually found in the subtle introduction of infinity into the picture or in obscuring the fact that the events considered are not equally probable. One paradox, in particular, has cast considerable discredit upon certain types of application of the theory of probability. This problem concerns the attaching of monetary value to chance, and the explanation is not wholly satisfactory even at the present time. This paradox is known as the St. Petersburg problem and may be stated as follows:[11] How much money should $A$ give $B$ as a fair price for the following offer? $A$ is to throw a coin until heads appears. If this happens on the first toss, then $A$ gets $1; if however, heads do not appear until the second toss, $A$ gets $2; until the third toss, $4; until the fourth toss, $8, etc.

Since the respective probabilities are 1/2, 1/4, 1/8, 1/16 etc. and the events are mutually exclusive it is clear that the sum of the products of the probabilities by the expected gains is the desired answer. To our astonishment it turns out to be infinity, so that even a million dollars would be inadequate to pay one who would make such a proposition. Casual trial of the problem will soon convince anyone that the answer is absurd. We have combined an infinitesimal probability of winning an infinite fortune in such a way that human credulity is shaken.

Where is the flaw in the reasoning? Various answers have been proposed, but none places the paradox in the same point. Daniel Bernoulli (1700-1782) gave a clever explanation by introducing the idea of *moral hope*.[12] Thus, he argues, the pleasure of gaining

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[11] This is similar to a problem proposed by N. Bernoulli to P. Montmort. See the latter's "Essai d'analyse sur les jeux de hazard," 1714, p. 402. For an account of the St. Petersburg problems see: I. Todhunter, *History of the Mathematical Theory of Probability*, Cambridge, 1865, and also A. Fisher (loc. cit.), pp. 51-53.

[12] From, "Specimen Theoriae Novae de Mensura Sortis," *Commentarii Acad. Petrop.*, Vol. 5 (1730-1731), 1738, pp. 175-192.

$1,000 is greater to one who has nothing than to a millionaire, hence the expectation from fortuitous events must be measured in comparison with what we already have. Introducing this idea into the problem he was able to reduce the answer to a reasonable figure. On this assumption Laplace estimated that if *A* has a capital of only $200 he could not pay *B* more than $9 for the offer.

Poincaré on the other hand puts the difficulty upon *B*. The fortune of this man is limited so that *A* cannot hope to win an infinite sum even though he should have the marvellous luck to throw tails indefinitely. By simple analysis Poincaré is able to show that if *B*'s fortune is only $1,000, *A* could afford to pay $6 per game, but that he could increase this to $16 if his opponent were a billionaire.[13]

Augustus De Morgan on the other hand regards the infinite answer as the sensible one and looks upon the game as *B*'s business.[14] If *B* should decide to play just a thousand games and then retire, his charge per game would be smaller than if he decided to run his business for a million games. The average loss per game, in other words, increases with the total played, and if the business is to be solvent for several generations a prohibitive price per game would have to be charged.[15]

It is a matter of more than casual interest to observe that the concept of *moral hope* introduced by Bernoulli in this discussion has had an extensive generalization in the theory of economics. It had focused attention upon the fact that economic behavior can be accounted for through its dependence upon the psychic reaction of individuals to their own personal wealth. To a rich man, for example, a small amount of money will have little significance, but to a starving man its possession may be the difference between life and death. The idea thus presented by Bernoulli has flowered under the hands of W. S. Jevons (1835-1882), M. E. L. Walras (1834-1910),

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[13] *Calcul des probabilitiés*, Paris, 1896, pp. 41-43.

[14] *Essay on Probabilitiés*, London, 1838, pp. 99-101.

[15] As De Morgan puts it: "A larger net would have caught, not only more fish, but more varieties of fish; and in two millions of nets we might have expected to have seen cases in which heads did not appear until the twentieth throw."

V. Pareto (1848-1923), and others into the modern theory of economic *utility*, or the calculus of satisfactions.[16]

The subtlety of the subject under consideration is now apparent, but the following example cited by J. M. Keynes in his treatise on probability focuses attention upon this aspect of its philosophy. It is instructive for the reader to reflect what decision he would have reached in the following proposition.[17]

The London "Daily Express" had offered a beauty prize, which, amounting to 400 pounds in value to each winner, was to be competed for on the following basis. Out of 6,000 photographs submitted in the contest, a certain number were to be selected and published in various districts into which the United Kingdom had been divided. The readers in these districts were then to decide by their votes which were the fifty most beautiful pictures. After this selection had been made the fifty winners were then to present themselves before a hardy gentleman, who, unmindful of the fate of Paris of Troy, was then to choose the twelve winners. Now it happened that one of the fifty beauties selected in the preliminary contest was not given ample opportunity to display her charms in competition with the others, so she brought suit against the Daily Express for damages since she had been defrauded of her chance to win one of the twelve prizes. The case was carried through two courts and the plaintiff was finally assessed damages computed at 12/50 of the value of a single prize.

### 5. *From Effect to Cause.*

We turn next to the consideration of another aspect of the theory of chance about which the storms of controversy have waged for many years. This is the so-called problem of *inverse probability*. In essence it postulates the possibility that one can reason from an event to its cause. If we know the cause of a certain act of nature it is easy to predict the behaviour of an object influenced by the same cause: but is the converse problem always true? If we are in the presence of an event can we then reason back to that which brought it to pass? The answer, in very general terms and subject to exception, is that if the process is a reversible one then the relationship between cause and effect is reciprocal with effect and cause. But alas, nature does not always act in this reversible way.

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[16] For an account of this development see the author's work: *The Theory of Econometrics*, Bloomington, Indiana, 1941, Chap. 3.

[17] The case is Chaplin vs. Hicks, 1911. See J. M. Keynes: *A Treatise on Probability*, London, 1921, p. 25.

The problem of inverse probability may be stated thus: We are in the presence of an event which we know has proceeded from one of a complex of mutually exclusive causes whose respective probabilities of producing the event can be numerically expressed. The question is this: What is the probability that the event proceeded from a particular specified cause?

A simple example will clarify this question. Suppose a black ball has been drawn from one of three bags, the first containing three black balls and seven white, the second five black balls and three white, the third eight black balls and four white. What is the probability that it was drawn from the first bag? This, one observes, is a procedure from effect to cause and there are subtle logical difficulties in the way. An answer, in the present case 36/191, is easily derived by a formal rule, but the interesting question is the philosophical justification of the rule.

The rule by means of which a numerical probability is attached to such a problem as that just stated is attributed to Thomas Bayes, an English clergyman, and was first published in the *London Philosophical Transactions* in 1763, two years after the author's death. It has been vigorously attacked by theory and paradox,[18] one of the most famous of the latter being due to the Danish actuary J. Bing, who shows by one mode of application of the rule that[19] "if, among a large group of equally old people, we have observed no deaths during a full calendar year then another person of the same age outside the group is sure to die inside the calendar year." The point to the controversy seems to be this: To obtain a rigorous answer we must have full information about the actual probabilities associated with the complex of mutually exclusive causes. For example, we may know that an event must proceed from a set of circumstances, but we may have no further information about them. Are we justified in assigning an equal probability to each cause? By no means. Shall we say that the probability of throwing a five in a single toss of one die is one-sixth because the die has six faces and we have no reason to believe that one side will appear more frequently than another? Such assumption on the basis of "insufficient reason" may lead to grave error since the die may be loaded on one side. The only safe principle to follow is what has been called the principle of "cogent reason" by which we assure ourselves that the die, typical of every quantity subject to statistical reason, is symmetric in every particular. On the basis of this knowledge we can assume with assurance that the probability of throwing a five must be one-sixth. The matter has been put admirably in the following statement of Arne Fisher:

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[18] R. A. Fisher, *Proc, Cambridge Phil. Soc.*, Vol. 27, 1930, p. 528, says: "I know only one case (Bayes rule) in mathematics of a doctrine which has been accepted and developed by the most eminent men of their time, and is now perhaps accepted by men now living, which at the same time has appeared to a succession of sound writers to be fundamentally false and devoid of foundation."

[19] See A. Fisher: *loc. cit.*, pp. 72-76.

> "We require not alone an exact enumeration of the various complexes from which the observed event may originate, but also an exact and complete information about the structure of such complexes in order to evaluate their various probabilities of existence. If such information is present, we can meet even the most stringent requirements of the general formula, and we will get a correct answer. But in the vast majority of cases, not to say all cases, such information is not at hand, and any attempt to make a computation by means of Bayes' Rule must be regarded as hopeless. We may, however, again remark that very seldom we are in complete ignorance of the conditions of the complexes, which is the same thing as saying that we are not in a position to employ the principle of equal distribution of ignorance in a rigorous manner. From other experiments on the same kind of event, or from other sources, we may have attained some partial information, even if insufficient to employ the principle of cogent reason. Is such information now to be completely ignored in an attempt to give a reasonable, although approximate answer? It is but natural that the mathematician should attempt to obtain as much of such information as possible and use it in the evaluation of the various probabilities of existence."[20]

Inverse probability finds one of its most important applications in the theory of statistical sampling. Thus, let us suppose that a sample has been taken from a statistical universe. By the term *universe* is meant the total group of things from which the samples are drawn. For example, if one measures the heights of 1,000 men, this is a sample from the total male population, which constitutes the statistical universe. The universe may be finite, as in the example, or infinite as in the case of most data which depend upon *a priori* probability. The theory of inverse probability enters into such a question as this: What can one say about the nature of the universe itself from a sample which has been obtained from it? Thus, if we know the average height of a thousand men, what can we then infer about the average height of all the men in the United States? Since we know in advance much about the nature of the distribution, the range of heights from the dwarf to the giant, it is clear that we can infer from the example a considerable amount about the universe itself. It is possible, for example, to set narrow limits within which, with a high degree of probability, the average of the universe itself lies. Contrast this inference, however, with the information which we would obtain from the average of a randomly chosen set of a thousand integers.

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[20] From *Theory of Probabilities*, second edition, 1922, p. 78.

It is interesting to apply the theory of inverse probability to the problem of statistical sampling. Suppose a merchant receives a shipment of 1,000 articles, each one of which is worth $10 if it is not defective. One hundred of these articles, chosen at random, are examined and found acceptable. Under these circumstances can we say that the shipment has a value of $10,000? The answer, by a rigid application of Bayes' rule, is that the probable value is $9,902, an estimate that would doubtless be quite summarily rejected by the shipper. But if there is added to our information the fact that the manufacturer employs the techniques of quality control, the situation is altered. We can now reject the principle of insufficient reason from which Bayes' answer is derived, and we shall be justified in paying $10,000 for the shipment.

The principle of *induction,* namely, the process of inferring a general principle from the observation of particular events, is obviously closely related to Bayes' rule. In the technique of Bayes there appeared for the first time a mathematical formulation of the logic of induction. Imperfectly understood by earlier logicians, who devoted their attention almost exclusively to deductive processes, the principle of induction awaited such systematic developments as those found in the *Philosophy of the Inductive Sciences* (1840) by William Whewell (1794-1866) and the *System of Logic* by J. S. Mill (1806-1873). Induction is the great principle of scientific discovery, since science proceeds, in general, by formulating theories on the basis of experiment. It will be clear that little progress could be expected from the use of intuition and logic alone. Bayes' difficulty, the attempt to draw conclusions on the basis of the assumption of insufficient reason, is the difficulty also of the inductive process. It is only by reducing ignorance through experiment, by replacing *insufficient reason* by *cogent reason* through the acquiring of pertinent facts, that any progress can be achieved by science. One may conclude, therefore, that Bayes' rule is an inductive process. But when it is applied without the support of adequate knowledge of the phenomena involved, any conclusion derived from it will be of doubtful value. On the other hand, if it is used in connection with experimental knowledge, then the rule is a powerful tool in attaining inferences of an inductive character.

An excellent example to illustrate the dangers inherent in the use of insufficient reason is furnished us by astronomy in what is called “Bodes' law.” This famous conjecture, given currency by J. E. Bodes (1747-1826), a German astronomer, was actually stated in 1772 by J. D. Titius (d. 1796) nine years before the discovery of the planet Uranus by Sir William Herschel (1738-1822). The “law” is formulated as follows: To the members of the sequence, 4, 4, 4, ....add successively the numbers 0, 3, 6, 12, etc. as shown below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0 | 3 | 6 | 12 | 24 | 48 | 96 | 192 | 384 | 768 |
| 4 | 7 | 10 | 16 | 28 | 52 | 100 | 196 | 388 | 772 |

The numbers in the last row, divided by 10, are, according to the “law,” the approximate distances of the planets from the sun, expressed in astronomical units, that is in terms of the distance of the earth from the sun.

When the “law” was first stated, it agreed very well with the known distances of the planets Mercury, Venus, the Earth, Mars, Jupiter, and Saturn, which are respectively in astronomical units: 0.387, 0.723, 1.000, 1.524, 5.203, and 9.536. The “law” attained considerable celebrity and belief in it was high, when Uranus was discovered and found to have the distance 19.191. The one discrepancy, namely the value 2.8 between Mars and Jupiter, caused much speculation and led to the search for a lost planet. This difficulty was soon swept away by the discovery in 1801 of Ceres, the first asteroid to be identified. The mean distances of the asteroids from the sun is very nearly the distance required by the “law.”

With this great success for an empirical law, which had been successful in eight cases, two of which were obtained after its formulation, caused wide acceptance for it in spite of the lack of any *a priori* basis or *cogent reason.* A search was then initiated by the astronomers for another new planet, which should be revolving in an orbit at a distance of approximately 38.8 astronomical units from the sun. The new planet Neptune was finally discovered in 1846 by the simultaneous computation of its position by J. C. Adams (1819-1892) in England and U. J. J. Leverrier (1811-1877) in France. But alas, the distance of the new planet was 30.071 astronomical units instead of the 38.8 required by the "law". It is interesting to record, however, that both Adams and Leverrier used in their computations the distance given by Bodes' law. An even greater failure was recorded by the discovery of Pluto, which has a mean distance of approximately 398 astronomical units.

From this example we see the dangers of relying only upon the principle of insufficient reason, where there exists no information upon which a more cogent judgment can be founded. And again we have an illustration of the great value of postulates and theories which, even though they may ultimately be changed, can form a basis for an *a priori* thread to guide the inductions obtained from empirical data.

### 6. *The Reliability of Testimony.*

An instructive and amusing application of Bayes' rule has been made to the probability of testimony. Most of our information is derived from secondary sources and it must frequently be a question in our minds as to how much reliability should be placed upon certain statements.

Let us suppose for concreteness that a ball has been drawn from an urn known to contain one white and nine black balls. A witness to the drawing assures us that the white ball was drawn. Can we then assume as certain that the white ball was actually drawn? Those who have had experience know that the reliability of the witness is an important factor and in some cases we should be loathe to place even a modicum of belief in the statement that the white ball was the one drawn. Since no mortal is wholly reliable in all events, being subject to human frailties such as faulty memory, faltering attention, and the like, the probability that he has told the truth is never exactly unity. But let us assume from the observation of an individual's characteristics with regard to accuracy, that we have ascertained that the witness tells the truth on the average of five times in six. With this reliability, then, what should be our measure of belief that the rather improbable event of the drawing of the single white ball from the ten in the urn actually occurred? Surely it will be larger than the *a priori* probability of 1/10th, since that is unaffected by the testimony of the witness. The actual observation of the event and a favorable report upon it should have some weight. The mathematics by which the answer is obtained cannot be given here, but it is interesting to observe that an application of the principle of inverse probability yields the answer 5/14, or a little more than one chance in three. At first sight this comparatively low probability surprises us, until we reflect that a witness whose reliability is not perfect is reporting on an event which is quite improbable.

An interesting application of this conclusion is to be found in the general lack of confidence in reports about psychic phenomena. A number of eminent men, distinguished scientists skilled in careful scrutiny of laboratory data, have investigated some of these affairs and reported their belief based upon ocular and instrumental testimony. But do we eagerly accept their faith? The answer is too well known to require comment. Their testimony so far has merely served to discredit them rather than to persuade us of the reality of the phenomena, because their admittedly high scientific accuracy is applied to reporting upon phenomena whose probability of happening, as far as the normal mind is concerned, is exceedingly small. If the probability of their being wrong were just equal to the probability of the reality of the event which they have witnessed, we might then give them a grudging half measure of belief. But if intuition demands that the probability of the actual reality of the event is a thousand times as small as the powers of observation of the witness, then the chance of its being an actual event is one in a thousand. The conclusion is inevitable. A tremendous volume of expert testimony must be amassed before the world will give even a small amount of belief to these affairs.

The following comment of Laplace upon the problem of testimony is well worth the reflection of those who probe into historical events and believe all that they find there.[21]

"The action of time enfeebles, then, without ceasing, the probability of historical facts just as it changes the most durable monuments. One can indeed diminish it by multiplying and conserving the testimonies and the monuments which support them. Printing offers for this purpose a great means, unfortunately unknown to the ancients. In spite of the infinite advantages which it procures, the physical and moral revolutions by which the surface of this globe will always be agitated will end, in conjunction with the inevitable effect of time, by rendering doubtful after thousands of years the historical events regarded today as the most certain.

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[21] Introduction to the *Théorie des probabilitiés*, p. lxxxii, Eng. translation (loc. cit.), p. 124.

"Craig has tried to submit to calculus the gradual enfeebling of the proofs of the Christian religion; supposing that the world ought to end at the epoch when it will cease to be probable, he finds that this ought to take place 1454 years after the time when he writes. But his analysis is as faulty as his hypothesis upon the duration of the moon is bizarre."

### 7. *The Normal Universe.*

We turn next to another consideration in which resides much of the power of the application of the laws of chance to physical affairs. Applying our definition and the two laws of independent and mutually exclusive events to a heterogeneous swarm of statistical units we encounter what is generally called the "normal scheme."

This can be illustrated by means of a simple machine described by Sir Francis Galton (1822-1911) in his *Natural Inheritance*.[22] The apparatus consists of a shallow box, covered on one side with a piece of glass, and containing a large number of pegs irregularly distributed over the surface of the back-board. At the bottom of the machine directly below the pegs a number of long narrow slots are made by equally spaced partitions. If, then, a quantity of small shot be introduced by means of a funnel at the top of the apparatus to a point midway between the compartments and directly above the pegs, an instructive phenomenon will be observed. The shot fall helter-skelter down between the pegs, striking now on this and now on that, bouncing back and forth in irregular courses, first to one side and then to the other, until all are lodged finally in the compartments arranged to receive them. An interesting fact is then to be observed. The final arrangement of the shot in the compartments conforms to a regular bell-shaped curve, the central slots containing the largest quantity and the end slots the least. Most of the pellets in falling through the pegs were deflected as much to one side as to the other and hence the tendency was for the largest number to accumulate in the central compartments. A few, however, suffering a succession of collisions which threw them constantly to one or the other side, finally landed in the outer slots. These are the statistical exceptions which represent the rare events found in all phenomena to which this calculus applies. The apparatus itself, illustrated in the figure, is called a Galton *quincunx*, the word quincunx referring to the arrangement of the pegs.

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[22] London, 1889, p. 63.

In this picture of the scurrying shot we find the model of statistical swarming. Here is the ideal norm to which phenomena must be subjected. Take at random a large group of men and measure them according to height, weight, girth, or any other physical unit. Some are short and some are tall; some are heavy and some are light, but the vast majority tend to accumulate around a definite average value. The deadly mean is found in all statistical populations; the dwarfs and the giants only are objects of curiosity and can capitalize upon their exceptional characteristics; the genius and the criminal are subjects for our interest, the latter, alas, being the more thoroughly scrutinized by popular attention. This observation may be, perhaps, the source of an ancient dictum; "that those things which are bizarre are beautiful."[23]

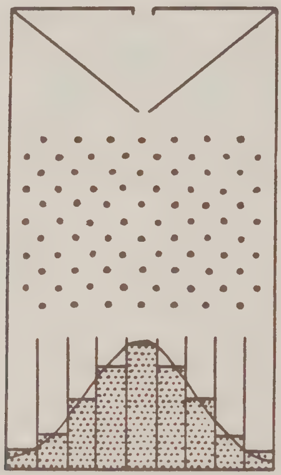

Figure 1. A Galton *quincunx* illustrating how a normal distribution can be derived.

Science in many fields directs its attention to the discovery of the monsters on the one hand and to the variations in the normal law on the other. We find, for example, two modal points instead of one in a distribution of the characteristics of a certain type of shellfish.[24] This leads us to the conclusion that the apparently homogeneous population is the mixture of two species whose mingling has not served to efface the primary characteristic of the two groups. Variations in the normal law are always to be regarded as evidences of correlations between types of things. The causes that are tending to produce the distribution are not all independent and of the same magnitude. There is evidence of exception to the postulate that the events occurring are wholly random. For example suppose that we should drop our shot through the machine previously described and suddenly discover that the compartments to the right were distinctly fuller than those to the left; our normal curve would appear skewed to one side. This mysterious exception to statistical expectation must have a cause. Some primary disturbance has been impressing its influence upon the pellets; the paths of the shot cannot be due wholly to a complex of uniform causes. We might find upon examination that we were using steel pellets instead of lead and that a magnetic field on the right was exerting an appreciable effect upon them.

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[23] For example see Thomas Browne: *Religio Medici*, part I, sec. 16: "There is no deformity but in monstrosity; wherein, notwithstanding, there is a kind of Beauty; Nature so ingeniously contriving the irregular parts, as they become sometimes more remarkable than the principal Fabrick."

The charm of the subject of statistics can only be appreciated after long contact with the delicate methods and the subtle deductions employed in it. The fundamental assumptions considered in the foregoing description of the theory have been hastily surveyed so that the applications which we contemplate and the conclusions that we shall draw may appear more striking proof of our contentions. The "sweet reasonableness" of the basic postulates appeals to our intuitions. We assume causality in nature, but we posit the impossibility of untangling the complex of causes which underlies the behaviour of individuals in a swarm of statistical units.

### 8. *Probability and the Theory of Gases.*

It was very natural then that Clerk Maxwell should have turned to the methods of probability when he undertook to study the motion of the particles of a gas. In the introduction to his historical papers on the dynamical theory of gases, which have been so powerful in the new physics, he makes the following statement of his problem:[25]

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[24] Karl Pearson: "Contributions to the Mathematical Theory of Evolution (III)," *Phil. Trans. of the Royal Society* (A), Vol. 185, part 1, 1894, pp. 71-110.

> "So many of the properties of matter, especially when in the gaseous form, can be deduced from the hypothesis that their minute parts are in rapid motion, the velocity increasing with the temperature, that the precise nature of this motion becomes a subject of rational curiosity. Daniel Bernoulli, Herapath, Joule, Krőnig, Clausius, etc., have shown that the relations between pressure, temperature, and density in a perfect gas can be explained by supposing the particles to move with uniform velocity in straight lines, striking against the sides of the containing vessel thus producing pressure. It is not necessary to suppose each particle to travel to any great distance in the same straight line; for the effect in producing pressure will be the same if the particles strike against each other; so that the straight line described may be very short. M. Clausius has determined the mean length of path in terms of the average distance of the particles, and the distance between the centers of two particles when collision takes place. We have at present no means of ascertaining either of these distances; but certain phenomena, such as the internal friction of gases, the conduction of heat through a gas and, the diffusion of one gas through another, seem to indicate the possibility of determining accurately the mean length of path which a particle describes between two successive collisions. In order to lay the foundation of such investigations on strict mechanical principles, I shall demonstrate the laws of motion of an indefinite number of small, hard, and perfectly elastic spheres acting on one another only during impact."

The beautiful analysis which Maxwell applies to his swarm of particles is one of the gems of physical literature. From it he and his successors have been able to derive most of the facts of thermodynamics and even the elusive concept of entropy finds a place among the statistical constants.

For the purposes of our discussion, however, we shall direct attention to a single phase of the problem. Let us visualize the swarm of atoms in a cloud of gas according to the picture suggested by normal intuition. The particles move with high velocities within the boundaries of the gas, frequently colliding with one another, shooting off at new angles to their former course, jostling helter-skelter in all directions.

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[25] "Illustration of the Dynamical Theory of Gases." *Phil. Mag.*, Vol. 19 (4th ser.), 1860, pp. 19-32; Vol. 20, 1860. In particular, p. 19.

Is it not a postulate founded upon the most secure intuitional belief to assume that the average velocities of the swarm of particles will be the same in every direction and that at any instant there will be approximately as many atoms moving toward the top of the gas container as toward any other side? Many attempts have been made to prove this proposition, but like the parallel postulate of Euclid it defies rigorous demonstration. Instead of seeking a proof we are perhaps more wise merely to assume that this very reasonable state of affairs exists within the swarm of particles and accept as intuitionally evident the following verdict of Maxwell, known in science today as the theory of the equipartition of energy:[26]

"It appears . . . that the velocities are distributed among the particles according to the same law as the errors are distributed among the observations in the theory of the 'method of least squares.' The velocities range from 0 to infinity, but the number of those having great velocities is comparatively small. In addition to these velocities, which are in all directions equally, there may be a general motion of translation of the entire system of particles which must be compounded with the motion of the particles relatively to one another. We may call the one the motion of translation, and the other the motion of agitation."

This statement of Maxwell defines what is called in the kinetic theory of gases the principle of the *equipartition of energy*. In the statistical swarm of molecular particles, there is no preferred axis along which the energy is distributed, nor is this energy concentrated in any particular velocity of the particles. The average velocity determines the temperature of the gas, and, by invoking the normal law of statistics which we have described above, we can infer the distribution of the velocities from the temperature. All is rational and in full accord both with the laws of chance and the concepts of classical dynamics.

### 9. *Probability and the Energy of the Ether.*

We return again to an old question: How does the luminiferous ether fit into the scheme of this rational picture? To put it differently, let us ask whether, if we regard the ether as the seat of energy and the medium for conveying it from place to place, can we build a rational picture of the distribution of this energy by means of the normal scheme derived from the postulates of the laws of chance?

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[26] *Loc. cit.*, pp. 23-24.

Thus far we have regarded energy somewhat in the light of a continuous entity and have revealed our convictions by speaking of its flow from place to place. To anticipate a little the material of the next chapter we shall see that the photo-electric effect has caused a profound change in our point of view. Energy may, in one sense, be regarded as discontinuous. We may think of it as moving about in packages called quanta, swarming hither and thither between material things. Luminous bodies radiate their energy in pellets and absorb it again in discrete quantities. If this, then, is to be regarded as the modern picture, what shall we say about the statistical behaviour of these quanta? Do they come under the same body of laws as the molecules and atoms of the gas? Are they statistical swarms subject to the normal scheme?

The most profound mystery of physical science rests in the conclusion that radiant energy is not amenable to statistical description in the same sense as the swarm of material particles with which we were concerned in the kinetic theory of gases. Applying to quanta the principle of the equipartition of energy, we are led to a distribution of radiant energy with respect to its wave length, which does not conform at all to the actual experimental evidence. As Jeans said in 1921, when the conclusions were much more novel than at the present time: "The state of things predicted (by the partition of energy according to classical models) is, however, so utterly different from that observed in nature that we are compelled to contemplate an abandonment, or at least a modification, of the classical mechanics."[27]

The mysterious ether,

> "A dagger of the mind, a false creation,
> Proceeding from the heat oppressed brain,"

again refuses to reveal its nature by submitting to what we customarily refer to as "the normal law." The reason why, in this age of physical triumph, we dare to speak of the new metaphysics is revealed by this fact more forcibly than by the failure of those who strove in earlier years to endow the ether with material properties. We turn from these speculations to a consideration of some physical facts.

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[27] *The Dynamical Theory of Gases*, 3rd ed., Cambridge, 1921, p. 365.

# CHAPTER 10

## THE QUANTUM THEORY AND WAVE MECHANICS

"Even the clearest and most perfect circumstantial evidence is likely to be at fault, after all, and therefore ought to be received with great caution. Take the case of any pencil, sharpened by any woman: If you had witnesses, you will find she did it with a knife; but if you take simply the aspect of the pencil, you will say she did it with her teeth."

Mark Twain in *Pudd'nhead Wilson.*

### 1. *On the Nature of Belief.*

Many years ago a king whose name has been forgotten dwelt in a kingdom whose history has been effaced by time. This obscure monarch undertook to determine the depth of the sea. The wisest of those who sought to answer this question for the king finally decided that the depth of the sea was *but a stone's throw.* Those who essay the task of exploring the relationship between the energy of ether and the electrons of ponderable matter seem frequently to phrase their conclusions in language of similar implication. Modern physical theories often sound as oracular as the utterances of Delphi. The current of the stream of speculative thought directed to the problem of the ether and its energy reminds one of the similar mystical stream of Coleridge's immortal dream as recorded in "Kubla Khan:"

"Five miles meandering with a mazy motion
Through wood and dale the sacred river ran,
Then reached the caverns measureless to man,
And sank in tumult to a lifeless ocean."

In the last two chapters we have discussed the birth of the electron and the basic assumptions of the mathematical theory which has been found, we dare not say most useful, but at least highly so, in the discussion of the activities of these ubiquitous particles. We have also turned our attention to a phenomenon that connected the behaviour of the electronic motions with the vast ocean of ether and its universal radiations. The photo-electric effect was discovered to present questions of strange difficulty and directed attention to another property of the ether more repugnant to reason than the denial of its elasticity, rigidity and density.

But in making our approach to the difficulties which beset modern physical thought we do not want to appear to adopt the point of view of those who believe that all things, to be understandable, must be capable of explanation in terms of mechanical models. To say that only twelve men in the world understand relativity does not mean that they belong to some esoteric society to whose members the secrets of nature are revealed in terms of things comprehensible. It means, if it means anything, that these men are familiar with the phenomena upon which the theory rests and the mathematical symbols by which the interpretation of these phenomena may be explored. The final conclusion regarding space-time and the framework of reality is as incomprehensible to their rational intuitions as to ours.

Roger Cotes, (1682-1716) who edited the third edition of Newton's *Principia,* made statements worthy of reflection in discussing the epistemological question: Whether the law of gravitation could be understood: "For causes usually proceed by a continuous binding together from the composite to the simpler. When one has arrived at the simplest cause it is impossible to advance further. Hence no mechanical explanation can be given of the simplest cause; for if this were the case, then this cause is not yet reduced to its lowest terms. Will you then call the simplest causes mystical and order them to be banished? If this be so you will in very truth banish those causes which depend upon the simplest, and those that depended upon the next, and so on until philosophy will at last be freed from any cause whatsoever."[1]

We return again to a consideration of the photo-electric effect described in chapter eight, which may be defined roughly in recapitulation to be the observation that, if monochromatic light be thrown upon a metal plate, electrons will be ejected from its surface with a velocity which depends wholly upon the frequency of the incident beam and not upon its intensity. The latter serves merely to eject more electrons but does not accelerate them. Since we shall have occasion to speak in further detail of the fundamental role of the curious property of light known as *frequency* we shall define it simply to be the value which represents the number of crests of a wave passing any given point in unit time. *Wave length,* another fundamental concept in the theory of radiation, is the distance between these crests. With these words we commit ourselves to the undulatory theory of radiation and are compelled to think in terms of some medium, however different it may be from ordinary matter, which conveys the energy from one place to another. From our previous experience with the theory of the ether we are now prepared to admit that metaphysical concepts may be forced upon us as explanations of the realities which we observe in the laboratory.

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[1] Newton's *Principia,* Editoris praefatis, 2nd ed., p. xxv.

### 2. *Failure of the Equipartition of Energy.*

It is well to approach our difficulties historically. Suppose that we begin as the discoverers of our present puzzle began, by creating some kind of a concept of the method by which energy is generated in a radiating body. Let us suppose that matter, from which ethereal waves are proceeding, is made up of electronic oscillators. We have shown in the phenomenon of the Crookes tube and in the evidence afforded by currents of electricity that there are numerous *free electrons* in the universe which are able to move about under the influence of impulsive forces impressed upon them. But it may readily be conceived, on the evidence of the existence of ponderable bodies, that there are also numerous *bound electrons* which are attached to their constituent atoms by forces the nature of which we but dimly understand. Let us assume that the phenomenon of radiation is an evidence of the activities of these bound electrons and that their oscillations are capable of setting up periodic disturbances in the ether.

What shall our mental picture be of this activity? As in all theories about the phenomena of the physical universe, our minds work better if we can obtain some modelized concept of the cause. Let us assume then that the bound electrons are tiny vibrating particles pictured, perhaps, as knobs upon the ends of a host of elastic rods. If these knobs are affected by a quantity of energy they create waves in the ether; if the waves of the ether beat upon them they are set in oscillation. There is mutual interplay, and the ceaseless flow of energy through the world is but evidence either of their ability to radiate or of their ability to absorb.

Now if a set of such oscillators were immersed in a medium resembling air we should expect something like this to take place. Gusts of wind blow upon them and they commence to weave back and forth like trees in a storm. The violence of the stream of air, which to preserve the analogy of waves must be considered to have regular periods of intermittence and intensity, increases the agitation of the rods and finally one of the knobs breaks loose from its attachment. And if the energy of the gale be conceived to increase still further then more of the knobs will be torn loose and their average energy of translation will grow with the violence of the storm; that is to say they will be hurled through longer and longer distances. But this, we have just observed, is not at all the case with the bound electrons in the stream of light, for these never fly from the security of their atoms with more than a certain velocity determined wholly by the character of the metal and the frequency of the incident beam. Hence our picture of the knobs is not consonant with experiment.

We might, however, look at the other aspect of the problem. Suppose our field of rods and knobs has been put into agitation by the wind and we then enclose it within a building whose walls are perfect reflectors of the waves of air. What should we expect to happen? According to Newtonian mechanics and the principle of the conservation of energy we should observe that, after a while, our waving knobs were losing their power of oscillation. Slowly the viscosity of the air would steal their kinetic energy, and in the course of time all the motion would have disappeared into the internal energy of the medium. But what if the contrary thing were really seen? What if we should observe, after a long period of time, that the field of rods which we had left in oscillation was still in vigorous motion? If we were to state that such was the evidence of our eyes we should certainly be disbelieved by anyone who based his judgment upon the principles of Newtonian mechanics. Is it not surprising, therefore, and very disturbing to those who want to believe in a rational universe, defined in terms of laws founded upon models comprehensible to our present chemical natures, to learn that electrons oscillate according to the second picture rather than according to the first?

The matter has been admirably phrased by J. H. Jeans who said after consideration of a picture similar to the one which we have just described:[2]

"To put the matter shortly; in all known media there is a tendency for the energy of any systems moving in the medium to be transferred to the medium and ultimately to be found, when a steady state has been reached, in the shortest vibrations of which the medium is capable. This tendency can be shown to be a direct consequence of the Newtonian laws. This tendency is not observed in the crucial phenomenon of radiation; the inference is that the radiation phenomenon is determined by laws other than the Newtonian laws."

Let us return for a moment to the picture of the oscillating knobs in the building the walls of which are perfect reflectors of the waves of air. According to the statement of Jeans, derived from comparatively elementary considerations, Newtonian mechanics demands that after a while the entire energy of the knobs shall be returned to the medium, since, by hypothesis, the confining walls are perfect reflectors. And in what form will this energy then be found? According to Jeans *it will exist in the smallest waves which the medium is capable of maintaining.* For example, if we were considering the energy of sound in a chamber of air, the familiar musical note *C* would have a wave length of about one and a third meters. But this note would soon disappear as an audible sound, and if we should trace its energy amid the jostling molecules of the air, we would find that it had finally degenerated into energy with wave length of the order of $10^{-7}$ centimeters.

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[2] *Report on Radiation and the Quantum Theory,* 2nd ed., 1924, p. 2.

If, then, we apply this thesis to electro-magnetic radiations, we are forced to consider the character of the medium which contains them. If this is the ether, then the limiting value for the wave lengths must be much lower than the same value for air since, as Jeans remarks, "whatever is regarded as certain or uncertain about the ether, it must be granted as quite certain that it approaches more closely to a continuous medium than to a gas. If it has any grained structure at all the distance between adjacent grains must be enormously less than $10^{-7}$ centimeters assumed for the corresponding distance in the gas. But even if this distance were as great as $10^{-7}$ centimeters, only one-millionth of the total radiant energy ought to be of wave length as great as $10^{-5}$ centimeters—entirely contrary to what is observed."[3]

Discussing this same problem ten years later, the failure of the classical hypothesis of the equipartition of energy when it is applied to radiation, Jeans turns a speculative eye upon the ether postulate itself. He remarks: "Thus, it may be argued, the difference in the cases we have been discussing may perhaps not be one of obedience or disobedience to the classical mechanics, but of the existence or non-existence of a surrounding medium. Our arguments may not prove the breakdown of the classical mechanics, but merely the non-existence of the ether. It is greatly to be wished that the question of the existence or non-existence of the ether could be settled by such simple considerations; unfortunately it cannot. The theory of relativity in effect requires that it shall be impossible to decide as to whether an ether exists or not, either by these or by any other purely mechanical considerations; the equations of radiation and absorption of energy are precisely the same whether the energy is radiated into, and absorbed from, an ether or empty space. The analysis which we shall now give will show that the existence or non-existence of an ether is wholly irrelevant to the question, so that if our analogies break down, it is not on the question of the reality of the ether."[4]

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[3] *Report on Radiation and the Quantum Theory,* 1st ed., 1914, p. 6.

### 3. *Is a Vacuum Empty?*

We must return again to the problem as old as science: What is radiant energy? What is its structure? In what does it reside? How does it get about from place to place? To essay the answers to these questions let us examine a small quantity of this elusive stuff with critical eyes. For our present purposes imagine that we have shut in a small quantity of space by means of an enclosure which is perfectly reflecting to radiation and let us exclude from it all particles of matter. In other words, let us assume that we have attained the perfect vacuum and let us now examine this little volume of empty space.

Our first discovery will be that the box is not empty as we may have imagined. For if we introduce into it a radiation thermometer we shall find at once that this particle of space contains a vigorous entity whose presence is made manifest by the fact that the thermometer will register a definite temperature.

If we examine the matter further we shall soon discover that the temperature which we have observed is highly characteristic of the entity which we are investigating. For we shall find that *the energy per unit volume, in other words the energy density of the box, is proportional to the fourth power of the temperature measured from the zero of the absolute scale*, that is, 273 degrees below zero centigrade. This remarkable proposition is known as the Stefan-Boltzmann law; J. Stefan (1835-1893) discovered it empirically, and L. Boltzmann (1844-1906) developed it from thermo-dynamical considerations.[5]

Continued investigation reveals the fact that the entity within the vacuum may be described in terms of periodic properties, for we may speak about its wave lengths and its frequencies. In this connection a beautiful discovery was made by W. Wien (1864-1928), who found that *in radiation of a uniform temperature there is one particular wave length which is favored above the rest since more energy is associated with it than with any of the others.*[6]

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[4] Second edition of Jeans' *Report*, 1924.

[5] "Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der elektromagnetischen Lichttheorie," *Ann. Phys. Chem.*, Vol. 22, 1884, p. 291.

This law of Wien, however, is only one aspect of a more general discovery, namely, the law of the distribution of the density of radiation throughout all wave lengths. For if this density be graphed along the vertical axis of a diagram and wave length along the other axis (See Figure 1), we shall discover that the resulting curve rises sharply from the short ultra-violet radiation to a maximum value and then declines more gradually as the wave lengths are taken further and further into the region of the infra-red. The curve has little resemblance to the bell-shaped curve of the normal distribution described in the preceding chapter. In fact the mathematical function which defines it is fundamentally different from the normal frequency function and the basic theory from which it is derived demands a type of statistical inference essentially unlike that which leads to the normal curve. In the figure the theoretical curve has been graphed for radiation which has an absolute temperature of 6000° absolute and this is compared with the similar curve derived from an analysis of the radiation from the sun. Biologists have commented upon the significant fact that our eyes

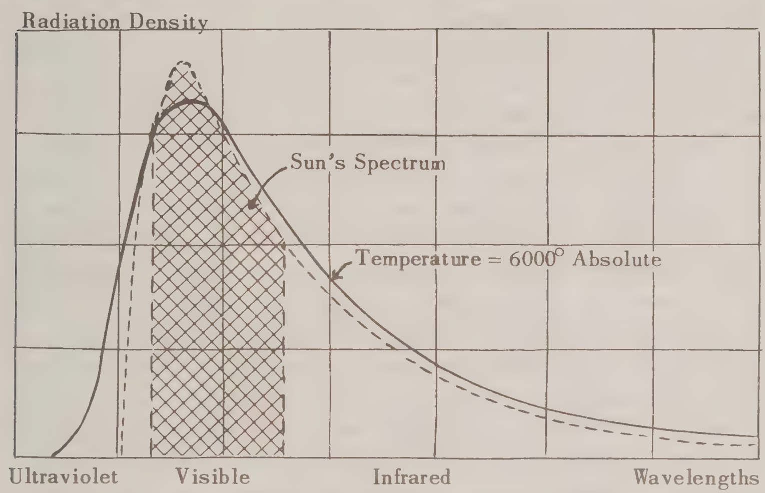

Figure 1. Radiation density as a function of the wavelength of the energy.

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[6] "Eine neue Beziehung der Strahlung schwarzer Körper zum zweiten Hauptsatz der Wärmetheorie," *Berlin Sitzungsberichte,* 1893, p. 55.

are so constructed that we see with that wave-length band which contains the Wien maximum value for the solar spectrum.

The discovery of the theoretical curve for the density of radiant energy was made by Max Planck (1858-1947), who in 1900 began his speculations on the nature of energy with this problem.[7] He succeeded in formulating the mathematical function which describes the distribution of energy and from it derived the laws of Stefan-Boltzmann and Wien as direct consequences. Now this, in a sense, might be regarded as something of a statistical achievement, the matching of the observed values with a mathematical curve, except that its theoretical justification led to a very revolutionary idea, that *the flow of energy must now be regarded as a flow of discrete entities, or quanta.*

Planck directed his investigations to the problem of the character of the radiation inside of a nearly-enclosed cavity maintained at an even temperature. As we have seen, this cavity is filled with an entity which is completely characterized by the temperature of the space and by its periodic properties. The former is uniform by the conditions of the experiment, but with the latter the case is otherwise, since it is distributed over a long range of wave lengths. Such a situation immediately suggests the statistical swarm studied in the preceding chapter, because waves of radiation are moving about in all directions with lengths varying between upper and lower limits. Consideration must also be paid to the bound electrons oscillating upon the walls of the cavity, since, in this instance, it is clear that we could not attain the state of perfect reflection postulated in our first abstract consideration of the energy-density of space.

The intriguing statistical problem presented by this picture formed the basis of Planck's musings, and, as we have indicated earlier, he soon found that the postulate that radiation was taking place according to Newtonian mechanics was wholly untenable. He argued that a continuous medium always extracts all the energy from any system of oscillating particles placed in it. If this were the case in radiation then no equilibrium between the oscillating particles composing the walls of the cavity and the energy of the intervening space could be maintained in which both retained their independent energies. After a while the bound electrons would cease to radiate and the wave lengths of the energy would continually be reduced to the smallest possible value within the capacity of the medium. If cosmic radiation be this ultimate in wave length, then all the energy in the enclosure with which we started would finally disappear in complete conversion into cosmic rays.

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[7] "Über irreversible Strahlungsvorgänge," *Annalen der Physik*, Vol. 1, 1900, p. 69, "Über das Gesetz der Energierverteilung im Normalspektrum," *Annalen der Physik*, Vol. 4, 1901, p. 553.

### 4. *Is Energy an Atomic Phenomenon?*

We come finally to the postulate of the quantum theory. Reflecting profoundly upon the difficulties which the classical picture presented and attempting to give some rational derivation of his distribution function for the density of radiation, Planck finally had a brilliant inspiration. The difficulties suddenly disappeared when one assumed that energy was not a continuous flow of something in and out of the ether, but rather a stream of discontinuous entities. Then one could infer that the vibrators in the walls of the cavity in equilibrium with radiant energy were not absorbing and radiating continuously. We thus reach the basic postulate: *Energy as it relates to this mutual exchange between matter and the ether is an atomic phenomenon; quanta of this mysterious activity of the universe are given out by the vibrating electrons; energy can never flow out of matter or into matter as though it were part of a continuous stream.*[8]

Man in his bolder moments has postulated the atomic character of matter and this theory has continued to gain in scientific stature over the centuries. But energy has been regarded otherwise as something akin to a continuous fluid. Although Newton with some caution advanced the corpuscular theory of light and the authority of his genius maintained prominence for the concept over many years, the experimental evidence of diffraction finally completely overwhelmed this proposition. Yet here again in the twentieth century after nearly a hundred years of triumph for the undulatory theory of light the atomic structure of energy appears in new guise and with a new name. We have here a strange reversal of the general scientific opinion. For that entity which surges into the falling ball; that which makes the wire from the battery glow, must be regarded, as in classical physics, as a flow of something moving like a stream from the storehouse of the ether into matter along the mysterious lengths of Poynting's vector. But the words of Lorentz must be recalled. "The flow of energy can, in my opinion, never have quite the same distinct meaning as a flow of material particles, which, by our imagination at least, we can distinguish from each other and follow in their motion."[9]

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[8] Planck's original theory assumed both discontinuous radiation and discontinuous absorption. In a later paper he modified his ideas to apply only to the radiation phenomenon.

How can one refrain from appearing mystical who advances the theory that energy is an atomic concept? The bold assumptions of Planck appeared in 1900 and their remarkable success in explaining the distribution of radiation attracted immediate attention. But what were these atoms of energy so strangely thrust into the ancient problem? Planck introduced a quantity $h$, now known throughout physics as Planck's constant, and showed that the phenomena which he had studied could be explained if the radiating electrons were assumed to be ejecting energy in little packages which contained multiples of the product $h\nu$, where $\nu$ is the frequency of the vibrations of the oscillating source and of the emitted radiation.

The idea caught fire in the mind of Einstein and in 1905 he rushed boldly forward with one of those speculations which have been so characteristic of his genius.[10] Where Planck had confined his inquiries to the interaction of matter and radiation within the confines of his cavities, Einstein postulated the quantum character of energy in its wanderings throughout space itself. The boldness of this idea is readily apprehended from the fact that the corpuscular theory of radiation was again to make its appearance less than a hundred years after the brilliant discoveries of Young and Fresnel had completely overwhelmed the followers of Newton.

Since the experimental evidence for this assumption was very tenuous at that time, Einstein turned his attention to the phenomenon of the photo-electric effect. Although the available data were both unreliable and scarce, sufficient evidence existed to present a problem which could not be resolved by classical theories as we have already indicated in the last chapter. A beam of light falls upon a metal plate and the photo-electric stream of electrons begins to pour forth. But the energy of the electrons is determined entirely by the frequency of the incident radiation, and not by its intensity. Einstein explained the phenomenon by assuming that each electron absorbs a single quantum of energy and flies off from the parent metal once the absorption has been completed. Since the energy of the quantum is $h\nu$ it is thus apparent why the intensity of the light has nothing to do with the energy of the photoelectrons. Assuming with Einstein that a minimum amount of energy

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[9] *Theory of Electrons*, 1909, p. 25.

[10] *Annalen der Physik*, Vol. 17, 1905, p. 132; Vol. 20, 1906, p. 199.

($w$) is necessary to free an electron from the surface of the metal, then the maximum energy which the photo-electron can have is defined by the famous equation

$$\tfrac{1}{2} mv^2 = h\nu - w \, ,$$

where $m$ is the mass of the electron and $v$ its velocity. Conversely the radiating electrons that are left behind, whatever their activity before and after they have ejected a quantum, are merely waiting until they have absorbed one of the indivisible packages of energy before they can re-radiate it into space. Many subsequent experiments have amply confirmed the daring theory of Einstein.

Years after the speculations which we have recorded above, the need for a special name for the packages of energy was clearly indicated. Called at first quanta, or corpuscles of radiant energy, they were finally named *photons*. This arresting designation was given them by G. N. Lewis (1875-1945) who wrote as follows:[11] "It would seem inappropriate to speak of one of these hypothetical entities as a particle of light, a corpuscle of light, a light quantum, or a light quant, if we are to assume that it spends only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains as an important structural element within the atom. It would also cause confusion to call it merely a quantum, for later it will be necessary to distinguish between the number of entities present in an atom and the so-called quantum numbers. I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name *photon.*"

### 5. *The Properties of Photons.*

How, then, are we to picture photons? Are we to regard them as small quantities of energy, tiny electrons of activity, flying about in space, hither and thither, jostling one another, flying away from impacts, in a word, forming a statistical swarm to be regarded in the same way as we regard swarms of material atoms and amenable to the ordinary laws of statistics? Have photons size? Can we assign a particle of space, defining its dimensions in terms of material units, within which a quantum of energy is to be found?

So far the answers to these questions have eluded those who have tried to find them. The problem is baffling. The ether in which the photon may be thought of as residing has receded further and further from material concepts. The photon appears to be similarly elusive. We shall briefly sketch a few of the experiments which show the puzzling nature of these packages of energy.

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[11] "The Conservation of Photons," *Nature*, Vol. 118, 1926, pp. 874-875.

One of the most beautiful of these is a simple experiment in diffraction. If we regard light as consisting of swarms of photons and picture diffraction as a statistical effect in which several photons unite at a single point to cause interference, then a reduction of the intensity of the light used in making diffraction photographs should have an appreciable effect upon the sharpness of the dark and bright bands. This experiment was performed by G. I. Taylor in 1909 with light the intensity of which was so reduced that two thousand hours were required to obtain a photograph.[12] The pattern obtained by Taylor was as sharp as that formed by light of much greater intensity. This conclusion gives us material for reflection. If diffraction is to be attributed to the statistical distribution of photons we must assume that our laws of chance apply to a swarm as tenuous as that consisting of one photon in 10,000 cubic centimeters, a hopeless hypothesis.[13] In commenting upon this situation H. A. Lorentz states what must be the verdict of rational intuition:[14]

"Now it must, I think, be taken for granted that the quanta can have no individual and permanent existence in the ether, that they cannot be regarded as accumulations of energy in certain minute spaces flying about with the speed of light. This would be in contradiction with many well-known phenomena of interference and diffraction. It is clear that, if a beam of light consisted of separate quanta, which, of course, ought to be considered as mutually independent and unconnected, the bright and dark fringes to which it gives rise could never be sharper than those that would be produced by a single quantum. Hence, if by the use of a source of approximately monochromatic light, we succeed in obtaining distinct interference bands with a difference of phase of a great many, say some millions, of wave lengths, we may conclude that each quantum contains a regular succession of as many waves, and that it extends therefore over a quite appreciable length in the direction of propagation. Similarly, the superiority of a telescope with wide aperture over a smaller instrument, in so far as it consists in a greater sharpness of the image, can only be understood if each individual quantum can fill the whole object-glass.

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[12] *Proc. Cambridge Phil. Soc.*, Vol. 15, 1909, p. 114.

[13] A different test by G. P. Thomson led to the same conclusion. *Proc. Royal Soc.*, Vol. 104 (A), 1923, pp. 115-120.

[14] Discussion on "Radiation" at the Birmingham Meeting of the British Association, 1913. See *Report on Radiation* (loc. cit.), p. 381.

"These considerations show that a quantum ought at all events to have a size that cannot be called very small. It may be added that, according to Maxwell's equations of the electro-magnetic field, an initial disturbance of equilibrium must always be propagated over a continually increasing space."

G. N. Lewis who, as we have said above, proposed the name photon to indicate more clearly the atomic nature of the quantum, gave six characteristics of the new entity. Remembering that the date was 1926, we shall find it instructive to consider them as follows: (1) In any isolated system the total number of photons is constant. (2) All radiant energy is carried by photons, although the photons are not actually the energy itself. (3) All photons are intrinsically identical. (4) The energy of any photon, divided by Planck's $h$, gives the frequency of the radiation carried. (5) All photons have the dimensions of action, or angular momentum, and are invariant with respect to the relativity transformations. (6) The frequency of a photon is not the same thing as the frequency of some physical system, but approaches it as the frequency diminishes.

We turn to a second experiment which has been called by K. K. Darrow the *inverse photo-electric effect.*[15] It is, as a matter of fact, a phenomenon of X-ray radiation. If, when light is thrown upon a metal plate, electrons are emitted with velocities which never exceed a very definite limit determined by the frequency of the radiation, should there not be a corresponding effect when a stream of electrons in a cathode tube bombard a metal target and excite radiation? The technical difficulties which were overcome and the delicate arguments employed in investigating this problem cannot be described here. *It is sufficient for our purposes to know that the inverse photo-electric effect is found in X-ray radiation in the existence of a maximum frequency of the rays.* The conclusion that we are to reach from this is precisely that derived from a consideration of Planck's oscillators and the photo-electric effect, namely that energy behaves as though it were concentrated in units of amount equal to $h\nu$. When an electron is stopped by impact against a metal it surrenders its units of energy, which reappears in equivalent radiation.

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[15] *Introduction to Contemporary Physics*, New York, 1926, p. 123.

Another question immediately presents itself. Radiation falling upon a metal plate generates a stream of electrons; electrons impinging upon a metal plate create radiation. What would be the effect if the electrons ejected from a target by bombardment of radiation were themselves allowed to fall upon another plate? This question was investigated during the earlier years of the present century and the answer discovered to be that the energy of the secondary stream of electrons depended wholly upon the energy of the primary ones and was independent of the metal targets and the distance over which the radiation traveled. This statement might at first sight appear to be a wholly reasonable one until one reflects that radiation may not be propagated as a stream of particles but as a wave. A wave spreads spherically through the ether and the intensity at any point on the wave front diminishes inversely as the square of the distance from its source. What a marvel is here discovered! Place a metal plate at such a distance from the source of the radiation that only the smallest portion of the wave front touches it. The amount of energy contained in the radiation at that point is an infinitesimal part of a single photon; yet in an instant the first photo-electron will jump out with its load of energy. This strange fact has led to the celebrated analogy of Sir William Bragg, (1862-1942) who stated that the situation, as far as material models are concerned, was as follows:[16]

"It is as if one dropped a plank into the sea from a height of 100 feet, and found that the spreading ripple was able, after traveling 1,000 miles and becoming infinitesimal in comparison with its original amount, to act upon a wooden ship in such a way that a plank of that ship flew out of its place to a height of 100 feet."

But in spite of the fact that we have experienced great difficulty in giving a space-concept to the photon, it is quite another thing when we ask the question: Is there any sense in which a photon may be regarded as having mass?

The clue to this significant speculation is found in Einstein's equation which identifies mass and energy. Since the energy of the photon is $h\nu$, we can equate this expression to Einstein's equivalent, namely, $mc^2$, and thus obtain the formula $m = h\nu/c^2 = h/(c\lambda)$, where $c$ is the velocity of light and $\lambda$ the wave length of the radiation. Substituting numerical values we obtain for what might be considered the mass of the photon the expression: $m = 2.19 \times 10^{-38}/\lambda$ grams. It is now a simple exercise to find the kind of radiation which would make this photonic mass equal to that of the electron. The calculation shows that this equivalence is effected if we assume $\lambda = 2.4 \times 10^{-10}$ centimeters, a wave length which is found in the region of hard gamma radiation.

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[16] "Aether Waves and Electrons," *Nature*, Vol. 107, 1921, p. 374.

### 6. *Bohr's Picture of the Atom.*

Before passing on to other aspects of our problem let us pause a moment and see what can be saved out of the wreck of Newtonian mechanics and classical belief. Let us assume that energy is corpuscular and that there is an ether which in some mysterious way acts as a medium in which the periodic properties of the entity may exist. Can we, in any way, form a picture of the method in which the electrons of matter are behaving? Previously we have thought of them as analogous to knobs upon elastic rods, but this concept can be replaced by a much more satisfactory one. The atomic picture which we are about to describe is due to Niels Bohr, a Danish physicist, who devoted his attention to the problem of constructing a model of the atom which would accord with the revelations of Planck and Einstein. In this he was eminently successful and the Bohr theory has been one of the striking successes of this remarkable period.[17]

The fundamental idea in the mind of the Danish physicist, apparently, was to save as much of the classical theory as possible while bringing his atomic model into line with the new quantum concept. In order to do this he first postulated that the principle of least action should be preserved. This mysterious principle, as we have indicated in the first chapter, for all its metaphysical origin and its remarkable resistance to rational interpretation, remains throughout physics the one unchangeable law. Einstein in developing his theory of relativity postulated it as a cardinal principle; in the wave mechanics of Schrödinger we shall see that it is the main justification for his metaphysical theory; no man is bold enough to whisper that it should be discarded, for with its denial the structure of physics would totter. Hence the theory of Bohr begins with the assumption of the validity of this principle.

Starting with the concept of the simple hydrogen atom, Bohr postulated that it should be regarded as consisting of a single positive proton as nucleus with a single negative electron revolving about it under the influence of an attractive force varying inversely as the square of the distance. So far there is no violation of Newtonian mechanics. But the novel element of the new atom is found in the assumption that the electron, while moving in its orbit, neither radiates nor absorbs energy. There are so-called stationary states within which the electron moves without disturbance either from or to the ether. In these stationary orbits the electron is unmolested and unmolesting, which is, of course, a violation of the principle which asserts that periodic motion sets up vibrations in the surrounding medium. In Figure 2 some of the possible orbits are shown for the hydrogen atom. Around the protonic nucleus the electrons move in co-planar ellipses of varying eccentricities, which are determined by the photons of energy which the electrons have absorbed.

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[17] "On the Constitution of Atoms and Molecules," *Phil. Mag.* (6), Vol. 26, 1913, .p. 1-25; 476-502; 857-875.

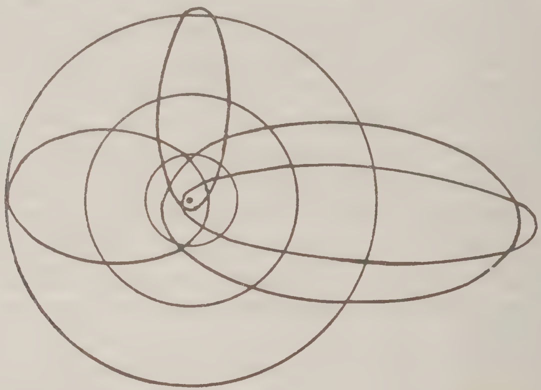

Figure 2. Possible electronic orbits in the hydrogen atom.

The cardinal postulate of Bohr was peculiarly adapted to the quantum hypothesis, which he introduced into the picture in the assumption that radiation takes place only when an electron moves from one stationary orbit to another, and that the loss or gain in energy in such a jump is exactly equal to $h\nu$ where $h$ is Planck's constant and $\nu$ is the frequency of the vibration. There is a good deal of mystery in this concept, however, for all of its success. How are we to reconcile with rational belief electrons which make their presence manifest only while in transit from one quantum orbit to another? What is the mechanism of such an atomic system and how is the energy transferred in these sudden erratic movements in which the radiating quantum is born? What is the condition of the electron when it is in transit? How long a time is required in these quantum jumps, or are they themselves definitive of time? Does time itself share the atomic structure of the universe and is its smallest unit the interval defined by the revolution of the electron in its nearest approach to the proton in the hydrogen atom?[18]

The great success of the Bohr atom was in its explanation of spectrum lines, which for beauty of interpretation and accuracy of prediction is unexcelled by few contemporary theories. Applying the simple postulates outlined above, first, that electrons move in systems derived from the assumption of the principle of least action and Newtonian force, second, that the radii of these orbits determine stationary states within which no radiation is possible, and third, that radiation occurs in discrete quanta generated by the electron in sudden, impulsive changes from one stationary orbit to another, Bohr derived his famous formula connecting wave lengths with a sequence of integers $(n,m)$:

$$1/\lambda = N\,(1/n^2 - 1/m^2).$$

In this equation we shall note that $N$ ( $=$ 109,677.69) is a number originally discovered empirically by J. R. Rydberg,[19] (1854-1919) and named in his honor but theoretically derived by Bohr, and $\lambda$ is the wave length of the radiation associated with the quantum integers $n$ and $m$. This is a beautiful result from several points of view but most of all, perhaps, from the fact that spectral lines are reduced to an aspect of the theory of integers, one of the most abstract and apparently most useless branches of mathematics if we speak only pragmatically.[20] In order to calculate the series of lines known as the Balmer series in the spectrum of hydrogen all we need to do is to set $n = 2$ in the Bohr formula and let $m$ range over the integers 3, 4, 5, 6, . . . etc. The extreme accuracy of the formula is seen in the fact that the observed values of the wave length corresponding to the first three integers (m) are 6563.07, 4340.64 angstrom units, and the calculated values are less by 0.03 in the first two wave lengths and greater by 0.02 in the third. What theory can hope for greater accuracy than this!

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[18] Norman Campbell has argued that time is a statistical phenomenon and that any temporal magnitude "represents a probability of transition characteristic of the individual elements." See *Phil. Mag.*, Vol. 1 (7th series), 1926, pp. 1106-1117. See also N. Bohr: *Nature*, Vol. 121, 1928, p. 588.

[19] "On the Structure of the Line-Spectra of the Chemical Elements," *Phil. Mag.* (5), Vol. 29, 1890, pp. 331-337.

[20] An illuminating discussion of this point, the utility of mathematics in general and the theory of numbers in particular, will be found in the small volume *A Mathematician's Apology* by G. H. Hardy, Cambridge, 1941.

The fruitful concept of Bohr was added to in essential ways by A. Sommerfeld and others. Sommerfeld developed the subject in one of the most masterful of volumes on the new physics entitled *Atomic Structure and Spectral lines.*[21] One of Sommerfeld's striking contributions to the Bohr picture was his formalization of the quantum conditions in terms of the periodic properties of the coordinates which describe the generalized momenta of the electrons. These conditions are mathematical in character and cannot be described with any more ease or precision than the condition of material behaviour, in some ways similar, known as the principle of least action. By means of his beautiful postulates Sommerfeld was able to make some headway in describing the spectra of more complicated atoms than the simple hydrogen model and also predicted, by introducing the relativity precession of the perihelion of the electronic orbits, the fine-structure of lines in the spectrum. This splendid prediction was amply verified when spectral lines were examined under the microscope and found to be resolvable into components of the predicted order of magnitude. The appearance of these precessing elliptical orbits of the hydrogen atom is shown

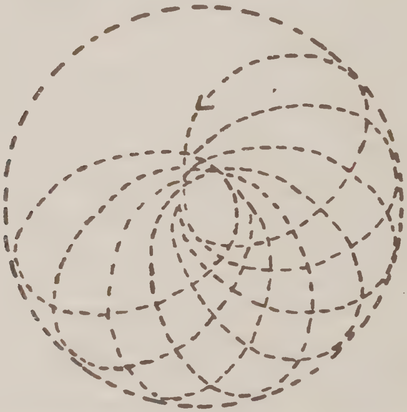

Figure 3. Relativity precession of the electronic orbits.

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[21] English translation of the 3rd edition by H. L. Brose, London, 1923.

graphically in Figure 3. Of other implications of this phenomenon we have spoken in an earlier chapter.

### 7. *Difficulties with Complex Atoms.*

The brilliant success of the hydrogen atom led at once to deeper explorations into the structure of those atoms in which more than one electron was involved. Here, however, difficulties multiplied rapidly. The supreme efforts of astronomers and mathematicians have not been able completely to surmount the difficulties met with in the three body problem, that is to say the problem involving the motions of three material objects moving under their mutual gravitational attractions. The theory of the moon according to the analytic methods of Charles Delaunay (1816-1872) contains one equation exceeding 170 pages in length. One does not lift such a weapon with greater ease than he would bend the bow of Odysseus. The second attempt to explore the atomic depths was directed to the helium model, in which a central double protonic nucleus was surrounded by two electrons. The difficulties encountered here have not been completely surmounted. H. J. Van Vleck who did fundamental work in this problem concluded that "as yet it appears possible to devise a satisfactory symmetrical model of the normal helium atom only with the aid of some such radical innovation as a reformulation of the quantum conditions or modification of the law of force between negative electrons."[22]

But in spite of the inherent difficulties of the problem, the attempt at investigation of these more complicated atoms has been very far from fruitless. Let us assume that we progress in the periodic table of Mendeléeff by adding one electron to the structure of an atom in order to obtain the atom of the next element above it. A corresponding addition must be made also to the positive charge of the nucleus. The atomic weight, which is almost entirely centered in the nucleus, is not explained by these additions, but this problem will be considered in another chapter. That the periodic table was discovered by a chemist was not a matter of accident, since the chemical properties of an atom depend upon the distribution of the electrons and not upon the nuclear mass. Atoms with the same electronic pattern are indistinguishable from one another by chemical means, although their physical characteristics may be quite different.

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[22] "The normal Helium Atom and its relation to the Quantum Theory," *Phil. Magazine* (6), Vol. 44, 1922, pp. 842-869.

It is in this way, by the addition of electrons and protons, that we can arrange the elements according to atomic numbers from the light hydrogen atom with its single electron to the heaviest atoms with their planetary systems just short of a hundred electrons. For more than a century and a half since its discovery in 1879 by Martin H. Kloproth (1743-1817), uranium with its 92 electrons was the heaviest of the elements. But the dream persisted that some day a new transuranic element might be discovered either in nature itself or in the crucible of the alchemists. This dream has been realized in a spectacular manner by the recent creation of six new elements, Neptunium, Plutonium, Americum, Curium, Berkelium and Californium, with patterns of 93, 94, 95, 96, 97 and 98 electrons respectively. Such is the marvel of the new alchemy!

The intricacies of the picture are greatly simplified by assuming that these electrons are effectively arranged in shells and that the chemical inertness of certain elements and the activity of others are due to the completeness with which the respective shells are filled. For example we may regard helium as having two electrons which compose the first shell; the second shell will include eight elements, which is completely filled in the atom of neon, a very inert element. The third shell, similarly, contains eight elements ending with argon, etc. It is not our purpose, nor could we hope within a single chapter, to elucidate all of the difficulties which this picture has produced nor the mental agilities of the host of theorists who have constructed and reconstructed the arrangement so as to make it more consonant with experimental evidence. The theory is enormously complex; there are many types of atoms and the vast amount of experimental data does not fit with suitable accuracy any model that has yet been constructed. In general it may be said that the picture of the shells with their full quota of electrons, accounting for the inertness of certain elements, and the presence of one or two valence electrons on the outside of completed shells in other atoms accounting for the chemical activity of these elements, is a rough approximation to the most generally accepted theory. The great probe into the mysteries of these minute quantities, from whose myriad swarms arise the massive edifices of the world, has been X-ray spectra and the swift particles from radium and other radio-active substances.

Because the mathematical difficulties inherent in the model of complex atoms could not be overcome otherwise, the shells of electrons were assumed to be coplanar rings as shown in Figure 4. This simplification of the problem proved to be unusually successful and numerous phenomena in spectroscopy were explained, at least approximately, by means of this atomic model. But if one were able to peer into the depths of an atom it is very unlikely that he would find the situation as simple as that which we have described. The electrons would in all probability be moving in elliptical rather than circular orbits, and perhaps even these might be greatly perturbed by the complexity of the interacting forces. It is also likely that the electrons would move in different planes oriented in various directions. It is a matter of much wonder, therefore, that the assumption of coplanar rings should have achieved such a large measure of success.

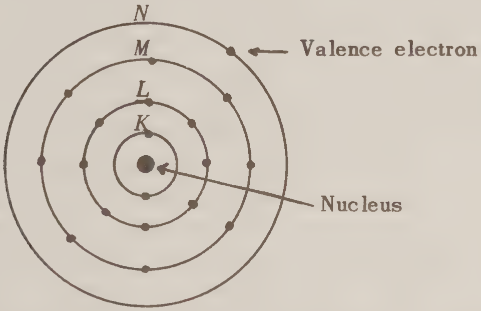

Figure 4. Model of the element potassium with two electrons in the *K* ring, eight each in the *L* and *M* rings, and one valence electron in the *N* ring. The atomic number of potassium is nineteen.

Another aspect of the complex atom was supplied by the concept of the *electron spin*. In order to account for certain fine-structure in spectral lines which was unexplained by the relativity precession of Sommerfeld which we have described above, S. Goudsmit and G. E. Uhlenbeck in 1925 assumed that electrons spin about axes through their centers much as the earth and other planets rotate about their axes.[23] This concept also proved to be very useful in explaining the magnetic properties of certain elements, since the spinning electrons would produce magnetic fields by virtue of their moving charges. Hence, if one were to assume that in certain shells some electrons spin in one direction and certain others in the opposite direction, and if one group of these spinning electrons was numerically greater than the other, then there would be a residual effect which would make itself evident in such a phenomenon as the observed magnetism of iron and other substances. Such is the mystery and the magic that one finds in this region so far below the limits of visibility!

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[23] For these pioneer studies see *Physica*, Vol. 5, 1925, p. 266; *Nature*, Vol. 117, 1926, p. 264.

### 8. *The Philosophy of the Compton Effect.*

We have shown in previous paragraphs that energy is to be thought of as being ejected by matter in the form of packages or quanta. We have set forth in as simple terms as the subject will permit some of the reasons which have led almost inevitably to this conclusion. But we have also pointed out that we must never at any time be unmindful of the phenomenon of diffraction and the apparent lack of any evidence to show that photons, once they have been ejected by the electrons, behave as swarms of spatial particles. The dilemma of modern physics is now before us. We have on the one hand, when we are regarding the relationship between matter and energy, a corpuscular theory of radiation, and on the other hand, when we consider light in the ether, the concept of a continuous medium propagating waves. We are like the shepherd of Aesop's fable, who would blow cold with his breath to cool his porridge, but would blow hot to warm his frozen hands.

Let us turn to another experiment, known as the Compton effect, in honor of the pioneer work of A. H. Compton both in the discovery of the phenomenon and in its elucidation.[24] We have seen that X-rays are produced if a stream of electrons is directed upon a metal target. What would happen if the radiation thus produced were allowed to fall upon another target? The answer is that secondary radiation, known as scattered X-rays, is produced, but curiously enough the new rays are of less penetrating power than the primary rays because of their reduced frequency. The phenomenon of scattered X-rays suggests that the original photons bombarding the metal plate have transferred only part of their energy to the electrons and have bounced off as smaller units.

In order to understand this statement let us consider what would happen if a photon should collide with a free electron, that is to say an electron detached from its atom. Some hypothesis will have to be made; hence we shall assume the simplest one consonant with Newtonian mechanics. Let this be the following: that both the energy and the momentum exchanged in the collision must be conserved. The photon in the collision will confer part of its energy and part of its momentum upon the electron, subject to the laws of such impacts, and will then fly off as a reduced quantum, the losses of which will be measured in terms of its reduced power of penetration. The intricate nature both of the experiments and of the mathematical interpretation of the phenomenon cannot be discussed here because of technical difficulties. It is sufficient to say that this quantum interpretation of scattered X-rays seems amply justified by the experimental facts. They lead again to the belief that energy actually goes about in packages, as earlier experiments indicated.

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[24] See *Bulletin of the National Research Council*, No. 20, 1922; also A. H. Compton: *X-Rays and Electrons*, New York, 1920, chap. 9. A careful account is also to be found in K. K. Darrow: *Introduction to Contemporary Physics*.

There is one feature of the theory that should be especially mentioned, however, because of its importance in the philosophical situation. When a photon strikes an electron we cannot predict the direction and velocity which each will take in its recoil. To be sure, if we know the direction in which one entity, say the electron, rebounds it is possible to predict the direction that the photon will take; but even though we knew accurately beforehand the initial conditions governing the collision it appears impossible at the present time to predict the subsequent paths of both the objects. It is a weird billiard game that one would play in which the object ball was an electron and the cue ball a photon. A novice would have an equal chance with the professional since the motions of the two balls would be wholly fortuitous.

A consideration of the implications of this remarkable fact has led P. W. Bridgman to the following melancholy conclusion about the nature of reality as it is revealed in the microcosm:[25]

"The same situation confronts the physicist everywhere; whenever he penetrates to the atomic or electronic level in his analysis he finds things acting in a way for which he can never assign a cause, and for which the concept of cause has no meaning, if Heisenberg's principle is right. This means nothing more nor less than that the law of cause and effect must be given up. The precise reason that the law of cause and effect fails can be paradoxically stated; it is not that the future is not determined in terms of a complete description of the present, but that in the nature of things the present cannot be completely described."

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[25] *Harper's Magazine*, March, 1929. Heisenberg's principle assumes that the behaviour of nature cannot always be described accurately, as in our inability to predict the mutual paths of the colliding electron and quantum. See Section 14.

### 9. *The Problem of the New Physics.*

Out of the new physics some facts are emerging which seem incontestable. We must modify our ideas about energy and the ether. We must have on the one hand a theory to explain diffraction and on the other a theory to account for the existence of photons. We should be careful, however, not to apply the laws of statistics to these photon swarms, because they are of a nature which defies such analysis. They may, to be sure, collide with quantities as tiny as an electron, but they may be regarded from certain aspects to be as large as balloons. To assume that photons are long tenuous filaments of inconceivably small cross section is in contradiction with certain experimental evidence. To assume that they are huge, broad affairs, as some experiments indicate, must strain our belief in the evidence which says that they may collide with an electron. To try to give them any spatial reality whatsoever leads inevitably to one contradiction or another. The difficulty is deeper seated; it goes back to the original problem of the ether.

"A question intimately involved in any attempt to find a physical basis for the quantum theory, is that of the degree of reality or substantiality with which the ether must be supposed endowed," says Jeans in his report to the British Association in 1913.[26] "The modern school of British physicists have been inclined to regard the ether as a completely real substance—indeed to some of them the ether is the primary real substance. At the other extreme stand the relativity school, who almost deny that the ether has any reality at all. There are innumerable intermediate positions between these two extremes. . . .

"If the ether is regarded as too devoid of substantiality to be treated as part of the dynamical system, it is necessary to fall back on the changes of energy in the mechanism by which the radiation is produced. Almost all the Continental physicists have regarded the quantum-theory from this point of view, which has the disadvantage that it is necessary to specify exactly the mechanism of radiation. Whatever this mechanism may be, and however many types of mechanism there may be, it is necessary that the energy of each should jump by quanta. So far no success has attended any explanation of why the energy of any type of mechanism should be incapable of changing except by quanta: assuming the fact, of course, the required results following, but the difficulty lies in making the assumption of the fact seem physically plausible."

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[26] *Report on Radiation,* 1st ed., pp. 87-88.

From these conclusions one would believe that physics is the science of reconciling the irreconcilable. How can waves be propagated with elements consisting of particles of energy? How can swarms of photons exist and still defy analysis by the laws of chance? Why does all energy appear to flow down toward a universal level and yet never arrive? How can photons be at the same time both large and small? How can matter affect other matter across non-material space?

Faced by the dilemma of a quantum theory and a theory of diffraction based upon the assumption of disturbance in a continuous medium, the bolder theorists commenced to seek a mathematical way out of the difficulties. The success of Clerk Maxwell, who showed that the ether of James MacCullah, discarded because it could not be reconciled with rational physical models, was the mathematical picture of electro-magnetic waves, and the later triumph of Einstein, who replaced gravitation by the mathematics of non-Euclidean geometry, gave courage to those who would essay the bold attempt. If physical pictures of reality cannot be made let us see what can be done by subjective principles.

### 10. *The Ether-Ring Theory of Quanta.*

The first theory that we wish to describe was due to Sir J. J. Thomson (1856-1940) and is not so wholly divorced from material models as is the later work of de Broglie and Schrödinger. The picture that the eminent English physicist suggested as a possible escape from the dilemma of the wave and quantum theories was that of the creation of rings in the ether by the stretching or contracting of tubes of forces connecting positive protons with negative electrons. Let us, for example, think of such a tube joining the two opposite particles. If the mutual potential energy is diminished the two particles approach one another; the tube in which the energy resides is contracted and, in order to escape, is shaped into a ring and thrown off into space with the velocity of light. Conversely, when the system, consisting of the proton and electron, gains energy it absorbs one of these hoops into its connecting tube of force which is correspondingly lengthened. These hoops with their unitary character take the place of the quanta of the Planck-Bohr theory. When a sufficient number of such rings are absorbed into the atom the electron is torn loose from its atom and the energy represented by the length of the tube disappears into the kinetic energy of the escape.

In order to explain the diffraction of light Thomson made the further assumption that these quantum loops are surrounded by a system of electrical waves of the ordinary type which vibrate with a frequency characteristic of the vibrations of the loops. This picture is summarized as follows:[27]

"... light on this view is made up of units, each of which contains a core in which the energy is concentrated; this core is surrounded by a system of electric waves which, though they have but little energy, give rise in cases where diffraction or interference occurs to electric and magnetic forces which deflect the paths of the cores without altering their energy. The core is supposed to vibrate in a definite period, and this period coincides with that of the electrical waves which surround it."

### 11. *De Broglie and the Wave Mechanics.*

We shall not comment upon the picture of Thomson which is suggestive of another that emerged from the speculation of this period and laid the foundation of the intriguing wave mechanics. This model, proposed the same year (1924) that saw the birth of the Thomson radiation, was suggested by Louis de Broglie, and if the metaphysical nature of its postulates can be surmounted it certainly achieves a remarkable success. The summary of de Broglie's paper exhibits the high ambitions of its author, who there says:[28]

"In the present paper it is assumed that the light is essentially made up of light quanta, all having the same extraordinarily small mass. It is shown mathematically that the Lorentz-Einstein transformation joined with the quantum relation leads us necessarily to associate motion of body and propagation of waves, and that this idea gives a physical interpretation of Bohr's analytical stability conditions. Diffraction seems to be consistent with an extension of the Newtonian Dynamics. It is then possible to save both the corpuscular and the undulatory characters of light and, by means of hypotheses suggested by the electro-magnetic theory and the correspondence principle, to give a plausible explanation of coherence[29] and interference fringes. Finally it is shown that every quantum must take a part in the dynamical theory of gases and how Planck's law is the limiting form of Maxwell's law for a light quanta gas."

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[27] *Philosophical Magazine*, Vol. 48, 1924, pp. 737-746.

[28] *Philosophical Magazine*, Vol. 47, 1924, pp. 446;457.

Surely the attainment of these results is sufficient to justify the existence of any paper. The theory of diffraction will be saved, the quantum theory will be maintained, and best of all, the laws of chance will be shown to apply to the statistical study of the swarming particles of energy. The only difficulty with this picture and the later extension of it by the methods of E. Schrödinger is that it is metaphysical in its final elements and incomprehensible to rational intuition. It is based upon the assumption of oscillations of a quantity entirely mathematical in character and incomprehensible to ordinary modes of thought, however satisfactorily Schrödinger has connected it with physical entities.

In a previous chapter we have commented upon various types of atoms. Pearson would have us believe that the atom was a squirt from a mysterious fluid entering our perceptions from the fourth dimension of space; Lord Kelvin advanced the theory that the atom was a vortex ring in the metaphysical ether; Bohr, incorporating the new knowledge of electrons in his theory, molded an atom in the image of a planetary system with curious stationary orbits, within which the members were without effect upon the external ether. We come at last to the most metaphysical atom of all which replaces all material concepts by a train of waves. The quantity that undulates is mysterious and elusive; the thing in which it undulates is so wholly transcendental that one never hears a question raised about either its nature or its existence.

De Broglie's idea arose out of a consideration of the two modern concepts of energy. According to Einstein the total amount of energy in a quantity of mass $m$ is equal to $mc^2$, where $c$ is the velocity of light; according to Planck and his successors the quantity of energy in a quantum is $h\nu$, where $h$ is the universal constant and $\nu$ is the frequency. Let us regard the equivalent energy in a particle of matter as being expressible in terms of these two concepts and let us equate them,

$$mc^2 = h\nu.$$

This simple appearing equation has bound up in it most of the mysterious features of modern physics. It is composed on the one side of the statement of a basic conclusion from the theory of relativity and on the other of the fundamental tenet of the quantum theory. Whatever mysteries are inherent in these two doctrines are inherited by the equation just written down.

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[29] The problem of *coherence* is that of explaining how several light quanta can be part of the same wave in order that diffraction and other optical phenomena may be accounted for.

Having made this beginning de Broglie reflected upon what would happen if the particle of mass $m$ should start to move. According to the theory of relativity the mass would increase with growing velocity and the term on the left of the equation would thus be augmented. On the basis of the same theory, however, the frequency would diminish. Hence the equality which held for the material particle at rest would no longer be true after motion had set in. While reflecting upon these strange consequences, de Broglie, in a moment of brilliant inspiration conceived the fundamental postulate of the theory which was to flower into the wave mechanics. Matter is not, as Kelvin thought, a vortex motion in the ether; it is a train of waves. *Associated with every particle there is an undulatory principle which spreads out continuously in all directions from it.* We dare not try to make the idea more concrete because all material analogies fail. The medium in which these waves exist is far less corporeal than the thinnest wraith left by the investigators of the nineteenth century; what this undulatory entity may be, wholly transcends our imagination to conceive.

Let us then regard matter as merely an aspect of an undulation which, when the particle is at rest, consists of a sphere of enveloping waves and when the particle is in motion, as a bodyguard of undulations which move with it, those in the rear disappearing as the matter progresses and new waves appearing in the direction of motion. It is possible, perhaps, to make a picture of the situation by thinking of a stone immersed in water and put in motion by some oscillatory device so as to set up a train of standing waves. If the stone then be given a rectilinear motion the train of waves will accompany it, those in the rear dying out and new waves appearing ahead.

And how can an atom be conceived in terms of this picture? Let us imagine the electrons in the Bohr atom. Around each of them is the train of waves, and as they move in their elliptical orbits these waves must arrange themselves in some kind of a stable configuration. It was the genius of de Broglie that saw how to account for the stationary states of the Bohr electrons by this method. If the waves accompanying the electron are to move around the orbit in any kind of a stable pattern it is necessary that their wave length be commensurate with the length of the orbit. In this way the beautiful discovery of Bohr, that the wave lengths of a spectrum can be calculated by means of a formula involving integers, may be apprehended. The total length of the orbits must contain whole multiples of the wave lengths of the constituent undulations.

Now the ideas advanced by de Broglie might have been regarded by his contemporaries as an interesting but profitless exercise in explanation were it not for one very arresting feature. In our discussion of the photon we were able to endow this elusive entity with a mass by the same equation which held the attention of de Broglie. Why then in similar manner should it not be possible to endow the electron with a wave length? For if in the equation given above we replace the frequency, $\nu$, by $c/\lambda$, where $\lambda$ is the wave length, we shall find that $\lambda = h/mc$. That is to say, the wave length of the photon is proportional to the reciprocal of its momentum ($mc$). Hence by the same argument it should be possible to assign to an electron a wave length proportional to the reciprocal of its momentum, that is to say, $\lambda = h/mv$, where $v$ is the velocity of the electron. The dramatic verification of this revolutionary idea will be discussed in a later section.

### 12. *Schrödinger and His Creation of Psi.*

Various other puzzling difficulties associated with the generation of radiation by matter were made plausible by de Broglie and his ideas rapidly gained adherents due both to the power of his point of view and to its exceptional novelty. They fell in particular under the scrutiny of E. Schrödinger, who in a series of brilliant papers in 1925-1926 seized upon the new concept of matter and rapidly moulded it into what is known today as the *wave mechanics.*[30]

The main point with Schrödinger is strangely like that of Einstein in the gravitation problem. To the latter, gravitational force appeared as a puzzling attribute of material things possessing properties impossible to reconcile with rational concepts. Hence, he avoided further trouble by removing the difficulties from matter and placing them upon the framework of space-time. Gravitation then became a problem in the geometry of four-dimensions, which manifests the peculiarities of its manifold only in the presence of ponderable bodies. In a similar way Schrödinger studied the de Broglie picture and decided that the presence of the material particle was not essentially necessary to it. Why not let the train of waves entirely replace the substantive entity? Gravitational force is an aspect of geometry; material particles are aspects of a vibratory property of space. Is there not something stirring in this picture? Do we not feel that reality further and further eludes our attempts to find it? Of what was Chesterfield thinking when he said: "Every man seeks for truth; but God only knows who has found it"?

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[30] "Quantisierung als Eigenwertproblem," *Annalen der Physik* (4), Vol. 79, 1926, pp. 361-376; 489-527; Vol. 80, pp. 437-490; Vol. 81, pp. 109-139.

It is of immense interest to see where Schrödinger takes hold of the problem. He begins with that mysterious postulate, the foundation of the description of all physical law, which we have called the principle of least action. Here stretching over the centuries from its mystical derivation by Maupertuis, through his interpretation of the behaviour of the Deity toward the world of material things, this strange necessity of thought reaches into the very nature of matter itself. More than that it assumes a dominating role in the structure of a new mechanics.

To explain the approach of Schrödinger in adequate terms would require considerable use of mathematical language. We can, however, state the matter descriptively as follows. The motion of material bodies, acting under impressed forces of any kind, may be accurately described by assuming that they follow paths determined by making the total action a minimum. Upon this assumption, as we have pointed out in the first chapter of the book, an adequate description of the behaviour of material bodies is always possible. But what is this mysterious action itself? It cannot be visualized. We cannot feel it, weigh it, or otherwise apprehend it in terms of our sensations. We can only calculate it. It appears, so to speak, to lack material handles. Through all the years since it was first stated by Maupertuis it has stood as a sort of spectre, always lurking in the background of every calculation, but disappearing once the desired motion has been obtained. It has moved the imagination of many men, but none has succeeded in explaining it. Schrödinger, fixing his attention upon this mysterious action, sought to find in it a revelation of the difficulties of the atom. His success exceeded his expectations, although the mystery of the entire situation can scarcely be said to have been dissipated by this penetration.

What Schrödinger found was that he could associate with each material particle a certain undulatory quantity involving a spatial rate of change of this mysterious action; he discovered that this vibrating quantity could be made to yield an accurate mathematical determination of many of the quantities involved in the quantum hypothesis, while at the same time it preserved the wave nature of light. Out of the equations of Newtonian mechanics which he derived in a straightforward manner from the principle of least action, Schrödinger was able to select a certain quantity involving action, or an equivalent expression in terms of kinetic and potential energy, which could be interpreted as the velocity of a family of wave fronts. This train of waves is the thing with which he replaced the corporeal particle.

Does the reader begin at last to think that modern physics is generating into the sophistry of the old schoolmen or does he believe that it is on the verge of revealing one of the most profound speculations yet made by the human race? Does it appear that modern dreamers are talking nonsense compared with which the metaphysics of the Middle Ages would be "as sensible as a dictionary"? Is there any impression of descending to a lower level of rationality when we review the reasons which led Duns Scotus to distinguish between three kinds of matter: "matter which is firstly first, secondly first, thirdly first," or the arguments advanced by Saint Thomas to settle the perplexing question: "whether a glorified body can occupy one and the same place at the same time as another glorified body"? We can only reply that those of the older age had no material experiments to control their imaginations; today the most searching gauntlet of physical fact must be run by those who would bring their dream into the tolerance of a skeptical age. "The wise man passes for a magician, the enlightened man for a heretic," says H. A. Taine of the mystics of that older metaphysics.[31] Today the dreamer is the sage, but his dreams must be built upon the solid rock of experience.

We return to the main theme, which is concerned, as far as simple language can tell the story, with an equation that defines the wave motion associated with matter. The unknown function whose undulations are the fundamental characteristic of material things has been designated by the Greek letter psi, $\Psi$, a wholly appropriate symbol under the dictum of Voltaire, previously quoted, that we should define that which we do not comprehend in a language which we do not understand. The quantity psi whose undulations are the essential characteristic of matter is a strictly mathematical symbol. As a matter of fact it is composed of two parts, one a real and the other an imaginary quantity in the mathematical sense of complex numbers. If the sign of the imaginary part of psi be changed and the new symbol multiplied by the old we obtain a real quantity which Schrödinger identifies with electric density. This is the only reality possessed by the mysterious function. It is just as elusive to physical definition as the concept of action which suggested it and in the same manner it fades out of the picture once the desired results of the quantum theory have been achieved. It resembles in this respect the catalytic agents with which chemists cause substances to unite and which they then remove unaffected from the reaction chambers. One is reminded of the story of the wise man who was called upon to distribute a legacy of eleven cattle between three sons, the first of whom was to receive a half, the second a fourth and the third a sixth. The learned judge added one of his own cattle to the herd, gave six to the first son, three to the second, two to the third and then returned his own animal to the field. The mysterious psi is like the addition from the wise man's herd; it retires from the scene once its labors have been performed.

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[31] *History of English Literature*, 1865, Book 1, chap. 3, sec. 6.

We may be able perhaps to make the matter no less incomprehensible to the lay mind if we explain in somewhat technical language the main tenet of Schrödinger's picture of the hydrogen atom. Let us think of the electron as being endowed with a potential energy calculated from Newtonian assumptions. We are not to regard the electron in any sense as a small material object, as the Bohr concept leads us to believe, but merely as an entity possessing the customary potential energy of Newtonian mechanics. Let us substitute this value of the potential energy into the wave equation,—an equation which is partially, though not entirely, definitive of the undulatory entity psi. By this statement we mean that psi is incompletely defined because only its partial derivatives are involved in the equation. Schrödinger, in order to give precise values to the mysterious psi, then assumed that it was to remain finite both at its origin in the atom and at an infinite distance from it. This peculiar postulate, entirely mathematical, is at the very heart of the Schrödinger concept as is evident from the title of his series of papers: "Quantization as a Problem in Characteristic Values." These characteristic values, imperative necessities arising out of the method of defining psi, are found to be precisely the energies associated with the electron in its stationary state in the Bohr atom.

It is perhaps unnecessary and inadvisable to go further in popular exposition of this involved subject. The point is accomplished if the reader has perceived the metaphysical basis for the new physics. In this maze of speculation he must not lose sight of the fact that Maxwell's equations, defining the propagation of electromagnetic disturbances through the ether of space, have essentially no greater reality than the equations of Schrödinger and of de Broglie. For all their plausible derivation from the picture of tubes of electric and magnetic forces Maxwell's equations must be regarded essentially as justifying the metaphysical belief of James Mac Cullah, whose choice among the elastic theories of the ether was regarded as the most untenable. The weird concepts at the basis of the wave mechanics can have justification only from their success in explaining in mathematical language what the laboratory technicians tell us they have seen. The keen-edged tools of mathematics must replace the desires of rational intuition and we must return to the melancholy belief that we can never completely understand the underlying structure of the universe. A new basis for epistemology is being forced upon us. Knowledge is not to be derived from material models; the light that comes to us from the petals of a rose is not the truth; the hard stone over which we stumble is only an illusion in our pathway and its reality exists in the mysterious undulations of the Schrödinger psi, the meaning of which is apprehended only through its behavior in an equation.

### 13. *The Matrix Theory of the Atom.*

Our story would not be complete without mentioning at least one other theory of the atom which has gained many adherents and has much success to justify its postulates. The principal founders of the so-called *matrix theory of the atom* are W. Heisenberg, Max Born, P. Jordan and P. Dirac.

The main postulate of this theory is contained in the following quotation from lectures given by Born during 1925-1926 at the Massachusetts Institute of Technology.[32]

"In the case of atomic theory, we have certainly introduced as fundamental constituents, magnitudes of very doubtful observability, as for instance, the position, velocity and period of the electron. What we really want to calculate by means of our theory and can be observed experimentally, are the energy levels and the emitted light of frequencies derivable from them. The mean radius of the atom (atomic volume) is also an observable quantity which can be determined by the methods of the kinetic theory of gases or other analogous methods. On the other hand, no one has been able to give a method for the determination of the period of an electron in its orbit or even the position of the electron at a given instant. There seems to be no hope that this will ever become possible, for in order to determine lengths or times, measuring rods and clocks are required. The latter, however, consist themselves of atoms and therefore break down in the realm of atomic dimensions. It is necessary to see clearly the following points: All measurements of magnitudes of atomic order depend on indirect conclusions; but the latter carry weight only when their train of thought is consistent with itself and corresponds to a certain region of our experience. But this is precisely not the case for atomic structures such as we have considered so far. . . .

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[32] *Problems of Atomic Dynamics*, Cambridge, 1926, pp. 68-69.

"At this stage it appears justified to give up altogether the description of atoms by means of such quantities as 'coordinates of the electrons' at a given time, and instead utilize such magnitudes as are really observable. To the latter belong, besides the energy levels which are directly measurable by electron impacts and the frequencies which are derivable from them and which are also measurable, the intensity and the polarization of the emitted waves. We therefore take from now on the point of view that the *elementary waves* are the primary data for the description of atomic processes; all other quantities are to be derived from them."

From this point on the theory is highly mathematical; the analysis of the vibratory motions, postulated as fundamental, is made through the manipulation of certain abstract entities of mathematics known as *matrices*. The theory is usually referred to in the quantum literature as the matrix theory which, from its very nature, precludes a popular description of its concepts. The classical theory of mechanics is modified to meet the conditions and the Bohr theory is made plausible by a beautiful though intricate mathematical analysis.

The theory of matrices has been extended in an important way by P. Dirac,[33] who introduced certain mathematical expressions called "Poisson brackets," which have properties similar to the matrices, but which are of a more general type. The exploration of these ideas belongs entirely to mathematical physics and we have made mention of them only to show how completely the physical picture has been submerged in subjective abstractions.

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[33] *Proc. Royal Soc.* (A), Vol. 109, 1925, p. 642.

### 14. *The Principle of Indeterminacy.*

A principle of very deep philosophical significance was announced in 1927 by W. Heisenberg[34] and has been more fully developed in a monograph on *The Physical Principles of the Quantum Theory.*[35] Its origin is in the observation that the conflict between the continuity view and the atomic view of nature is not one of words which may be cleared up by a re-definition of terms, or of contradictory experiments which further investigation will reconcile, but that it is a deepseated struggle of fundamental principles.

In order to reconcile the quantum hypothesis with the views of the continuity postulate of wave motion, Heisenberg assumed that there exists in all physical actions a *region of uncertainty* which is fundamentally associated with our measurements of space and velocity. If we attempt to specify the position of an elementary particle with greater exactness, then the determination of its velocity or its momentum becomes more uncertain. The area of this region of uncertainty, that is to say, the elementary unit of space, by the elementary unit of momentum, is not smaller than Planck's constant, $h$.

"This uncertainty relation specifies the limits within which the particle picture can be applied," says Heisenberg. "Any use of the words 'position' and 'velocity' with an accuracy exceeding that given by the equation (stated in preceding paragraph) is just as meaningless as the use of words whose sense is not defined."[36]

The importance of this proposition cannot be over-estimated, because it attacks the very foundations of causality and determinism upon which science rests its structure. As we shift from molecular to molar physics this basic postulate gains in validity, but the vital fact still remains that a complete indeterminacy is in the roots of nature. The situation is not unrelated to the statistical postulate that although a complete indeterminacy governs the toss of a single coin, the law of large numbers asserts itself when the tosses are increased without limit.

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[34] *Zeits, Phys.*, Vol. 43, p. 172.

[35] Eng. trans., Chicago, 1930.

[36] *Physical Principles of the Quantum Theory*, p. 15.

The following excerpt from a paper of Niels Bohr gives a penetrating appraisement of the philosophical importance of the indeterminacy principle:[37]

"This postulate implies a renunciation as regards the causal space-time coordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably. This appears, for example, clearly in the theory of relativity, which has been so fruitful for the elucidation of the classical theories. As emphasized by Einstein, every observation or measurement ultimately rests on the coincidence of two independent events at the same space-time point. Just these coincidences will not be affected by any differences which the space-time coordination of different observers otherwise may exhibit. Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately every observation can of course be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions, entails that for every particular case it is a question of convenience at what point the concept of observation involving the quantum postulate with its inherent 'irrationality' is brought in."

### 15. *The Diffraction of Particles.*

We turn finally to a consideration of a great triumph for the wave mechanics and the general point of view that in these new concepts of matter and radiation we are actually discovering an unexplored territory for whose examination the ideas of the older physics, however seasoned by time and experience they may be, are unsuited. This victory was an experiment performed in 1927 by C. J. Davisson and L. H. Germer of the Bell Telephone Laboratories.[38]

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[37] "The Quantum Postulate and the Recent Development of Atomic Theory," *Nature*, Vol. 121, 1928, pp. 580-590; in particular p. 580.

[38] "Diffraction of Electrons by a Nickel Crystal," *Physical Review*, Vol. 30, 2nd ser., 1927, pp. 705-740.

The interpretation of these interesting discoveries indicates that the wave nature of the electrons as predicted by the theory of de Broglie and Schrödinger is factually revealed.

In order to appreciate the theoretical significance of this statement let us return to the wave equation and see what the character of the wave train associated with a beam of particles would be. We have already described in an earlier section how one reaches the conclusion that the wave length of such a beam is given by the relationship: $\lambda = h/mv$, where $mv$ is the momentum of the individual members of the beam. The incomprehensible character of this conclusion is further enhanced when one computes the *velocity* of the wave associated with this wave length. For if we designate this velocity by $u$, it turns out that $u$ is defined by the equation, $uv = c^2$. Since $v$ for material particles is always less than the velocity of light, it follows that $u$ must be greater than this velocity, a conclusion which adds but another puzzle to the mysterious situation. However, if we assume that we are dealing here with experimental facts and not with metaphysical fancies, then the wave lengths of beams of material particles might be exhibited by the phenomenon of diffraction. To test the matter we compute the wave length of an electron accelerated by a potential difference of 100 volts and we discover that this wave length is of the order of 1.2 Angstrom units, a value which lies in the region of typical X-ray radiation. Hence a beam of electrons should exhibit characteristics similar to those found in X-ray phenomena.

It was not with the intention of checking the fundamental postulates of the wave mechanics that Davisson and Germer originally undertook their experiments, but rather to explore the structure of matter as revealed in the deflections caused in the path of bombarding electrons. If a stream of these tiny particles be directed against a metal target, electrons are scattered in all directions from the area under bombardment. The question first proposed by the investigators was this: What information could be discovered from these streams of electrons with regard to the arrangements of the atoms from which they were deflected?

Now it is a matter well known to technical workers that the wave character of X-rays is beautifully revealed through phenomena incident to their scattering by the surface of a crystal. This work, highly technical in character, is due essentially to M. von Laue and Bragg who showed the existence of diffraction in the reflection of

X-ray radiation.[39] If a beam of X-rays is directed toward a crystal such as zincblende the phenomenon of diffraction is revealed. The crystal has the surprising property of acting as a diffraction grating. A grating is a device made by tracing a number of very fine equidistant lines upon a plate by means of a fine diamond point. These lines are opaque to light so that the radiation which passes through the plate in the intervals between them will be reunited to form images. The discoveries of Laue showed that diffraction beams caused by the impinging of X-ray radiation upon a crystal were found both on the side of the transmitted beam and on the near or incident side of the target. The latter, in particular, are arranged in a regular array about the point of incidence. The wave length of the diffraction beams is entirely characterized by the wave length of the incident beam. The exploration of these concepts involves highly technical considerations and for our purpose is unnecessary.

It is sufficient to say that when a crystal surface is bombarded by a beam of electrons of sensibly uniform velocity the same phenomenon exhibited in X-ray radiation is observed. For certain critical speeds the electron beam generates a scattered stream of particles which issue from the incident side of the crystal in sharply defined beams analogous to those found in the X-ray diffraction.

The actual arrangement of the diffraction rays is not identical with those of X-rays, but the assumption that we are observing a true diffraction phenomenon enables us to calculate the *wave length* of the incident beam. A remarkable fact emerges. The length of the wave conforms to the predictions of de Broglie and Schrödinger. It is inversely proportional to the velocity of the stream of electrons. This important result was anticipated by W. Elsasser in a suggestion made in 1925 that the wave character of electron beams might be discovered in a study of their interaction with crystals.[40] It is the great achievement of Davisson and Germer that they were the first to show the experimental fact.

The conclusions of the American physicists have also been verified by others. In particular George Thomson, son of J. J. Thomson, whose great achievement was to show the material character of electrons, has given clear evidence of this wave property of matter. He obtained diffraction patterns similar in appearance to the one shown in chapter four in connection with the undulatory theory of light by passing electrons through thin films of crystalline materials.[41] One of his patterns obtained by photographing a beam of electrons which had passed through a piece of gold foil is exhibited in the accompanying plate. The evidence is conclusive. For if the particles had been scattered in all directions, as one would assume on the basis of the classical theory of statistical swarms, then the pattern would have shown a bright central area which would have shaded off gradually and continuously to the edge of the plate. But if electrons are diffracted in the same way as X-rays, then we should expect to find alternate areas of light and dark bands which depend upon the wave lengths of the particles of the incident beam. And this, as one sees, is exactly what was shown by the photographic plate.

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[39] See Compton's: *X-rays and Electrons* (loc. cit.), pp. 16-25.

[40] *Naturwiss*, Vol. 13, 1925; p. 711.

The triumph of the wave theory as it applied to beams of electrons turned immediate attention to the possibility of exhibiting the wave character of all material particles. For theoretically a wave length could be assigned to any moving object by means of the de Broglie formula. For example, a hydrogen molecule, moving under a thermal agitation of 200° centigrade, would have a wave length of 0.82 of an Angstrom unit, which puts it into the region of hard X-rays. Similarly an alpha particle from radium, because of its relatively high velocity, has a wave length of about $6.6 \times 10^{-5}$ of an Angstrom unit, which puts it into the region of cosmic radiation. But if we apply the formula to as large an object as a 22-rifle bullet with a velocity around 32,000 centimeters per second, then we reach the astonishing conclusion that the de Broglie wave length is of the order of $10^{-23}$ of an Angstrom unit, which places it far below anything yet discovered in the field of radiation.[42]

Many ingenious experiments have established the truth of the wave theory. A stream of helium atoms under thermal agitation was reflected from the face of a crystal and the diffraction pattern thus obtained corresponded exactly with that obtained when light is reflected from a ruled grating. When the beam of atoms was "monochromatic," that is to say, when it contained atoms all of the same velocity, the diffraction pattern became sharper in exact accord with the tenets of the wave theory.[43] Experiments with protons have not yielded results as satisfactory as those obtained with electrons, atoms, and molecules, but in spite of some difficulty in the interpretation of observed patterns, nothing inconsistent with the wave theory of particles has been observed.[44]

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[41] *Proc. of the Royal Soc.*, (A), Vol. 117, 1928, p. 600; (A), Vol. 119, 1928, p. 651.

[42] From a table of such values in J. D. Stranathan's *The "Particles" of Modern Physics*, Philadelphia, 1940, p. 540.

Shortly after the wave properties of electrons had been demonstrated, these newly discovered attributes were incorporated into a device known as an *electron microscope*, which has provided man with one of his most remarkable experimental tools. In this instrument beams of electrons are focused, not as in the case of light by a glass lens, but by means of either an electric or a magnetic field of the proper design. While the focusing of the beam has no relationship to the wave character of electrons, the resolving power of the instrument depends essentially upon this property. The magnificence of the achievement is found in the fact that the electron microscope has yielded magnifications as great as 500,000 times, whereas the very best of optical systems is inferior to a magnification of 4,000 times. The scientific progress of the race is spectacularly indicated by comparing the first crude microscopes of Hans and Zacharias Janssen created in Holland between 1590 and 1609 with these magnificent instruments which bring even molecules within the range of visibility. The first suggestion of the use of electric and magnetic fields as a focusing device for electrons was made by H. Busch in 1926, who calculated the focal length of a magnetic coil along whose axis a beam of electrons was made to pass.[45] The first description of an electron microscope was published simultaneously in 1932 by M. Knoll and E. Ruska of

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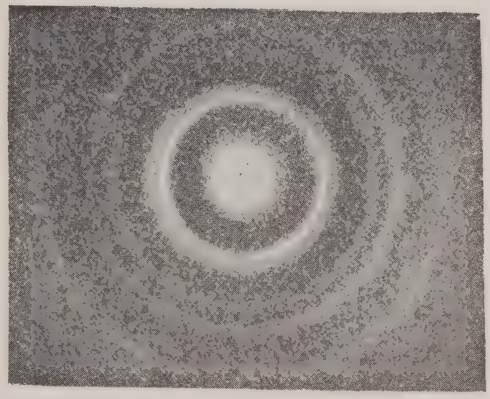
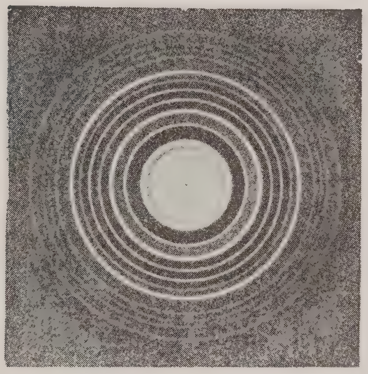
[43] See experiments of I. Estermann and O. Stern: *Zeits. für Physik*, Vol. 61, 1930, p. 95. See also the results of these same physicistsand R. Frisch: *Phys. Zeits.*, Vol. 32, 1931, p. 670 and *Zeits. für Physik*, Vol. 73, 1931, p. 348.

[44] See, for example, A. J. Dempster: "Reflection of Protons from Crystals," *Physical Review*, Vol. 34, 1929, pp. 1493-1494; "Reflection of Positive Ions by Crystals," *Ibid.*, Vol. 35, 1930, pp. 1405-1406. H. J. Yearian: "Diffraction of Protons of Medium Energy by Vapour Molecules," *Journal of Chem. Phys.*, Vol. 8, 1940, pp. 24-28.

[45] "Berechnung der Bahn von Kathodenstrahlen im Axialsymmetrischen electromagnetischen Felde," *Annalen der Physik*, Vol. 81 (4), 1926, pp. 974-993.



*(a)*



*(b)*

PLATE VIII

(*a*) The classical electron diffraction pattern obtained by G. T. Thomson in 1931. The pattern was produced by passing electrons through a gold foil. (*b*) A diffraction pattern which shows that the rings obtained by electron diffraction compare favorably in sharpness with those obtained from light. L. H. Germer obtained the pattern in 1939 by passing electrons through a caesium iodide film obtained by evaporation.

the Technische Hochschule in Berlin[46] and by E. Brüche and H. Johannson of the A. E. G. Forschungsinstitute in Berlin.[47] The former used magnetic focusing in their instrument and the latter electric focusing. Since the resolving power of a microscope depends upon the frequency of the wave length used, it is clear that this will be increased as the velocity of the electrons in the beam is increased. To attain very high resolution it has been necessary to accelerate the electrons through large potential differences.[48]

### 16. *What Shall We Believe?*

What are we to believe? In the last chapter we have reached the climax of that long period of investigation which started with the musings of Democritus and Lucretius and arrived at the suggestive tenets of the wave mechanics. We see in all this the battle between those who would look upon nature as a continuous phenomenon and those who would regard it as divisible into primary units. Those who hold to the belief that the substratum of phenomena is an ether stretching throughout all space and interpenetrating all matter are essentially of the former faith; those who see in the electron, indivisible and individual, the ultimate nature of the universe are clearly of the latter tenet. Inscrutable nature, forever holding out her promises and forever withdrawing them again, smiles at our efforts and says: "There is truth in the views of both." When we study light in the neighborhood of matter and view the interaction between it and radiating electrons, then the atomists are favored. But when we are dealing with light and other radiation remote from matter then the continuists have the better of the argument. Radiation is a wave; phenomena such as diffraction, in which light plus light may equal darkness, are supportable only on the assumption that periodic disturbances are occurring at the point in question. But the old thought forever intrudes: Disturbances in what? There is no answer to the question and perhaps there never can be in terms acceptable to human intuition. We become the new agnostics who realize the presence of something in nature which can never be understood in terms of those things to which we wish to reduce comprehensible experience.

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[46] "Das Elektronenmikroskop," *Zeits. für Physik,* Vol. 78, 1932, pp. 318-339. See also: "Beitrag zur geometrischen Elektronenoptik," *Annalen der Physik,* Vol. 12 (5), 1932, pp. 607-661.

[47] "Elektronenoptik und Elektronenmikroskop," *Naturwissenschaften,* Vol. 20, 1932, pp. 353-358. See also E. Brüche: "Elektronmikroskop," *Ibid.,* Vol. 20, 1932, p. 49.

[48] For an extensive and authoritative account of electron optics and its applications see L. M. Myers: *Electron Optics,* New York, 1939, xviii, 618 p.

The facts that seem to emerge from the investigations of these later years may be summarized as follows:

First: The interaction between matter and radiant energy is not explainable in terms of the ordinary concepts of Newtonian mechanics. Energy which does not disappear from vibrations of long wave length into vibrations of the smallest wave length capable of being set up in the medium is not in accord with the models of normal Newtonian intuition.

Second: The difficulties of the Newtonian ether may be resolved if we assume that energy is not a continuous phenomenon but is atomic. We may solve our problem by introducing quanta of energy which are characterized by the frequency of their periodic activity and a universal constant whose nature is still obscure.

Third: The material particles may be replaced by mathematical concepts derived from the study of the vibrations of a mysterious function associated with the quantity of action the minimizing of which describes the activity of natural objects. In other words particles, in some of their behaviour, act as periodic vibrations similar to radiations; radiations, in some phenomena, assume atomic properties and may be regarded as possessing the attributes of matter.

Do not these concepts form a basis for a new metaphysical philosophy of nature? Do we not see emerging from these strange concepts, forced upon us by the most delicate and accurate technique of modern laboratories, a whole new foundation for knowledge? Shall we ever be able to integrate our experience into a system of belief which the future may take with assurance as the foundations of its new structure? Or are we to believe with Heraclitus that nature is naught but a ceaseless change; that nothing is secure; that the world of experience is but shifting sand upon which no permanent edifice of thought may be erected?

The present status of the material particle is disturbing; that which was a solid has become a wraith and that which was a wraith has attained material attributes; we are faced by a strange dualism in nature which surprises us much as that excellent branch of geometry which assures us that for every theorem depending upon points and lines there is a dual theorem in which points take the role of lines and lines emerge as points. But the real difficulty is not so much in this as in our inability to make any headway with the mystery of the ether. Einstein throws the mystery of gravitation into conceptual experiences derived from an incomprehensible four-dimensional geometry; de Broglie and Schrödinger reduce the experience of light and electrons to a baffling activity of a Greek letter understandable, if it be understandable at all, in terms of mathematical analysis. Surely there is much truth in the chance remark of Hamlet:

> "There are more things in heaven and earth, Horatio,
> Than are dreamt of in your philosophy."

# CHAPTER 11.

# THE NEW PARTICLES.

"Nature, therefore, carries on her operations by imperceptible particles."

Lucretius in *De Rerum Natura,* Book I, line 328.

"Time is the wisest of things, for it finds out everything."

Thales.

## 1. *Recapitulation of the Situation.*

In our survey of the history of the atomic concept we saw how step by step from the early musings of Democritus and Lucretius to the more recent speculations of Dalton, Avogadro and their contemporaries, science, slowly at first but with an ever accelerating pace, began to gain increasing confidence in the idea. The mystery of the search as it now appears in modern physics centers around the problem of the relationship of an apparently continous flow of energy with a material world composed of discrete particles.

Before continuing further with a description of the discoveries of the new physics, let us recapitulate the situation as it now appears to us. Exploring the depth of the atom we have identified three quantities. Two of these possess the properties of matter as we have understood them, and the other the properties of a continuous undulation. But certain remarkable experiments have shown that the material particles behave under suitable conditions like waves. The property of diffraction which they possess is an incontestable proof of this. On the other hand the immaterial quantity assumes some of the aspects of a material entity.

The first of these three quantities is the *electron,* a particle which carries with it a negative charge of electricity and which has a small mass of the order of $9 \times 10^{-28}$ of a gram. Because of its relatively insignificant mass, the electron can be given very high velocities, which approach the velocity of light itself. But this upper limit is not attained for as the speed increases so also does the mass of the electron in accordance with the law derived from the theory of relativity. The beta particle, which we found to be a constituent of radio-active transformations, was soon identified as a high-speed electron. Its special designation as a beta ray, which it has retained in nuclear physics, is used for convenience to indicate that the particle is an electron which has emerged from the nucleus of an atom during radio-active phenomena.

In recent physics the need has arisen for a new unit of energy to describe the activity of these minute particles. This new unit is called an *electron volt* and it designates the energy possessed by an electron when it has been accelerated by an electric field of one volt. This bit of energy is exceedingly small since it is equivalent to $1.60 \times 10^{-12}$ of an erg. It corresponds to the energy which would be possessed by an electron moving with a velocity one five-hundreth that of light. Since the unit is often inconveniently small it is frequently replaced by the *megavolt*, denoted by the symbol *Mev*, which is equal to a million electron volts. This corresponds to the energy which an electron would possess if it moved with a velocity 0.94 times that of light. If the mass of one electron were completely dissolved into energy by the mass-energy relationship of Einstein it would generate approximately a half megavolt of energy.

The second of these constituents of the natural world is the *proton*, a particle which carries a charge of electricity equal to that of the electron, but opposite to it in sign. The proton, however, is about 1840 times as heavy as the electron, having a mass of the order of $1.67 \times 10^{-24}$ grams. This greater weight puts difficulties in the way of giving to the proton velocities as great as those created for the more mobile electron. The alpha particle which we have encountered as one of the constituents of radio-active transformations, is identified as the nucleus of the helium atom, that is to say, the helium atom from which the two planetary electrons have been removed. It carries a positive charge just double that of the proton, but its mass is approximately four times as great as that of the proton. In nuclear physics the rest-energy of the proton has become a familiar unit. This is equal approximately to 931 megavolts, or 1840 times the rest energy of the electron.

The third quantity which we have investigated is the *photon*, the fundamental constituent of electro-magnetic radiation, a quantum of energy with which is associated a characteristic frequency. In some respects the photon is to be regarded as a kind of particle, although its fundamental properties are those of radiation. Its velocity is that of light and it is unaffected by magnetic fields. It carries no charge although its nature is electro-magnetic, nor will it affect the electric charge of any particle of matter which it chances to encounter. The precise location of a photon cannot be given, and it has baffled experimenters who have endeavored to determine its size and shape. The gamma rays of radioactive transformations are photons. The quantity of energy associated with the photon is written in the form

$$E = h\nu,$$

where $h$ is Planck's constant equal to $6.55 \times 10^{-27}$ erg-seconds, and $\nu$, the frequency of the radiation, is equal to $c/\lambda$, where $c$ is the velocity of light and $\lambda$ the wave length of the radiation. In spite of the undulatory nature of the photon, a mass has been assigned to it by equating energy to mass according to Einstein's law. It is significant to observe that when the frequencies are those of hard gamma radiation, the mass of the photon is approximately equal to that of the electron.

These quantities were the playthings of the physicist prior to the year 1932 when fateful and momentous decisions were in the making. Cosmic radiation on the one hand and nuclear bombardment by more and more powerful devices on the other were leading science into an enlarged knowledge of the relationship between matter and energy.

### 2. *The Drama of the Positron.*

The first of the great discoveries was made in 1932 by C. D. Anderson, who announced the existence of an electron with a positive charge. This strange particle was identical with the electron in all its physical measurements except for the sign of the unit of electricity which it carried. This new addition to the family of particles was promptly called a *positron.*

The long delay in the discovery of the positron is readily explained by the strange fact that, while the electron is a stable entity, the positron has a life-span which is a very small fraction of a second. This brevity of existence is accounted for by the great affinity which the positron appears to have for the electron, and since these particles are abundant everywhere the positron almost instantly combines with one of them.

The most spectacular aspect of the discovery resides in the fact that when the positron disappears in union with its negative companion, in general *two photons of gamma radiation reappear in their stead.* Thus we have a certain glimpse of the mechanism by means of which matter disappears in the creation of energy, a clue to one of the greatest mysteries of the nature of the material world.

The discovery of the positron was a drama in itself. It came about as a concomitant of the investigation of cosmic radiation, which we shall consider in some detail in another chapter. It is sufficient for our present purpose to observe that one of the most fruitful methods of investigating cosmic and other radiation phenomena is by a study of the cloud tracks in a Wilson expansion chamber placed between the poles of an electro-magnet. As the particle enters the chamber its rectilinear path is converted into the arc of a circle by means of the magnet. From the radius of the circular path and the strength of the magnetic field, both the velocity of the particle and its energy can be readily estimated. This method has already been described in connection with the determination of the charge and the mass of the electron.

Now it happened that the study of cosmic radiation disclosed the existence of particles of vast energy. Hence magnetic fields of increasing intensity were required to investigate these strange new phenomena. But prior to 1932 the highest energies that could be measured were of the order of 15 megavolts. It was obvious that a more powerful tool had become a necessity. This was provided by a new apparatus constructed by C. D. Anderson and R. A. Millikan at the California Institute of Technology. The new magnet required a generator of approximately one thousand horsepower for its operation and the device was then able to measure particles with energies as great as 6,000 megavolts. So powerful was the new apparatus that the tracks of some of the electrons ejected by the gamma rays of radium with energies of the order of a megavolt were converted into circles of very small radii.

When Anderson then turned this new tool to the study of cosmic particles the energies of which had been too great to come within the range of earlier apparatus, he discovered in 1932 the cloud-track of a particle which appeared to have the same mass as an electron, but whose charge of electricity was positive.[1] The evidence is found upon a photographic plate (See Figure 1) which showed the tracks of two particles, apparently emerging from a common point of origin. One of these curved to the right and the other to the left. Anderson identified one of the paths as that of an electron with an energy of 27

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[1] "Energies of Cosmic-Ray Particles," *Physical Review*, Vol. 41, 1932, pp. 405-421.

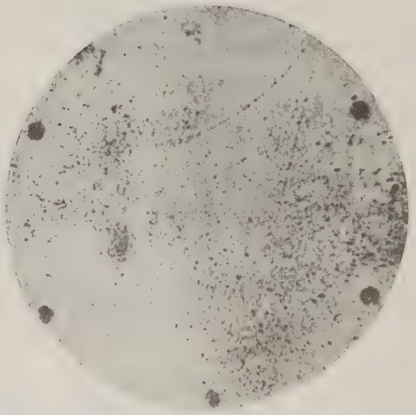

Fig. 1. Cloud chamber tracks obtained by C. D. Anderson in 1932 showing the common origin of two particles, one with a positive and the other with a negative charge. The track with the smaller curvature was assumed to have been made by a positron.

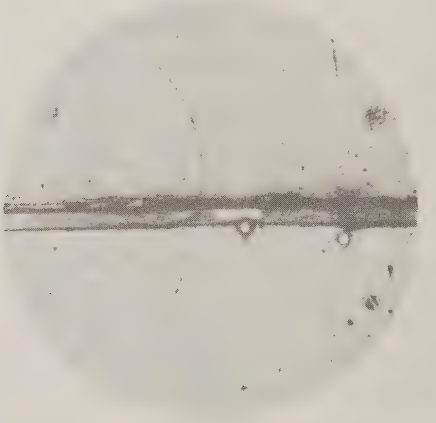

Fig. 2. A classical exhibit which was made by C. D. Anderson in 1933 to demonstrate the direction of motion of the particle making the track. From the known orientation of the magnetic field the charge on the particle was shown to be positive. The conclusion derived from this was that the track was made by a positron.

megavolts and the other probably as that of a positron with an energy of 450 megavolts.

Since the idea presented by this discovery and its interpretation was revolutionary, new evidence for the conclusion was required. In particular it was necessary to demonstrate that the track of the positron was not, in fact, that of another electron which had entered the apparatus from a different direction to that of the companion electron. This would account for the difference in curvature of the two particles.

The proof presented was as simple as it was elegant. Since the problem consisted in demonstrating the common direction of motion of the two particles, a sheet of lead was introduced into the apparatus. This exerted a retarding influence upon the velocities of particles which penetrated it, a fact made evident from the change in the curvature of the paths. The greater the curvature the slower the velocity. Inspection of the cloud chamber track shown in Figure 2 gives incontestable evidence that the particle passed through the sheet of lead from below. The orientation of the magnetic field left no room for doubt also that the particle carried a positive charge of electricity. And in this way the positron entered the family of the new particles of modern physics.[2]

The identification of the positron was soon followed by the discovery of a method by means of which an abundant supply could be created. In 1934 Irene Curie and her husband F. Joliot were investigating the phenomenon of artificial radioactivity. This they were able to create by bombarding boron, magnesium, or aluminum with alpha particles emitted by polonium. New unstable elements were formed which regained their stable state by the emission of a positron. Here then was at hand an unlimited source of these ephemeral creations and one no longer needed to rely upon the random gifts from cosmic radiation.[3]

### 3. *The Significance of the Positron.*

The astonishing and profound significance of the positron was soon discovered when the two primary facts about it were subjected to careful scrutiny: (1) its phenomenally short life, and (2) the coincidence of its track with that of an electron.

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[2] C. D. Anderson: "The Positive Electron," *Phys. Review*, Vol. 43, 1933, pp. 491-494. See also *Science*, Vol. 76, 1932, pp. 238-239.

[3] *Comptes Rendus*, Vol. 198, 1934, p. 254.

Abundant evidence was soon available that *when the positron disappeared this annihilation was accompanied also by the disappearance of an electron. The two particles displayed a fatal affinity for one another which resulted in their almost instantaneous destruction.* But since, as we have seen in an earlier chapter, matter cannot disappear without a compensating quantity of energy reappearing to take its place, the annihilation of the positron-electron pair should result in the creation of one or more photons of equivalent energy. What this energy should be is readily ascertained from Einstein's equation: $E = Mc^2$. In the first section of this chapter we have shown that the mass of a "rest electron," that is to say, an electron of low velocity, is equivalent in energy to approximately half a megavolt. If we make use of the de Broglie formula described in Section 11 of Chapter 10 to compute the characteristic frequency associated with this energy, we find that the photon which results from the annihilation should have a wave length of the order of 0.024 of an Angstrom unit. This would place it in the region of hard gamma radiation. But since the electron disappears only by coalescence with a positron we should expect the creation of two photons of the indicated order of energy. However, the mechanism of the transformation is still unknown and it often happens that only one photon is generated by the annihilation. In this case the photon would have an energy equivalent to a wave length of half the former size, namely 0.012 of an Angstrom unit.

Experimental evidence had been available since 1930 that gamma radiation of the two wave lengths indicated above was more abundant in nuclear absorption phenomena than could be accounted for by the theories available at that time.[4] Hence a perfectly satisfactory explanation was now possible and its significance was revolutionary. The interpretation gave meaning to the mysterious negative energy solutions of Dirac which we have mentioned earlier in the book, the holes which could be filled up only by the disappearance of a bit of positive matter, the electron.[5] In a word *the ephemeral positron, combining with the*

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[4] C. Y. Chao: "The Absorption Coefficient of Hard $\gamma$-Rays," *Proc. of the Nat. Acad. of Science*, Vol. 16, 1930, p. 431. See also *Proc. of Royal Soc.*, Vol. 135 (A), 1932, pp. 206-213.

[5] See Section 6, Chapter 8.

*stable electron, creates the elusive photon.* To P. M. S. Blackett and G. P. S. Occhialini belongs the credit for first suggesting the theory that the annihilation of the positron-electron pair was responsible for the anomalous abundance of gamma radiation of the two wave lengths given above. Thus they write:[6]

"On Dirac's theory the positive electrons should only have a short life, since it is easy for a negative electron to jump down into an unoccupied state, so filling up a hole and leading to the simultaneous annihilation of a positive and negative electron, the energy being radiated as two quanta. . . .

"It is not unlikely that positive electrons may be produced otherwise than in association with the penetrating radiation. Perhaps the anomalous absorption of gamma radiation by heavy nuclei may be connected with the formation of positive electrons and the re-emitted radiation with their disappearance. The re-emitted radiation is, in fact, found experimentally to have an energy of the same order as that to be expected for the annihilation spectrum."

The question of the number of photons emitted by this process of material annihilation was one of immediate interest, for sometimes apparently two appeared and at others only one. The answer was found through a consideration of the principle of the conservation of momentum. For a simple calculation shows that two photons must result when the positron-electron coalescence occurs in a space otherwise free of other particles; but that when a third particle is present, especially a massive atomic nucleus, then the principle of conservation can be maintained only if a single photon is created.

Since in this manner a glimpse of half the mechanism of the energy-matter transformation has been revealed, the other half now challenges attention. Is the process of annihilation a reversible one? Or is all matter in a process of dissolution which will end ultimately in a wholly radiant universe? The evidence of stellar evolution, which we shall discuss in more detail in Chapter 13, controverts this proposition. New stars are being created from cosmic dust, if the evidence has been properly interpreted, and the cosmic dust in its turn must be created from the radiant energy of the stars that are slowly dissolving into space. If this should prove to be the mechanism of the universe, then one should expect to find evidence in some phenomenon for the annihilation of photons and the creation of positron-electron particles to replace the vanished energy.

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[6] "Some Photographs of the Tracks of Penetrating Radiation," *Proc. of Royal Soc.,* Vol. 139 (A), pp. 699-726.

The first evidence for this transformation was discovered when it was observed that both positrons and electrons were ejected from some of the heavy elements when the gamma rays from thorium-C″ fell upon these metals.[7] When the energy of the gamma photon was measured it was found to be of the order of 2.62 megavolts and the kinetic energy of the positron-electron pair of the order of 1.6 megavolts. The difference between these two values, namely about 1.02 megavolts, was almost exactly the energy-equivalent of the masses of the two particles. Many extensive experiments on this problem have tended to confirm the conclusion that photons, under certain conditions, can be converted into particles. But it must also be observed, that this existence of the created matter has been of the most ephemeral character. For the very short life of the positron has resulted in an almost instantaneous return to its prior state of the newly-created matter.

Contemplating the universe of stars, and attempting to formulate a systematic theory of their evolution, astronomers have found an impelling necessity to postulate the creation of new matter. The most logical origin for this transmutation of energy into matter appears to be in the almost empty regions between the galaxies. In such regions, where the temperature would approximate absolute zero, we have a laboratory in which entropy has nearly vanished. An engine, working between the great temperatures found in the interior of stars, and the zero-temperature of intergalactic space, would have perfect efficiency. It is an attractive idea, subject neither to experimental proof nor to rejection on the basis of philosophical argument, that the universe, which never attains its heat-death, maintains its energy on the one hand by the annihilation of matter in the interior of stars, and recovers its material existence by the reconversion of photons into stable matter in the empty regions of space.[8]

### 4. *Discovery of the Neutron.*

Contemporaneous with the investigation of the positron and the discovery of its properties, studies were directed to another series of remarkable phenomena. These resulted finally in the discovery in 1932 of a new and perhaps an even more important particle. This unknown member of the enlarging atomic family was the neutron, the identification of which was ultimately to prove to be one of the most portentious events in human history. For the neutron was the key which, after a quest of more than fifteen centuries, finally unlocked the door to the mysteries of the transmutation of atoms. The explanation of the phenomena which led to the discovery of this remarkable particle was made by the British physicist James Chadwick in such a convincing manner that he was awarded the Nobel prize in 1935.

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[7] For the experimental evidence see C. D. Anderson and S. H. Nedermeyer: "Positrons from Gamma-Rays," *Phys. Review*, Vol. 33, 1933, p. 1034.

[8] For a recent review of the evidence for a continually rejuvenating universe and the relationship of the problem of the creation of new matter in an expanding universe, the reader is referred to *Cosmology*, by H. Bondi, Cambridge University, 1952.

The *neutron* is a particle without a charge, which has a mass approximately equal to, but slightly larger than, that of the proton. The ratio of its mass to that of the proton is about 1.009. Its size is probably about that of the proton, and it behaves in many respects as if it were a proton whose charge had been neutralized by combination with an electron.

The neutron was anticipated by Rutherford as early as 1920 when he considered the possibility of such a particle existing in nature. Thus in his Bakerian lecture of that date he said: [9]

"Under some conditions, however, it may be possible for an electron to combine much more closely with the H nucleus, forming a kind of neutral doublet. Such an atom would have very novel properties. Its external field would be practically zero, except very close to the nucleus, and in consequence it should be able to move more freely through matter. Its presence would probably be difficult to detect by the spectroscope, and it may be impossible to contain it in a sealed vessel. On the other hand, it should enter readily the structure of atoms, and may even unite with the nucleus or be disintegrated by its intense field, resulting probably in the escape of a charged H atom or an electron or both."

As early as 1930 evidence was presented which pointed toward the existence of an unknown particle. W. Bothe and H. Becker in a study of the radioactivity of polonium found that when alpha rays from this substance fell upon certain light elements a very penetrating radiation was generated.[10] Since the penetration was much greater than that previously observed for other particles, exceeding even that of the hard gamma rays of thorium-C″, it was natural to assume that a new type of photon was being observed with an exceedingly short wave length. Moreover all attempts to form ionizing tracks in a cloud chamber failed.

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[9] "Nuclear Constitution of Atoms," *Proc. of Royal Soc.*, Vol. 97 (A), 1920, pp. 374-400.

On this assumption the mechanism of the bombardment appeared to be this. The nucleus of beryllium absorbed an alpha particle and thus became carbon, ejecting a photon of hard radiation in the process. Computation of the mass-energy transformation showed that the photon should have an energy of 10.5 megavolts, which was a magnitude of the proper order since the observed energy was between the limits of 7 and 15 megavolts.

But difficulties soon began to accumulate around the numerical estimates of the energy possessed by the new radiation. When the supposed photon was allowed to pass through various kinds of material before entering an ionization chamber, unexpected results were observed. If the material was rich in hydrogen molecules, then the ionization observed was greatly increased. This increase was traced to the energies of recoil protons, which had been ejected from the hydrogen. So great was this ionization that energies as large as 50 megavolts had to be attributed to the original radiation.

Here, indeed, was a mystery, but it was intensified by observing that when other recoil nuclei were used instead of hydrogen, very different estimates of the energy of the new radiation were obtained. When the effect of nitrogen nuclei was measured, the estimate of the original energy of the radiation was increased to 90 megavolts. In other words, *the energy of the new photon appeared to depend upon the energies of the recoil nuclei,* and varied from one substance to another.

In order to extricate themselves from such an impossible situation, the physicists went to unusual lengths. Faced by a mystery in which the new photon failed to fit into the traditional pattern, and stirred doubtless by the radical theories that had developed in the preceding decade to explain the activities of radiation, these physicists suggested that the long-cherished laws of the conservation of energy and the conservation of momentum might be abandoned. The very fact that such a radical suggestion met with some support shows how deeply stirred was the scientific world by these strange and mysterious phenomena.

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[10] *Zeitschrift für Physik,* Vol. 66, 1930, p. 289; and *Naturwiss.,* Vol. 19, 1931, p. 753.

But very fortunately for science, it was not necessary to abandon at this time these fundamental laws, for the difficulties were finally resolved in a very satisfactory manner by Chadwick. Reflecting upon the inconsistencies of the estimates which we have described above, Chadwick abandoned the assumption that the new radiation was a photon of high penetrating power and replaced it by another. All the phenomena could be accounted for *if the ray was an uncharged particle*, that is to say, a neutron. By means of the very laws of the conservation of energy and the conservation of momentum, whose abandonment had been contemplated, the energy of the neutron, as estimated from the energies of recoil nuclei, was readily and consistently computed. Chadwick showed that if the origin of the neutral particle was beryllium and if its energy was obtained from the observed ionizing effect of the hydrogen nucleus, then the energy of the neutron should be of the order of 4.8 megavolts. Similarly, if the same source was used, but if the energy of the neutron was computed from the recoil effects of collision with nitrogen nuclei, then the energy of the neutral particle was of the order of 5.7 megavolts. The difference between the estimates was merely one of statistical error, a conclusion that was immediately verified by other investigators.

### 5. *New Aspects of the Nucleus.*

With the discovery of the neutron we are now in possession of one of the most closely guarded secrets of the internal structure of atoms. For now we see that the building-blocks of matter consist of at least three fundamental particles: (1) the mobile electron with its negative charge; (2) the heavy proton with its positive charge; (3) the heavy neutron without a charge.

It now became apparent that the classification of the elements which had been used by the chemists from the beginning of their science was one based upon the number of protons in the nuclei of the atoms. Hitherto it had been assumed that it was based upon the excess of protons over electrons. Since this excess charge was balanced by the shells of electrons which we have discussed earlier in connection with Bohr's model of the atom, it was customary to think of the elements as characterized by the number of electrons in these enveloping areas. However, it was clear that the nuclear charge was the more fundamental unit since the chemical characteristics of the atom were unchanged when one or more of the electrons were stripped from the shells in the process of ionization.

The number of units in this excess charge of the nucleus, or if one prefers, the number of electrons in the outer shells of the non-ionized atom, is called the *atomic number* of the element. Since hydrogen has a single electron, helium two electrons, iron 26, lead 82, uranium 92, these integers are the respective atomic numbers of hydrogen, helium, iron, etc. Hence the chemists have said until the recent discovery of transuranic elements, that there have been 92 fundamentally different kinds of matter, the so-called chemical elements; in the long history of investigation of chemical compounds they have never found any reason to modify this classification. Any substance with an atomic number of 82 will act like lead in every chemical reaction, since chemical reactions depend entirely upon the properties of the electrons about the nucleus. In this modern period of science we might almost say that chemistry is the science of electronic affinities in order to distinguish it from physics, which is concerned in a very essential manner with the mass properties of the elements.

It is possible, however, to make a new classification of the elements in terms of their *atomic weights*. For convenience, oxygen of atomic number 8 has been assigned arbitrarily an atomic weight of 16. The arbitrariness of this is found in the fact that 16 is an integer; the physical reason for the choice is indicated by the observation that a cubic centimeter of oxygen weighs approximately, but not exactly, sixteen times as much as a cubic centimeter of hydrogen. If, now, we assume that the primary elements are actually constructed from the lightest element hydrogen by the addition of particles of sensibly equivalent masses, then we should expect to find between hydrogen of atomic weight unity and uranium of atomic weight 238 a large number of undiscovered new elements. Subtracting 92, the atomic number of uranium, from 238, its atomic weight, we find that there are at least 146 of these unknown elements. Find them, says the physicist to the chemist. But the latter replies that this is impossible since there are only 92 electrons available in the structure, and the science of chemistry depends entirely upon their configurations.

Another curious aspect of the problem was also apparent and had been a matter of frequent conjecture. The atomic weights of the chemical elements were not all integers. Thus, with the assignment of an atomic weight of 16 to oxygen, the atomic weight of hydrogen turned out to be the fraction 1.008 instead of unity. Chlorine, one of the most anomalous of the elements, had an atomic weight of 35.45 instead of 35. Most of the elements, however, had atomic weights sufficiently close to integers to give high plausibility to the "building-stone" theory of their structure. This observation was originally made by William Prout in 1815, as we have mentioned earlier, and the mystery of the almost integral values of the atomic weights had stimulated much careful measurment and an abundance of speculation since his first formation of the hypothesis.

The simplest way to explain the mystery of the atomic weights is to assume that a certain number of neutrons have been added to the electron-proton combination which determines the atomic number. For example, the element sodium has an atomic number of eleven. It is thus the eleventh chemical element and it contains eleven electrons in the region about its nucleus. The negative charge is neutralized by the presence of eleven protons within the nucleus itself. But these eleven protons are not sufficient to account for the atomic weight of 23, and the weight of the electrons is insignificant since 1840 would be required to make up the mass of a single proton. Hence we must assume within the nucleus the presence of twelve neutrons which contribute nothing to the chemical characteristics of sodium and nothing to the electrical charge of the element. However, they do add twelve units of mass to the atomic weight. There remains still a slight discrepancy, although this is very small, since the atomic weight of sodium is actually 23.05.

Contemplating this interesting situation, physicists and chemists began to suspect that elements indistinguishable chemically from one another might actually possess different masses by reason of an excess or deficiency in neutrons. There might exist, for example, a heavy and a light chlorine in nature and the relative abundance of these separate atoms might account for the anomalous value of the atomic weight. For example, if 75 per cent of natural chlorine were actually an element with an atomic weight of 35 and the remaining 25 per cent consisted of an element with an atomic weight of 37, then natural chlorine would have a composite weight of 35.5, which is approximately the one observed.

This logical deduction, an inevitable consequence of the building block theory of the elements, proved to be correct. The two chlorines were given the name *isotopes.* This word, derived from the Greek signifying "in the same place," indicated that the new elements occupied a unique position in the periodic table so far as their chemical attributes were concerned, but differed from one another with respect to their atomic weights.

### 6. *The Discovery of Isotopes.*

Many years before the discovery of the neutron as a fundamental unit in the structure of the nucleus, there had been accumulating abundant evidence for the existence of isotopes. As early as 1910 F. Soddy in connection with an investigation of radioactive elements made the following prophetic statement:[11]

"The recognition that elements of different atomic weights may possess identical properties seems destined to have its most important application in the region of inactive elements, where the absence of a second radioactive nature makes it impossible for chemical identity to be individually detected. Chemical homogeneity is no longer a guarantee that any supposed element is not a mixture of several of different atomic weights, or that any atomic weight is not merely a mean number. The consistency of atomic weight, whatever the source of material, is not a proof of homogeneity."

Vindication of the position taken by Soddy was soon forthcoming in a brilliant discovery made by J. J. Thomson in 1913 in the course of a study of positive rays.[12] Deflecting the rays by means of electric and magnetic fields, Thomson obtained parabolic tracks of the constituent particles which depended upon the ratio $m/e$, where $m$ was the mass and $e$ the electric charge of the particle. In examining the tracks of neon (atomic weight 20.2) he found unmistakable evidence for the existence of two components, one with an atomic weight of 20 and another with an atomic weight of 22.

The epoch-making discovery of Thomson attracted immediate interest, since the age-old problem of the multiplicity of the elements had been put into a new light and the postulate of their essential stability had been challenged. The new devices, capable of diverse applications, were soon invented to provide conclusive and abundant evidence for the existence of elements chemically indistinguishable from one another, but with different atomic weights. One of these devices was invented by A. J. Dempster (1886-1950)[13] and the other by F. W. Aston[14], both of whom contributed fundamentally to the new theory of the elements.

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[11] *Annual Report of the Chemical Soc.*, 1910, p. 285.

[12] "Rays of Positive Electricity," *Proc. of Royal Soc.*, Vol. 89 (A), 1913, pp. 1-20.

The method employed by Dempster was similar to that used earlier in the determination of the ratio $e/m$ for the electron. A variable potential was employed to accelerate the particles. After they had attained a relatively uniform velocity, their paths were deflected by means of a fixed magnetic field. Any variation in mass then becomes conspicuous from the observed deviations in these paths. On the other hand, the apparatus designed by Aston made use of a variation in velocity to detect the differences in mass. By means of an ingenious arrangement of electric and magnetic fields, he was able to separate particles of slow velocity from those of high velocity, and thus detect the variation in their masses from this separation. Because these two instruments are designed to form a spectrum of mass variation among a group of particles, they have been referred to as *mass spectrographs.*

With the discovery of the isotopes it has now become necessary to invent a new symbol for the elements which will take account of this enlarged view of them. The old designation of 92 classes is no longer sufficient. Hence there has come into general use the symbol

$$_Z E^A$$

where $E$ is the name of the element, $Z$ is the atomic number, which is equal to the positive charge of the nucleus, and $A$ is the atomic weight. Thus if $E$ is chlorine, we would now write $_{17}\mathrm{Cl}^{35}$ and $_{17}\mathrm{Cl}^{37}$, where the subscript 17 refers to the positive charge of the nucleus and hence counts also the planetary electrons of the atom, and where the superscripts refer to the atomic weights 35 and 37 respectively. These indicate that the nucleus consists of 17 protons and 18 or 20 neutrons in the two cases.

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[13] "A New Method of Positive Ray Analysis," *Physical Review*, Vol. 11, 1918, pp. 316-325.

[14] *Proc. Cambridge Phil. Soc.*, Vol. 19, 1919, p. 317; "A Positive Ray Spectrograph," *Phil. Mag.*, Vol. 38, 1919, pp. 707-714.

Since all quantities with the same value of $Z$ are indistinguishable chemically from one another they have been called isotopes. But it is clear that the possibility exists that different chemical elements might have the same atomic weights. That is to say, they might differ chemically from one another but be indistinguishable as far as measurements by the mass spectrograph is concerned. That such, indeed, is the case is seen from the existence of such quantities as the following:

$$_{18}A^{40},\ _{19}K^{40},\ _{20}Ca^{40};\quad _{78}Pt^{196},\ _{80}Hg^{196};\quad _{82}U^{239},\ _{93}Np^{239},\ _{94}Pu^{239}.$$

The first three quantities behave chemically like argon, potassium, and cadmium respectively, but their atomic weights are identical. Similarly the next two are the chemical elements platinum and mercury, but no difference between them can be determined by the mass spectrograph. Of the last three elements uranium is the first and it is followed by the two newly discovered elements, neptunium, and plutonium. These quantities can be separated from one another by chemical means, but as far as their mass is concerned they are identical. Such elements which differ in atomic number, but which have the same atomic weight are called *isobars.*

Here at last we begin to find a fundamental difference in the hitherto stable and unchanging characteristics of the elements. Several hundred isotopes were rapidly identified of which less than three per cent were found to be radio-active. But with the rapidly accumulating knowledge of the new substances from a wealth of experiments in artificial radio-activity, there seems to be at the present time little doubt that this list can be almost indefinitely extended.

### 7. *The Packing Fractions.*

But the mystery of fractional atomic weights was not resolved completely by the discovery of the isotopes, although in most cases the alchemist's dream of creating the elements from fundamental building stones was practically realized. The enigma of hydrogen with its atomic weight of 1.008 was not solved by the discovery of a rare isotope with an atomic weight of 2.015.

With increasing accuracy in the measurement of atomic weights both of the natural elements and of their isotopes separately, it soon became evident that an unexplained residual of weight remained.

In 1927 Aston suggested that this small variation in weight from the expected integral value might be occasioned by the binding energy within the nucleus itself.[15] For example, if the helium nucleus consists of two protons and two neutrons in close association, then the weight should be less than four times that of the hydrogen nucleus which consists of a single proton, since part of the mass of the helium might conceivably be absorbed into the "packing" energy. To measure this Aston then introduced the concept of the *packing fraction*, which he defined to be the ratio of the excess or deficiency of the atomic weight of the element from its characteristic integer with respect to the characteristic integer itself. Thus for hydrogen with an atomic weight of 1.008 and an atomic number one, the packing fraction was $+ 0.008$. Similarly for chlorine-35, by which we mean the chlorine isotope of atomic number 35, with its atomic weight of 34.9803, the packing fraction is the difference: $34.9803 - 35 = -0.0197$, divided by 35. This yields the value -0.0006. Obviously the packing fraction for oxygen was zero, since the origin of the system of atomic weights assumed a value of 16 for this element.

If we denote the atomic weight of an atom by $W$, its characteristic integer by $A$, and its packing fraction by $f$, then clearly we shall have the following relationship between these quantities:

$$W = A + fA = A(1+f).$$

Thus for chlorine-35 we have the numerical identity: $34.9803 = 35(1 - 0.0006)$.

The significance of this concept of the packing energy may be illustrated by an example. Suppose that a process were known by means of which an oxygen atom could be combined with a fluorine atom to form chlorine, the characteristic integers of which are respectively 16, 19 and 35, that is, $16 + 19 = 35$. But the atomic weights are respectively 16.0000, 19.0045, and 34.9803. The difference between the sum: $16.0000 + 19.0045 = 35.0045$ and the number 34.9803 equals 0.0242, which is the excess atomic weight that must be accounted for in the theoretical transmutation which we are considering. If we designate respectively the characteristic atomic numbers by $A_1$, $A_2$, $A_3$, the atomic weights by $W_1$, $W_2$, $W_3$ and the packing fractions by $f_1$, $f_2$, $f_3$, we can express the difference in atomic weights of the transformed oxygen and fluorine and of the resulting chlorine by the following equation:

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[15] "A New Mass-Spectrograph and the Whole Number Rule," *Proc. of the Royal Soc.*, Vol. 115, A, 1927, pp. 487-514.

$$W_1 + W_2 - W_2 = A_1(1 + f_1) + A_2(1 + f_2) - A_3(1 + f_3)$$
$$= A_1f_1 + A_2f_2 - A_3f_3 = 0.0242.$$

Now the excess mass represented by this number would presumably disappear into energy of an amount equal to 0.0242 $c^2$, where $c$ is the velocity of light. To represent this in the unit of megavolts which we have introduced in the first section of this chapter, it is sufficient to multiply 0.0242 by 931, which is the energy equivalent in megavolts of the rest-mass of the proton. We thus obtain the very considerable energy per atom transformed of 22.5 megavolts. In other words, if we could create an atom of chlorine by combining an atom of oxygen with an atom of fluorine, the transmutation thus effected would yield a return of 22.5 megavolts of energy. The correctness of this view will be shown later by means of certain remarkable experiments. It is sufficient for our present purpose to indicate the great importance of the concept of the "packing energy" of the nucleus as revealed by the variation of the atomic weight from its characteristic atomic integer.

Aston, in the development of his theory, determined in 1927, with the greatest accuracy provided by the instruments available to him, the values of the atomic weights and hence the packing fractions of a number of the elements. From these values he determined a curve in which the packing fraction was plotted against the mass numbers of the elements. While the curve thus obtained is obviously not to be regarded as a continuous one since the quantities on the lower scale are integers, there is conclusive evidence of a general relationship between the packing fraction and the mass, or characteristic, numbers of the elements. More recent data, obtained with greater precision than that possible for Aston, demonstrated the essential correctness of his views.

In the first of the accompanying figures (See Figure 3 ) the packing fraction is shown for the elements with atomic numbers up to and including 40 where the most recent data is used. The most interesting aspect of this curve is found in the minimum divergence of the atoms with masses which are some multiple of four.

Such atoms may actually consist of an integral number of helium atoms and the fact that the packing fraction is so small suggests that such atoms may be unusually stable ones. As one goes beyond $A = 20$ this effect tends to disappear.

Another unusual aspect of the packing fraction is observed from both Figures 3 and 4, namely that from $A = 20$ to $A = 165$ (approximately) the packing fraction is negative, while before $A = 20$ and after $A = 165$ it is positive. The significance of this is found in the observation that if a heavy element could be broken into two lighter ones, then a residual amount of energy would be released in the transmutation. On the other hand, at the lower end of the periodic table energy would be generated only if a heavier element were created out of lighter ones. For example, there would be an energy release if the mercury isotope of atomic mass number 200 (Hg-200) could be disintegrated into the isotope of tin of mass number 120 (Sn-120) and the isotope of krypton of mass number 80 (Kr-80). We have just seen from the numerical example given earlier that the theoretical creation of an atom of chlorine (C1-35) out of oxygen (0-16) and fluorine (Fl-19) would generate energy of the order of 22.5 megavolts. If we reverse these processes and attempt by modern alchemy to create mercury out of tin and krypton, this might be accomplished only by a large input of energy. The same is true if we should try to disintegrate chlorine into simpler parts.

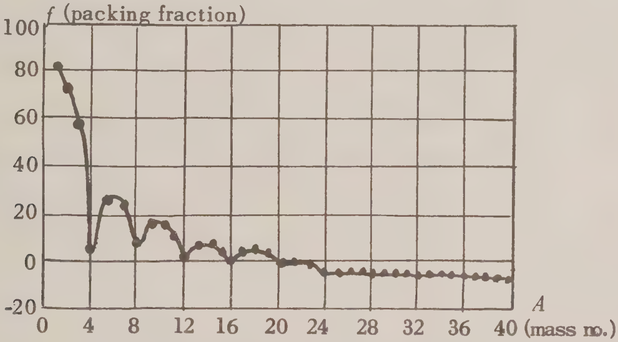

Figure 3. This diagram shows the values of the packing fraction: $f = (\frac{W}{A} - 1) \times 10^4$, where $W$ is the atomic weight and $A$ the mass number, for the light elements.

Still another interesting aspect of the Aston theory of the packing fraction is found in the scale itself, which is obviously determined by the arbitrary choice of the mass number of oxygen as $A = 16.0000$. A slight discrepancy enters the picture, however, since oxygen consists of three isotopes of mass numbers 16, 17, and 18 respectively. It is fortunate, indeed, that the first of these comprises 99.76 per cent of the natural element and hence the choice of 16 as the base of the system of atomic numbers creates only a slight variation between the physical and chemical scales of the atomic weights. If account is taken of the two heavier isotopes in their proportions of 0.04 and 0.20 of one per cent respectively, then natural oxygen as compared with the isotope of atomic number 16 has an atomic weight of 16.0044. In most cases this slight variation from 16 is not important, but in energy transformations of the type which we have been considering it attains more significance since the residual weights are multiplied by the factor $c^2$, where $c$ is the velocity of light. From this example we see the shifting sands upon which absolute values in the physical sciences are based. The invariables in one generation become the variables of the next.

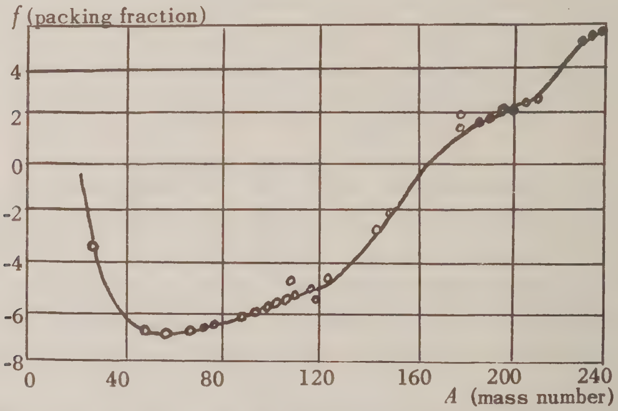

Figure 4. Values of the packing fraction, *f*, for the heavy elements according to Dempster.

Some idea of the magnitude of the output of energy involved in these transmutations of the elements can be obtained from a simple computation. We have seen above that the creation of an atom of chlorine from atoms of oxygen and fluorine would release 22.5 megavolts of energy. Since there are about $1.72 \times 10^{22}$ atoms of chlorine in a gram, this would mean that the creation of a single gram of chlorine out of the lighter elements would generate $3.87 \times 10^{23}$ megavolts of energy. But since $2.25 \times 10^{19}$ megavolts are equal to a kilowatt hour of energy, we see that the creation of a gram of chlorine by the processes of modern alchemy would release $1.72 \times 10^{4}$ kilowatt hours. If we compare this with the total energy output of the electrical machinery of the United States in the year 1950 ($33 \times 10^{10}$ kilowatt hours), we reach the startling conclusion that the creation of about 21 tons of chlorine out of the lower elements would yield energy equivalent to this vast national production. Hence we see that the great problem now before the world of science is to devise some means by which it can unlock this tremendous treasure of the atomic nucleus.

## 8. *Deuterium, Tritium, and Other Isotopes.*

As soon as the astonishing fact had been established that almost all of the natural elements consisted of two or more isotopes, there was a search for methods by means of which these new physical elements might be separated from one another. Since the isotopes of a single natural element were chemically indistinguishable from one another the methods to be employed in the separation were necessarily those of physics rather than those of chemistry.

One of the most important of these methods is that of the mass spectograph itself, which, as we have said earlier, was used in proving the existence of the isotopes in the first place. Since the device segregates atoms of different atomic weights by bending their paths into circles of varying curvature through the use of electric and magnetic fields, this method is sometimes referred to as that of electromagnetic separation.

A second method which has proved successful in the separation of isotopes is that of thermal diffusion, a method introduced by H. Clusius and G. Dickel. This depends upon the observation that if a gas or liquid consisting of two or more isotopes is placed between two vertical parallel walls one of which is kept hot and the other cold, in general the heavier isotope will tend to concentrate near the cold wall and the lighter isotope near the hot wall. In practice the apparatus actually consisted of a chamber with cold walls and a tube through the center which was kept hot, thereby establishing a temperature gradiant. Convection currents set up by the gradiant carried the lighter isotope inward and upward, while the heavier isotope moved outward and toward the bottom of the apparatus. Successful separation of chlorine isotopes and of the light and heavy uranium hexafluorides has been effected by this means.

A third method of separation of isotopes is that of the diffusion of gas into an evacuated space through porous membranes. The rate of this diffusion depends upon the absolute molecular velocity of the gas and this is inversely proportional to the square root of the molecular weight of the atom. Hence if a gas which consisted of two isotopes, let us say, chlorine 35 and 37, were allowed to diffuse in the manner described there would be a concentration of the lighter isotope on one side of the barrier and a concentration of the heavier isotope on the other. It is clear that the heavier the gas the smaller would be the residual effect, since in the case of uranium 235 and 238 the ratio of the square roots would be 1.0064, whereas in the case of chlorine the ratio is 1.0282. These fractions are both small and hence a great many passages through the barrier would be required to enrich the gas in one or the other of the two isotopes. Nevertheless this was one of the methods used conspicuously in the separation of uranium-235 from the natural metal in the construction of the atomic bomb.

Fractional distillation is another method that has been successfully employed in separating isotopes. The rate of evaporation from a liquid surface is inversely proportional to the square root of the mass of the atom. Hence if a liquid containing two isotopes were evaporated, the part vaporized would contain more of the lighter isotope than the residual liquid which would consequently be richer in the heavier isotope.

A fifth method depends upon the use of centrifugal forces, the familiar centrifuge separator which is well known in the removal of cream from milk. The gas or liquid is rotated rapidly and the lighter material tends to accumulate near the center of the centrifuge, while the heavier moves toward the peripheral surface. The factor of separation depends upon the ratio of the masses of the two substances. In general, because of technical difficulties in obtaining sufficiently high speeds, this method has not been very successful as a device for the separation of isotopes.

Several other methods have been used which for the most part have very special applications. One of these, separation by chemical exchange reaction, depends upon the fact that in some chemical transformations one isotope will tend to concentrate in the gaseous phase of the compound while another will tend to concentrate in the liquid phase. Photochemical means have also been employed and also electrolysis, the latter, in particular, being particularly effective in the separation of the isotopes of hydrogen.

Among the spectacular discoveries in this field, especially in its appeal to the general public, was that of heavy water, or in more scientific terms, the discovery of the hydrogen isotope $H^2$. The existence of heavy hydrogen could scarcely have been suspected from the agreement of the physically and chemically determined atomic weights of this element. But with the discovery of two heavy isotopes of oxygen, which, as we have indicated above, introduced a small discrepancy when the atomic weight of oxygen was assumed to be exactly 16, the apparent agreement failed. It was then belived that this discrepancy must be accounted for by the existence of one or more isotopes of hydrogen. A systematic study of the problem by H. C. Urey, F. G. Brickwedde, and G. M. Murphy[16] resulted in the discovery of heavy hydrogen, $H^2$, with an atomic weight of 2.0147, nearly twice that of the primary isotope, $H^1$, with an atomic weight of 1.0081. The new isotope was naturally a very small part of natural hydrogen, subsequent investigation showing that its relative frequency was only 0.02 of one per cent. It is a matter of considerable historical interest to observe that the apparent agreement of the physically and chemically determined atomic weights of hydrogen, and the subsequent apparent discrepancy resulted from a small experimental error in the original work of Aston. When this was cleared up, lo! the two atomic weights again agreed as they did in the first place.

Heavy hydrogen has been given the name *deuterium* and its nucleus is called a *deuteron*. The great importance of the deuteron is found among other things in its existence as another particle available for nuclear bombardment. It is a particle intermediate between the nucleus of hydrogen and the nucleus of helium, which, as we have said before, was originally called an alpha particle from its appearance in the radio-activity of radium. A third radio-active isotope, $H^3$, has also been discovered to which the name *tritium* has been given. It is very rare and only a trace is found in ordinary water.

*Heavy water* is the name given to water formed from the abundant isotope of oxygen, namely $0^{16}$, and deuterium. Since there are three stable isotopes of oxygen and two of hydrogen, and since water consists of two atoms of hydrogen and one of oxygen, it is clear that nine different heavy waters might exist, eight of which will exceed the weight of ordinary water. The specific gravity of heavy water at $25^0$ centigrade relative to ordinary water is 1.1079 and there are significant differences between the two waters in most of their physical characteristics.

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[16] *Physical Review*, Vol. 40, 1932, p. 1.

In nature heavy water occurs only in the ratio of two parts to 10,000, as we have indicated above, so the separation presented a problem of unusual difficulty. Fortunately, however, it was discovered that electrolytic cells used in the separation of oxygen and hydrogen contained relatively high concentration of heavy hydrogen. Thus the process of electrolysis was employed successfully in creating almost pure heavy water in considerable abundance.

### 9. *The Meson.*

As we have already described in a preceding section, C. D. Anderson in 1932 while investigating the phenomenon of cosmic radiation examined certain cloud tracks which appeared to have a common origin and hence discovered the positron. Just four years after the announcement of this important find, Anderson in collaboration with S. H. Neddermeyer again described another particle which appeared to have some of the properties of an electron, but was definitely something else.[17]

Although the new particle appeared to carry a negative charge of electricity, it was very penetrating, but did not have the same ionizing power of the electron. The inference was that the new particle, while possessing an electric charge equal to that of the electron, had a much greater mass, something, in fact, of the order 200 times. This, however, would still leave it about nine times lighter than the proton.[18]

As soon as sufficient evidence had accumulated to verify these remarkable discoveries, a name was sought for the new particle. At first it was called the heavy electron. Then the name *baryton* was introduced, namely "the heavy particle" from the Greek [barus = heavy], because of its mass. This designation was soon replaced by the term *mesotron*, or the intermediate particle. The recent tendency has been to contract the name into *meson*.

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[17] *Science* Supplement, Vol. 84, 1936, p. 9.

[18] C. D. Anderson and S. H. Neddermeyer: *Physical Review*, Vol. 50, 1936, p. 270.

As we have already said the discovery of the meson came about through a study of the cloud-tracks of cosmic radiation. These tracks indicated an ionization greater than that which would be caused by a normal electron, but the range and the curvature of the paths showed also that the particles could not be normal protons. Hence the conclusion was reached that there existed an intermediate particle which was more massive than the electron, but less massive than the proton. Moreover some experimenters have presented evidence to show that the charge of the meson is positive, and others have shown that it is negative. Hence we are able to infer that there exists a particle with a charge which is equal in magnitude to that carried by the electron, but which may be of either sign.

It is very interesting to observe that a particle closely resembling the meson was predicted on the basis of theory a year before the meson was observed in the laboratory. This remarkable analysis was made by H. Yukawa, who was investigating the nature of the field of force which presumably existed in the atomic nucleus. In the course of this study, Yukawa found it necessary to introduce a new particle which had a mass intermediate between the electron and the proton. This particle was very unstable with a half life of the order of $0.25 \times 10^{-6}$ seconds.[19]

A great deal of uncertainty has prevailed as to the actual mass of the meson, estimates ranging from 100 to nearly 1,000 times the mass of the electron. On account of these divergent results it was suggested that there may be no characteristic mass for the meson. But a careful analysis of the data from many laboratories finally led to the conclusion that most of the energy was carried by two particles, now called respectively the mu-meson and the pi-meson. The first of these, which accounts for most of the cosmic ray phenomena observed in the lower atmosphere, has a mass equal approximately to $200m$, where $m$ is the mass of the electron, and has a life of the order of $2 \times 10^{-6}$ seconds. The second meson has a mass about equal to $300m$, and its life is less than $2 \times 10^{-9}$ seconds. There also exists some additional evidence to indicate that there may also be other particles with meson characteristics which have masses that vary from $700m$ to $1{,}000m$. The evidence here, however, is based upon the interpretation of a number of individual observations and has not yet received general acceptance.

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[19] H. Yukawa: "Interaction of Elementary Particles," *Proc. Phys. Math. Soc. of Japan*, Vol. 17, 1935, pp. 48-57.

The evidence available at the present time indicates that the meson is a radioactive particle. The first evidence for this came from a comparison of the times of absorption of mesons first in air and then in such substances as water, carbon, lead, etc. An equivalent layer of air absorbed the meson more rapidly than the other materials. The interpretation given this experiment was the logical one that, since the time of passage through the air is greater than the time of passage through the other absorbing media, more of the meson disintegrated in the air.

In a phenomenon so fragile there are difficulties in interpretation and the deductions which have been made may be materially changed as experimental evidence accumulates. Many attempts have been made to detect the particles which result from the disintegration of the meson. The best evidence at the present time appears to lead to the conclusion that an electron is one of the components of the disintegration of the mu-meson. Conjecture also affirms that another component may be an elusive particle to which has been given the name *neutrino*. This particle, if it exists, for the evidence which creates it is a theoretical argument based upon tenuous data, has no charge and a negligible mass. Direct evidence is wanting since its lack of charge precludes the possibility of ionization tracks. With this note of mystery and conjecture we shall leave the subject of the meson and await the future discoveries in this promising field to yield a more satisfactory picture.[20]

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[20] For extensive discussions of the present status of the problem of the meson the reader is referred to the following: (1) J. D. Stranathan: *The Particles of Modern Physics*, Philadelphia, 1942, Chapters 12 and 13; (2) D. J. X. Montgomery: *Cosmic Ray Physics*, Princeton, 1949, viii + 370 p.; (3) J. G. Wilson (Editor): *Progress in Cosmic Ray Physics*, Amsterdam and New York, 1952, xvi + 557 p.; (4) *Cosmic Radiation*, Colston Research Soc. and University of Brystol, London and New York, 1949, viii + 189 p.

# CHAPTER 12.

# THE PHILOSOPHER'S STONE.

"That stone, or like to that, which here below
Philosophers in vain so long have sought;
In vain, though by their powerful art they bind
Volatile Hermes, and call up unbound
In various shapes old Proteus from the sea."
Milton in *Paradise Lost,* Book 3.

"The changing of bodies into light, and light into bodies, is very comfortable to the course of Nature, which seems delighted with transmutations. Water, which is a very fluid, tasteless salt, she changes by heat into vapour, which is a sort of air; and by cold into ice, which is hard, pellucid, brittle, fusible stone; and this stone returns into water by heat, and vapour returns into water by cold . . . And among such various and strange transmutations why may not Nature change bodies into light, and light into bodies?"

Newton in *Opticks* (1704), Query 30.

## 1. *The Dream of the Alchemist.*

One of the first authentic accounts of man's long search for the secret of transmuting the elements is found in the commentaries of Synesius (c. 373-c. 414), friend of Hypatia of Alexandria and student of "the holy art of Egypt." The quest was continued by the Arabs after the fall of Alexandria. Gibbon in his *Decline and Fall of the Roman Empire* says:[1] "But the most eager search of Arabian chemistry was the transmutation of metals, and the elixir of immortal health: the reason and the fortunes of thousands were evaporated in the crucibles of alchemy, and the consummation of the great work was promoted by the worthy aid of mystery, fable, and superstition." The search for the philosopher's stone is thus an adventure which has endured for more than fifteen centuries, and has never been far outside the vision of physical science.

The first phase of the quest may be said to have ended on August 6, 1945 in what was undoubtedly the most dramatic, important, and terrible event in human history. On that day the city of Hiroshima, a naval base with a population in excess of 300,000 was wrecked by an atomic bomb. Exploding with a force approximately two million times as great as that of the most powerful explosive hitherto created by the arts of chemistry, this deadly weapon spewed a blast of energy 60,000 feet into the stratosphere and terminated abruptly the hostilities that had raged for nearly four years. This violent event had been preceded less than a month before by a test of the new weapon on the desertlands of New Mexico some 120 miles southeast of Albuquerque.

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[1] Chap. lii.

The philosophers had found their stone; the first dream of alchemy had been achieved. For the incredible energy of the atomic bomb was derived from a process by means of which heavy atoms were broken into lighter ones. It was not the transmutation so long sought for of mercury and lead into gold, the production of a precious metal from those more abundantly provided by nature. But rather it was derived from the separation into several fragments of the nucleus of uranium, the heaviest atom in the periodic table of the elements. And the product sought for by these modern alchemists was not the several particles into which uranium had been broken, but rather the vast storehouse of energy which had bound these parts together.

When the alchemists many centuries ago began their search for the ultimate nature of matter, the *prima materia* so called, employing the relatively feeble energies of chemical reactions in their explorations, little could they have foreseen in the wildest flights of their imagination the dramatic termination of their quest. A revolution in the medium of exchange, a temporary panic in the treasuries of the world, a beggaring of princes were the extreme consequences which could have followed from a successful consummation of their experiments. As Ben Jonson puts the matter in his satiric play *The Alchemist* (1610):

> "This night I'll change
> All that is metal in my house to gold:
> And early in the morning, will I send
> To all the plumbers and the pewterers
> And buy their tin and lead up."

The will-o'-the-wisp that led them on was the reasonable assumption:

> "Ay, for 'twere absurd
> To think that nature in the earth bred gold
> Perfect i' the instant: something went before.
> There must be remote matter."

Perhaps unfortunately for the development of science the alchemist was generally regarded as a knave, a dealer in black magic, and his art a fraud. "I'll believe," says one of the sceptics in Jonson's play, "that Alchemy is a pretty kind of game, somewhat like tricks o' cards, to cheat a man with charming." In spite of this, however, the fundamental postulate of alchemy, the possibility of transmuting the elements, was tentatively held by a number of leading scientists. The most famous alchemist of the early sixteenth century was Paracelsus (1493-1541), whose opposition to medieval scholasticism contributed significantly to the early progress of science. Even Sir Isaac Newton and G. W. Leibniz did not reject the possibility of transmutation and Robert Boyle (1627-1691), sometimes called the father of chemistry, was a firm believer in the ultimate success of alchemy.

The difference between these early alchemists and their modern successors, who finally vindicated the principle itself, is found in the power of the tools which they employed. The most efficient reaction of chemistry, such as that found in high explosives, has been able to yield no more than 1,500 calories of energy per gram of substance employed. Atomic fission, however, involves energies of the order of twenty billion calories per gram. It was not the dream of the alchemist that was at fault, but the incredible nature of the intrinsic energy of the atoms. The highest achievements of modern science, supported by a fund of two billion dollars, were necessary to complete the quest.

## 2. *High-Speed Particles.*

In tracing the story of atomic alchemy we must begin with a description of some of the methods which have been devised for attaining particles of high velocity. For we have seen in the original investigations of Rutherford that his success depended almost entirely upon the velocity of the particles which he used in the bombardment of different atoms. The most powerful of these were the alpha and beta particles derived from radium and other natural radioactive elements. But the highest energies that could be attained in this manner were those from thorium-C', which, although developing the very considerable energy of 8.8 megavolts, was very much below the demands of the later alchemists.

Several highly important devices have been invented to secure these high-speed particles. The first is the *cyclotron*, an instrument designed and constructed by E. O. Lawrence and first described in 1930.[2] The Nobel prize was awarded to Lawrence in 1939 for his notable invention. The second of these devices is called the *Van de Graaff accelerator* after its inventor, R. J. Van de Graaff, who described the machine in 1931.[3] The third invention was the *induction accelerator*, or as it is more commonly called, the *betatron*, which is designed primarily as an accelerator of electrons. This device was the invention of D. W. Kerst, who perfected it in 1942.[4]

The cyclotron consists of two semicircular hollow metal boxes, which are placed between the poles of a powerful electro-magnet so that a strong magnetic field exists perpendicular to their faces. An alternating potential is applied between the two boxes. Let us suppose that an ion is introduced at the point C (Figure 1). It is subject at once to a potential which drives it along a circular path because of the presence of the magnetic field. Now the fundamental principle upon which the cyclotron operates is found in the fact that the time required for the ion to describe a complete semicircle and return to the opening between the boxes *is independent of the radius of the semicircle.* Hence every ion, whatever path it is describing, will reach the opening at the same instant. It must be observed, however, that this is true only if the ions are of equal

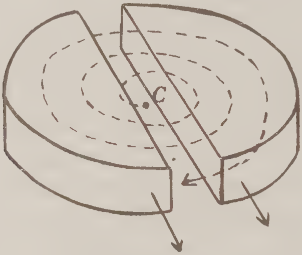

Figure 1. Schematic Diagram of the Path of a Particle in a Cyclotron.

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[2] *Science*, Vol. 72, 1930, p. 376.

[3] *Physical Review*, Vol. 38, 1931, pp. 1919 et seq.

[4] *Physical Review*, Vol. 61, 1942, p. 93.

mass and if the change of mass with velocity is negligible, since the time is a function of the mass. Under these assumptions it then follows that if the alternating field is properly adjusted so that the direction of the field changes at the moment the ions reach the opening between the boxes, they will be accelerated further. Each time of circuit, then, the ions describe wider and wider arcs until they reach the edge of the boxes where they pass through an exit slit in the form of a beam. The electric and magnetic fields act together in such a manner that the beam consists of ions of quite uniform energy.

This method of acceleration by the continued use of a relatively small potential is called the method of *resonance*, since by resonance we mean the phenomenon in which large vibrations are caused by the constant application of a relatively small force. Thus a large ship will sometimes roll heavily in a light sea when the period of the waves is equal to the natural period of the ship. It is to prevent damage to a bridge by resonance that a column of marching men is required to break step when crossing such a structure. Hence it is that by the constant application of the alternating potential the ions are steadily accelerated at each circuit until they reach their maximum velocity at the outer edge of the apparatus.

Since the effectiveness of the cyclotron in creating high-speed particles depends essentially upon the strength of the electro-magnet, this part of the device has been steadily enlarged. This unit is very massive and weighs in a recent cyclotron as much as 4,000 tons. Incredibly high energies have thus been imparted to the particles introduced into them. Because of the fundamental use of resonance in the cyclotron, this apparatus is not used in the acceleration of the electron. The reason for this neglect of the smallest of the particles is found in the fact that the time required for an ion to describe a semicircle and return to the opening between the boxes depends linearly upon the mass. Since high-speed electrons have a great variability in mass with velocity, it is clear that adequate resonance conditions would become impossible at high levels of energy. On the other hand the more massive positive particles such as the proton and the alpha ray acquire high energies at relatively low velocities and the troubles of maintaining resonance conditions are greatly reduced.

The Van de Graaff accelerator avoids the difficulties of resonance since it gives high velocity to particles by imposing a direct potential gradient upon them. This potential is developed on the principle of the electro-static machine. By means of a belt made of some flexible insulating material, charges of electricity are carried to two large metal spheres, negative charges being delivered to one and positive charges to the other. In this way, depending both upon the size of the spheres and their insulation, a very great potential difference is established between them. For example, the large machine constructed at Massachusetts Institute of Technology was able to develop a potential difference in excess of five million volts. The particles are acted upon by this great potential gradiant throughout the length of an accelerating tube, sometimes as long as 15 or 20 feet, which contains a source of ions at one end and the target at the other. It is obvious that many technical difficulties are present in the operation of a machine which maintains potential differences of so high an order. The Van de Graaff machine working at its highest efficiency could accelerate ions to an energy in excess of 45 megavolts.

Although modern alchemy has need principally for high-velocity positive particles, considerable attention has been given also to the acceleration of electrons. As we have stated elsewhere, these particles present difficulty in their highly variable mass. A few figures will illustrate the situation. When an electron has a velocity 0.9 times that of light it has a kinetic energy of about 0.66 of a megavolt; at $0.99c$, where $c$ is the velocity of light, its energy is 3.11 megavolts, but when its velocity is $0.999c$, the energy has increased to 10.78 megavolts. At the lower figure the mass of the electron is 2.29 times its rest mass; at the second figure the ratio has increased to 7.11 and at the upper figure to 22.4. This kinetic energy must not be confused with what is called the energy equivalent of an electron, namely the value $Mc^2$, where $M$ is the rest mass. This is about 0.509 megavolts. The values given above are computed from the formula:

$$\text{K.E.}_\circ = Mc^2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1\right],$$

which was introduced in Section 12 of Chapter 5 as the modern definition of kinetic energy.

A very effective device for accelerating electrons has been devised by D. W. Kerst as we have stated earlier in this section. This apparatus, called an induction accelerator, or betatron, induces an alternating magnetic field in the orbit of the electron. This creates a high electromagnetic force in the orbit and the electron is accelerated in consequence. Since the efficiency of the device is not impaired by the increasing mass of the electron, very high velocities have been imparted to these particles. Some of the energies thus obtained have been as great as 20 megavolts.

Equipped with these new and powerful instruments we are now ready for the great adventure, the transmutation of the elements. We are prepared to explore the mysteries of the nucleus wherein resides the problem of atomic transmutation, just as in the mysteries of the electron-shells about the nucleus reside the problem of chemical transformation. The adventure consists in bombarding different atoms with any one of the new particles, namely, photons, electrons, protons, neutrons, positrons, deuterons, and alpha particles.

### 3. *Recapitulation of the Older Alchemy.*

In a previous chapter we have indicated how the discovery of radio-activity stimulated the old dream of the alchemists, the transmutation of the elements. For radium, uranium, and other naturally radioactive substances were actually achieving a slow but ever-so-certain disintegration of the atoms which composed them.

The pioneers in the older alchemy were Sir Ernst Rutherford and his associates who began to explore the consequences of bombarding the elements with such high-speed particles as were then available to them. The first achievement in this field belongs unquestionably to Rutherford who in 1919 first succeeded in demonstrating artificial disintegration of matter. Using alpha particles from radium he studied the effect of their collision with the atoms of hydrogen, nitrogen, and oxygen. He observed that protons were knocked out of nitrogen by these heavy particles and hence nitrogen was in effect transmuted by the bombardment.

In the light of present knowledge it is now believed that what actually happened in the transformation was the addition of the alpha particle, namely ${}_2\mathrm{He}^4$, the helium nucleus, to ordinary nitrogen, namely ${}_7\mathrm{N}^{14}$, from which came heavy oxygen, ${}_8\mathrm{O}^{17}$, and a single proton, namely ${}_1\mathrm{H}^1$, the hydrogen nucleus. In modern nomenclature we can describe the transformation in the following symbols:

$${}_7\mathrm{N}^{14} + {}_2\mathrm{He}^4 \rightarrow {}_8\mathrm{O}^{17} + {}_1\mathrm{H}^1 .$$

The proton was observed to have a range longer than could be accounted for by the speed of the bombarding particles. The energy in fact was supplied to it by the trigger mechanism of the transformation.

A new phase of the adventure began with the creation of faster particles. One of the most significant results in these preliminary investigations was an experiment in 1932 in which J. D. Cockroft and E. T. S. Walton bombarded lithium with protons.[5] These particles appeared to trigger-off alpha particles, and the astonishing thing was that the alpha particles were observed to have energies of the order of 8.6 megavolts, whereas the original protons had energies which ranged from ten to seventy per cent of a megavolt. Here, then, was the first glimmering of a mechanism by means of which a large energy could be obtained for the expenditure of a small one.

It is important in what follows to understand the transformation which we have just described. Representing it by means of symbols which we have used before, we can write it in the following way:

$${}_3\mathrm{Li}^7 + {}_1\mathrm{H}^1 \rightarrow {}_2\mathrm{He}^4 + {}_2\mathrm{He}^4 .$$

In other words the addition of a hydrogen nucleus (a proton) to an atom of lithium resulted in the transmutation of lithium into two helium nuclei. But if we examine the actual transformation from the point of view of the atomic weights involved, we observe the following remarkable fact. The atomic weight of lithium (7.01818) plus the atomic weight of hydrogen (1.00813) is a total of 8.02631. On the other hand the atomic weight of the two helium nuclei is only 8.00778. *Hence there is a net loss of 0.0185 in atomic weight in the transformation.* If we replace the three masses involved, that of lithium ($M_1$), of hydrogen ($M_2$), and of helium ($M_3$), by their energy equivalents according to Einstein's equation, namely by $c^2 M_1$, $c^2 M_2$, and $c^2 M_3$ respectively, where $c$ is the velocity of light, then the energy generated by the transformation, which we represented by $\Delta E$, will be given by the equations:

$$\Delta E = c^2(M_1 + M_2 - M_3).$$

But since we are concerned with atomic energies, this expression can be replaced by the following formula:

$$\Delta E = c^2 m(W_1 + W_2 - W_3),$$

where $m$ is the mass of the hydrogen nucleus and $W_1$, $W_2$, and $W_3$ are respectively the atomic weights of lithium, hydrogen, and the two helium nuclei. A simple calculation shows that $c^2m$, the rest energy of the hydrogen nucleus, is approximately equal to 931 megavolts.

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[5] *Proceedings of the Royal Society*, Vol. 137 (A), p. 229.

Hence, in the computation of nuclear transformations of the kind which we have been describing, we can use the formula

$$\Delta E = 931(W_1 + W_2 - W_3),$$

where the energy is expressed in megavolts.

Returning to the original transmutation, we now see that the energy released by the disintegration of lithium is equal to $\Delta E = 931 \times 0.0185 = 17.2$ megavolts. Since this energy is divided equally between the two helium nuclei it is now apparent why the alpha particles of such astonishing energy (8.6 megavolts) were observed as concomitants of the transformation.

The acquiring of large energies from minute quantities of matter focused renewed attention upon the accuracy of the atomic weights. Although there had been an increasing desire to "push back these decimal points," to develop techniques by means of which higher accuracy could be attained in the determination of the atomic weights, this problem now became one of vital significance. We have already described in the preceding chapter how the classical work of F. W. Aston resulted in the determination of atomic weights with errors that had been pushed back as far as the fourth decimal place. These studies now assumed increasing importance in the quest of the alchemists.

Since the real key to the release of energy in atomic transmutations is found in the packing fraction, it is desirable to represent the increment of energy in this form. The packing fraction, as we described it in the preceding chapter, is equal to the difference between the atomic weight (W) and the characteristic integer (A), divided by the characteristic integer. Expressed in symbols, where the packing fraction is designated by f, this relationship may be written as follows:

$$W = A + f \cdot A.$$

Returning then to the formula for the energy increment as given above, we can replace the atomic weights by their equivalents in terms of the packing fractions and thus obtain the expression

$$E = 931\ (f_1\, A_1 + f_2\, A_2 - f_3\, A_3),$$

since in every transformation we have $A_1 + A_2 - A_3 = 0$.

Shortly after this spectacular discovery, where the output of energy was many times that expended, lithium was subjected to a bombardment by deuterons. This experiment yielded as before two high speed alpha particles but in addition a single neutron. If we designate a neutron by the symbol ${}_0n^1$ to indicate that the particle has no charge, but unit mass, then we can represent the reaction symbolically as follows:

$${}_3\mathrm{Li}^7 + {}_1\mathrm{H}^2 \rightarrow {}_2\mathrm{He}^4 + {}_2\mathrm{He}^4 + {}_0n^1 .$$

The release of energy in this case is given by

$$\begin{aligned}\Delta \mathrm{E} &= 931\ (7.01818 + 2.01473 - 8.00778 - 1.00893)\\ &= 931 \times 0.01620 = 15.08 \text{ megavolts.}\end{aligned}$$

Many experiments of this type were performed and the evidence became overwhelming that the atom could be disintegrated and that the energy released by the transformation could be interpreted accurately by means of Einstein's equation relating mass and energy. The world was thus upon the threshold of its greatest discovery. But the clouds of a world-wide conflict were also just appearing on the horizon at this dramatic moment.

### 4. *Atomic Fission.*

With the discovery of neutrons and a rich source in the bombardment of lithium by alpha particles, a new and powerful tool had finally come into the hands of the nuclear adventurers. For the neutron, as we have said before, has a very special advantage over the other particles in its lack of an electric charge. It is not diverted from its target by the electrical field which surrounds and shields the heart of the atomic nucleus.

In 1939 the investigation took a different and spectacular turn. Prior to this momentous year, in all nuclear transformations the element was transmuted by the ejection of a proton, a neutron, a deuteron, or an alpha particle. Hence the new element differed only slightly both in its atomic number and its atomic weight from the original element. Thus lithium became helium, boron became lithium and helium, sodium transmuted into florine, magnesium became sodium, etc. The energies obtained from the reaction, although large in comparison with those used in the process of transformation, were relatively small. But in the experiments which we are about to describe the atom of uranium was actually split into several fragments, of which two were of approximately equal masses.

The story begins about 1934 with the quest for new *transuranic* atoms, that is to say, atoms with nuclei which had electric charges in excess of the 92 possessed by uranium. This investigation was initiated by Enrico Fermi, Italian physicist, shortly after the discovery of the neutron. Fermi exposed a large number of elements to the bombardment of neutrons and thus created many new radioactive substances. The mechanics of the situation seemed to be that the nucleus of an element picked up a neutron, which made it unstable. The nucleus then ejected a beta particle, that is to say, a high speed electron, and the positive charge was thus increased by one. This, of course, resulted in the formation of an element one unit higher in the table of the elements.

Why, reasoned Fermi, should not the same process when applied to uranium of atomic number 92, result in a new transuranic element of atomic number 93? The process might even be continued to elements 94, 95, 96 and beyond, which would thus result in the construction of a whole new hierarchy of elements.

Acting on this line of reasoning Fermi and his associates exposed uranium to the neutron stream and found as a result a strange activity. Uranium was divided into several radioactive parts which ejected beta particles and which had half-lives of varying durations. Four such parts were identified, the longest half-life being 90 minutes. It was natural to suppose that one or more transuranic atoms would be discovered among these particles, and they were subjected to chemical investigation to determine their nature. But this is a problem that is easier proposed than solved. A very delicate chemical technique is obviously required to identify such minute quantities of matter as those involved in the transformation. However, the results of these studies showed very definitely that the new substances did not belong to any of the elements between atomic numbers 86 and 92. It was natural to suppose that man had finally progressed into the region of the transuranic atoms and for a while it was tentatively believed that the new substances were the elements 93, 94, 95, and 96. But there were certain anomalies in the situation and by 1935 it became increasingly evident that the new radioactive substances could not belong to the group of heavy atoms.

Among those actively engaged in the study of artificial radioactivity were Irene Curie and her husband F. Joliot,[6] who in 1933 gave the first demonstration of the possibility of exciting such activity. They found that when boron, magnesium, and aluminum were bombarded by alpha particles from polonium, these elements emitted positrons and this emission continued for some minutes after the bombardment had ceased. Since these early experiments more than 300 new artificial substances had been discovered by 1940 and many more have been produced by the recent activity in this promising field.

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[6] *Comptes Rendus,* Vol. 198, 1934, p. 254; *Nature,* Vol. 133, 1934, p. 201.

In 1938, while investigating the residues from the bombardment of uranium, Irene Curie and P. Savitch discovered a new active body with a half-life of 3.5 hours, which in the technique of micro-chemistry was precipitated with lanthanum as carrier.[7] At last the great secret stood revealed before them, but its magnitude blinded them to the truth. They reached the following conclusion: "From these chemical properties, it appears that this body can be only an isotope of actinium, or a new transuranic body possessing chemical properties very different from those of the homologous superiors of rhenium and platinum." A little later, not quite satisfied with this interpretation, they decided: "It seems then that this body can be only a transuranic element possessing properties very different from those of other known transuranic elements, a hypothesis which raises difficulties of interpretation." The truth of the matter was that the substance was actually an isotope of lanthium, *for uranium had been split into fragments,* not transmuted into an atom of higher atomic number.

Shortly after this momentous experiment O. Hahn and F. Strassman in Germany repeated the work of Curie and Savitch.[8] They discovered that certain of the radioactive elements from the irradiated uranium could be precipitated with barium as a carrier. These active elements finally transformed into an element that could be precipitated by lanthanum. This evidence pointed to the conclusion that the active particles were ${}_{56}\mathrm{Ba}^{140}$ and ${}_{57}\mathrm{La}^{140}$. So incredible did this result appear that the two scientists published their conclusion with evident hesitation. Thus they said: "As chemists we must truly say with respect to the new substance that it is not radium, but barium; for elements other than radium or barium are out of the question." And in this way was one of the greatest discoveries of mankind announced to the world. The Nobel prize for chemistry was given to Hahn in 1944 for his fundamental work in this mysterious problem.

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[7] "Sur le radioelement de period 3.5 heures forme dans l'uranium irradié par les neutrons," *Comptes Rendus,* Vol. 206, 1938, pp. 906-908, 1643-1644.

[8] *Nature,* Vol. 143, 19, pp. 239, 276, 471, 637, 852.

The publication of these results led to a great activity. Lise Meitner and O. R. Frisch while in exile in Denmark from Germany[9] also announced the proper interpretation of these experiments of the neutron bombardment of uranium. Almost simultaneously F. Joliot reached the same conclusion[10], another example of the coincidence of discovery as soon as science has attained the general level from which it may be made. To this phenomenon, the fragmentation of the uranium atom, was given the name of the *fission of uranium.*

The fact of fission was announced dramatically in the United States by Niels Bohr at the Fifth Conference on Theoretical Physics held in Washington Jan. 26, 1939. Bohr discussed the significance of the interpretation of the experiments as it had been communicated to him before he left Denmark by Meitner and Frisch.[10] Upon learning of these exciting speculations, a number of the scientists present initiated experiments to confirm the results. This they were soon able to do and a considerable output of papers on the subject resulted in the ensuing months.[11]

The startling aspect of the fission experiment was the unusual quantity of energy released in contrast with the amount which had been observed by the earlier alchemists in the transmutations which we have described in preceding pages of this book. The size of this energy output can be estimated from the following considerations. Let us assume that natural uranium, which consists principally of the isotope ${}_{92}U^{239}$, can be split into two parts, and let us assume that these are ${}_{58}Ce^{140}$ (cerium) and ${}_{44}Ru^{99}$ (ruthenium). The atomic weights of these three elements are respectively 239.1338, 139.9580, and 98.9416.

When these figures are substituted in the energy-mass equation expressed in the unit of megavolts, namely,

$$E = 931\ (W_1 - W_2 - W_3),$$

where $W_1$, $W_2$, $W_3$, are the atomic weights respectively of uranium, cerium, and ruthenium, we obtain

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[9] *Comptes Rendus,* Vol. 207, 19, p. 341.

[10] See N. Bohr: *Physical Review,* Vol. 55, 1939, pp. 418-419; Meitner, Hahn, Strassman: *Zeitschrift für Physik,* Vol. 106, 1937, p. 249; Vol. 109, 1938, p. 538.

[11] For a bibliography and discussion of these papers consult L. A. Turner: "Nuclear Fission," *Reviews of Modern Physics,* Vol. 12, 1940, p. 150.

$$E = 931\ (239.1338 - 238.8996) = 931 \times 0.2342$$
$$= 218 \text{ megavolts.}$$

That is to say, the process of fission has released 218 megavolts of energy as against 17.2 megavolts in the lithium bombardment by alpha particles which we have described in the preceding section.

The process which we have described is a gross simplification of what actually takes place in uranium fission. For the atom splits into a complicated array of fragments and not into two parts as we have assumed in the computation just described. These fragments, however, fall into two definite ranges of atomic numbers. The first range is from 35 to 44 with atomic weights varying from 82 to 100; the second group of substances are found in the range of atomic numbers from 51 to 58 with atomic weights varying from 127 to 150. As one may well imagine, the phenomenon is accompanied by a violent radioactivity, while the various nuclei are reaching stable states after the fission.

### 5. *The New Elements.*

The discovery of nuclear fission marked the beginning of an intensive investigation of this remarkable phenomenon. The numerous fission-created isotopes within the ranges of atomic numbers mentioned above and their radioactive properties presented complicated and intriguing problems. Moreover, thorium (Th) of atomic number 90 and protoactinium (Pa) of atomic number 91 were found to be fissionable with fragments approximating the same range of atomic numbers as uranium. The rate of fission was also found to vary with the speed of the neutrons used in the bombardment. This fact was ultimately to prove to be of the very highest importance. Thus it was found that only very fast neutrons were able to cause fission in thorium, protoactinium, and uranium-238, the abundant isotope of uranium, while neutrons of thermal velocity, called slow neutrons, caused fission in uranium-235, a rare isotope of uranium, but not in the other elements.

Perhaps the most astonishing discovery in this new adventure of the alchemists was the fact that the capture of a thermal neutron, namely one with a low velocity, by uranium-238 led to the creation of the new and unstable isotope uranium-239. This in turn, being highly radioactive, emitted a beta ray, that is to say a nuclear electron, and hence transmuted itself into a new element one number higher in the periodic table of the elements. To this new atom was given the name of *Neptunium*, and it was designed by the symbol ${}_{93}Np^{239}$. That is to say it is the first transuranic element, the 93rd in the list, with a positive charge of 93 units for its nucleus, and an atomic weight of 239. To put the matter otherwise, neptunium has a nucleus consisting of 93 protons, $239 - 93 = 246$ neutrons, and 93 electrons in its enveloping shells. Neptunium, however, is an unstable element and ejects beta particles in a vigorous radioactivity. Its half-life is very short.

This degeneration of neptunium by electron emission thus leads in its turn to the creation of a second transuranic element to which has been given the same plutonium. It has an atomic number of 94 and an atomic weight of 239, which makes it an isobar of neptunium and uranium-239. That is to say we have created ${}_{94}Pu^{239}$. This new element has nucleus of 94 protons and $239 - 94 = 246$ neutrons, surrounded by a peripheral configuration of 94 electrons. Plutonium, unlike neptunium, is a stable atom, that is to say, one with a long half-life. Its radioactivity consists in the emission of alpha particles.

The nature of plutonium was a matter of the deepest interest to physical science for it was the first stable transuranic element ever discovered by man. Its isotope of atomic weight 238, namely ${}_{94}Pu^{238}$, was the first to be made and studied. It is an alpha-ray emitter with a half-life of 50 years and it is created by the bombardment of uranium-238 with deuterons. The first knowledge about the properties of the new element was released from the secrecy of wartime researches in the now classical report of *Atomic Energy* by H. D. Smyth.[12] This document contained the following statement: "From its position in the periodic table, plutonium might be expected to be similar to the rare earths or to uranium, thorium, or osmium. Which of these it will resemble most closely depends, of course, on the arrangement of the outermost groups of electrons and this arrangement could hardly have been predicted. On the whole, plutonium turned out to be more like uranium than like any of the other elements named and might even be regarded as the second member of a new rare-earth series beginning with uranium. It was discovered fairly early that there were at least two states of oxidation of plutonium. (It is now known that there are four, corresponding to positive valences of 3, 4, 5, and 6)." The preparation of plutonium salts by the techniques of microchemistry showed that it was possible to separate plutonium from other metals by means of its chemical properties. This discovery was ultimately to prove to be a matter of very great importance.

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[12] This document, *Atomic Energy for Military Purposes*, Princeton, 1945, was the official report of the development of the atomic bomb.

Recently four other new transuranic atoms have been created, corresponding respectively to the atomic numbers 95, 96, 97, and 98. To them have been given respectively the names of Americum, Curium, Berkelium and Californium. The properties which they have, their atomic weights, the character of their radioactivity, their chemical characteristics, are still hidden under the cloak of secrecy which envelopes such potentially dangerous substances. Are they, like uranium-235 and plutonium, fissionable by thermal neutrons, or do they degenerate rapidly by alpha radiation into stable members of the periodic table with smaller atomic numbers?

### 6. *The Chain Reaction.*

Here then in the phenomenon of uranium fission existed the possibility of attaining a vast supply of energy from a relatively small input. No longer were the alchemists in search of the magic formula which would convert mercury into gold, but the quest had shifted to one of power. The treasure in the depths of the atom was not the yellow glow of gold, but the generation of kilowatt hours of energy.

The rewards promised by this possibility are readily understood from a few figures. We have already described the computation by means of which the conclusion has been reached that the fission of one atom of uranium will produce 218 megavolts of energy. Making use of modern estimates of Avogadro's number, that is to say, the number of molecules in a cubic centimeter of gas at standard temperature and pressure, we can readily compute from this basic constant the number of atoms in a gram of uranium. This gives us the large number $2.56 \times 10^{21}$ which, when multiplied into 218 megavolts, is the energy output of the fission of a gram of uranium. To make this incomprehensibly large figure more understandable we can reduce it to kilowatt hours by dividing by $2.25 \times 10^{19}$, which is the number of megavolts in a kilowatt hour. We thus reach the astonishing result that the fission of a single gram of uranium will yield 24,800 kilowatt hours of energy, which is more energy than is used by the average family for all its electrical lighting in ten years. Its magnitude is better appreciated when one reflects that the fission of approximately 14.7 tons of uranium would be sufficient to generate the entire output of electrical energy in the United States for a single year, the electrical productions of the year 1950 being used in making this estimate. But fantastic as these figures are, they are dwarfed by those derived on the assumption of a complete annihilation of matter. The energy output per gram of uranium or any other element would then be a thousand times greater than that obtained from the fission of uranium. Although the achievement of the annihilation of the atom may be cherished as a future dream of science, we have certainly attained more than enough power for the good of the human race in the one-tenth of one per cent obtained from the process of nuclear fission.

The next question that presents itself is the practical one of how this vast energy can be released in such a continuous and orderly way that it may be used to do the work of the world. But at the moment of discovery of nuclear fission a vast war was engulfing the nations of the world and the problem shifted to that of finding means by which the energy could be released explosively. Essentially both problems are the same, for both depend upon the discovery of what is called a *chain reaction*, or a self-perpetuating series of nuclear fissions such that one that is completed supplies the agents of reaction for the next. The difference between the two problems is merely the difference in the speeds of the reactions.

Thus in the burning of coal a chemical reaction is set in operation which continually maintains itself. The carbon oxidizes and the heat that is developed is sufficient to keep the process in continuous operation until all the coal has been reduced. The orderly burning of coal as it is used in industrial production can be readily converted into an explosive chain reaction by mixing finely powdered coal with oxygen. The chemical reaction is exactly the same, but its speed is greatly increased and the coal is oxidized in an explosive blast.

In a sense a chain reaction is the nuclear equivalent of the combustion process observed in such chemical reactions as the burning of coal. In the fission of uranium by the bombardment of neutrons, other neutrons are released which in their turn produce fission, and this again releases more neutrons. But there is a continual attenuation of the neutron density and in a few moments the reaction is at an end. It is much like trying to ignite wet wood with a match. So long as the stream of neutrons was supplied from an external source the reaction would continue, but the process itself was not self generating. To contrive such a process was the problem forced upon the world of science by the exigencies of the war.

In contemplating the creation of a chain reaction we are faced in the beginning with the problem of discovering what happens to the neutrons released by an atomic fission. In the first place they may have such high velocities that they will escape entirely from the uranium. Or again they may be retained within the uranium mass either by the uranium itself or by impurities without causing another fission. Such an absorption is spoken of as a non-fission capture. Or the neutrons may actually penetrate the nuclei of the atoms and create once more the desired fission. It is this last possibility that will perpetuate the chain reaction.

It is clear at the outset that the first necessity is to retain as many of the fission-creating neutrons as possible within the uranium mass. This central problem is described as follows in the *Report* of H. D. Smyth to which we have referred earlier: "The relative number of neutrons which escape from a quantity of uranium can be minimized by changing the size and shape. In a sphere any surface effect is proportional to the square of the radius, and any volume effect is proportional to the cube of the radius. Now the escape of neutrons from a quantity of uranium is a surface effect depending on the area of the surface, but fission capture occurs throughout the material and is therefore a volume effect. Consequently the greater the amount of uranium, the less probable it is that neutron escape will predominate over fission capture and prevent a chain reaction. Loss of neutrons by non-fission capture is a volume effect like neutron production by fission capture, so that increase in size makes no change in its relative importance.

"The critical size of a device containing uranium is defined as the size for which the production of free neutrons by fission is just equal to their loss by escape and non-fission capture. In other words, if the size is smaller than critical, then – by definition – no chain reaction will sustain itself. In principle it was possible in 1940 to calculate the critical size, but in practice the uncertainty of the constants involved was so great that the various estimates differed widely. It seemed not improbable that the critical size might be too large for practical purposes. Even now estimates for untried arrangements vary somewhat from time to time as new information becomes available."

Since experiment showed that fission capture by the active uranium-235 was most efficient for thermal neutrons, that is to say, neutrons of low speed, the first consideration was to find a method for reducing the high velocities of the fission neutrons. It had been known for some time that this reduction in speed could be accomplished by passing the neutrons through substances with low atomic weight. Among the most efficient of these substances, which are known as moderators, were hydrogen, deuterium, beryllium, and carbon, since none of them showed any marked tendency to capture the neutrons as they passed through. The final choice of a moderator was carbon in the form of graphite.

To achieve a working mechanism large lumps of uranium were imbedded in a matrix of graphite. This lattice arrangement of active and moderating material is commonly referred to as a *pile*. In order for it to work efficiently the loss of neutrons by non-fission capture by impurities in the pile had to be reduced to a minimum. This meant such a careful preparation both of the uranium and the moderator that only a few parts in a million of extraneous material remained. The first pile was operated December 2, 1942. Smyth in his *Report* says: "So far as we know this was the first time that human beings ever initiated a self-sustaining nuclear chain reaction." This pile, erected under the direction of E. Fermi, operated with natural uranium.

Numerous exciting problems confronted those who constructed the first pile. Its critical size, that is to say, the volume within which as many neutrons would be created as were captured, was a matter of conjecture. About this problem Smyth in his *Report* makes the following statement: "The original plan called for an approximately spherical pile with the best materials near the center. Actually control measurements showed that the critical size had been reached before the sphere was complete, and the construction was modified accordingly. The final structure may be roughly described as an oblate spheroid flattened at the top, i. e., like a door knob. It was desired to have the uranium or uranium oxide lumps spaced in a cubic lattice imbedded in graphite. Consequently, the graphite was cut in bricks and built up in layers, alternate ones of which contained lumps of uranium at the corner of squares. The critical. size was reached when the pile had been built to a height only three quarters of that needed according to the most cautious estimates. Consequently only one more layer was added . . . . The pile contained 12,400 lbs. of metal."

In order to safeguard the experiment in a field so fraught with explosive possibilities where a chain reaction might develop which could not be controlled, strips of cadmium or boron steel were inserted in the pile. These materials are neutron absorbers and hence by inserting them into the pile it was possible to bring the emission of neutrons below the critical conditions. In Smyth's *Report* it is said: "The control strips were placed in a suitable 'retard' position from the start and the neutron intensity was measured frequently. This was fortunate since the approach to critical conditions was found to occur at an earlier stage of assembly than had been anticipated."

The ultimate goal of the pile was the production of plutonium in quantities sufficiently great for military purposes. Natural uranium consists of three isotopes: ${}_{92}U^{234}$, ${}_{92}U^{235}$, and ${}_{92}U^{238}$. Of these the last comprises 99.3 per cent of natural uranium, the second seven tenths of one per cent and the first only six-thousandths of one per cent. A fourth uranium isotope, namely ${}_{92}U^{239}$, is highly unstable and does not occur in the natural metal. Fissionable uranium is U-235, but U-238 is used in the creation of plutonium as we have described earlier. In this process a neutron is captured by U-238, which transforms into the radioactive U-239. This in turn emits a beta particle, which yields neptunium, and this short-lived element transmutes into plutonium by the emission of another beta particle. Hence the self-generating pile which we have just described would supply from the fission of U-235 enough neutrons to continue the fission and also provide neutrons for the transmutation of U-238 into plutonium.

Two major difficulties arose in the operation of the pile, one associated with the heat generated by the fission of U-235 and the other associated with the radioactive particles which accompany the disintegration of the atom. A pile large enough to produce a kilogram of plutonium per day must release energy at a rate of approximately 1,000,000 kilowatts. The magnitude of this figure may be appreciated from the fact that the total capacity of the electricity-generating plants of the United States is only 26 times this figure. In the pile of commercial size which was finally constructed at Hanford in the state of Washington it was necessary to carry off this vast energy by diverting great quantities of water from the Columbia River through the pile. The temperature of the river was actually raised four degrees by this excess energy.

The most troublesome feature of the pile was the large quantity of radioative material generated by the fission of uranium-235. Each atom smashed by the neutron yielded some thirty elements among which were radioactive xenon and radioactive iodine. The disposal of this dangerous material so that no harm could come either to those working in the plant or to the inhabitants of the surrounding country was one of the most difficult problems connected with the manufacturing of plutonium.

The final piles were actually very large blocks of graphite with holes in which could be inserted cylinders of metallic uranium. These cylinders were sealed in aluminum cans which protected the metal from corrosion by the water which continuously flowed through the piles. Because of the dangerous substances generated in the manufacturing process these tubes were removed by remote control for the final separation of the plutonium.

### 7. *The Atomic Bomb.*

Although the laboratory creation both of uranium-235 and of plutonium was solved satisfactorily early in the study of nuclear physics, the problem remained to establish large-scale production of these substances for military purposes. Under the general designation of the Manhattan District Project, established with unlimited authority and adequate funds on June 18, 1942, the work of manufacturing both uranium-235 and plutonium was undertaken. Three plants were established on a tract of land in the Tennessee Valley. One of these was to effect the separation of uranium-235 from the natural metal by an adaptation of the mass spectograph to large scale production, the so-called electro-magnetic method of separation. A second plant was set up to use the method of diffusion separation in the same process. A third unit was devoted to perfecting the methods for the creation of plutonium and to study the many problems associated with its manufacture.

It is one of the miracles of modern science that almost three years after the founding of the Manhattan District Project, the delicate problems had been solved and sufficient material was at hand so that mankind awaited its most spectacular experiment. This was the detonation of the first atomic bomb.

"A weapon has been developed," says the Smyth *Report*, "that is potentially destructive beyond the wildest nightmares of the imagination; a weapon so ideally suited to sudden unannounced attack that a country's major cities might be destroyed overnight by an ostensibly friendly power. This weapon has been created not by the devilish inspiration of some warped genius but by the arduous labor of thousands of normal men and women working for the safety of their country."

The first test of the new atomic weapon was made at 5:30 a. m. July 16, 1945 in the desertlands of New Mexico. "Here in a remote section of the Alamogordo Air Base 120 miles southeast of Albuquerque," says a war department release published in the Smyth *Report*, "the first man-made atomic explosion, the outstanding achievement of nuclear science, was achieved at 5:30 a. m. of that day. Darkening heavens, pouring forth rain and lightning immediately up to the zero hour, heightened the drama.

"Mounted on a steel tower, a revolutionary weapon destined to change war as we know it, or which may even be the instrument to end all wars, was set off with an impact which signalized man's entrance into a new physical world. Success was greater than the most ambitious estimates. A small amount of matter, the product of a chain of huge specially constructed industrial plants, was made to release the energy of the universe locked up within the atom from the beginning of time. A fabulous achivement had been reached. Speculative theory, barely established in pre-war laboratories, had been projected into practicality."

The bomb itself consisted of a chain reaction of either pure uranium-235 or plutonium with fast neutrons instead of those of thermal velocity. The explosion was automatically achieved by the rapid assembling chunks of the active element, each of which was below the critical mass, into a mass of critical size. One may conjecture that this was perhaps of the order of 100 kilograms. By the instantaneous fission of such a large particle a terrible explosion ensued which released in the fraction of a second something of the order of two and a half billion kilowatt hours of energy. Thus in a very small moment of time and within a radius of a few inches there was delivered an amount of energy equal to about two per cent of the

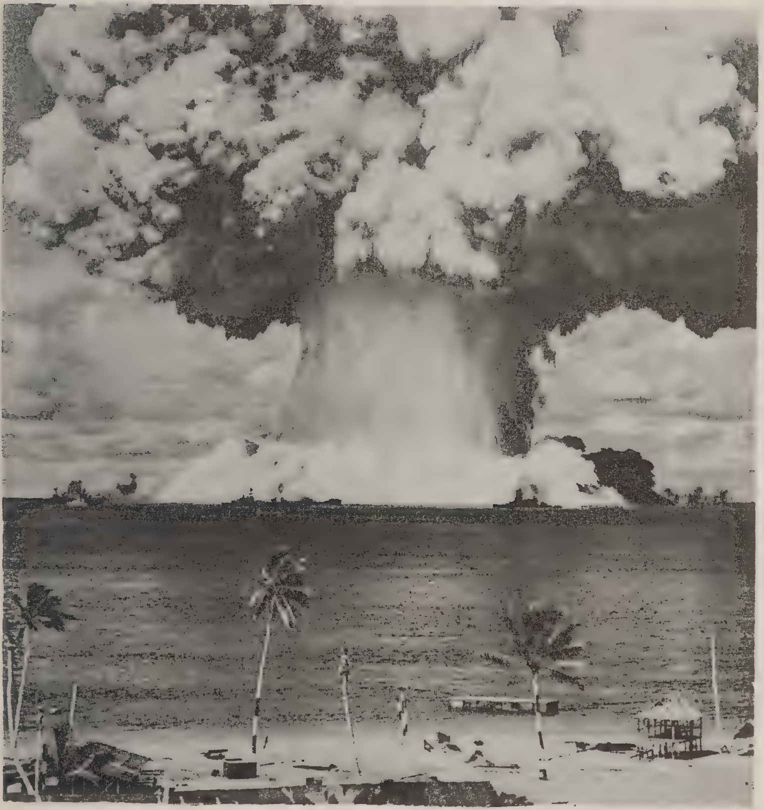

PLATE IX

The Atomic Bomb. "The effects could well be called unprecedented, magnificent, beautiful, stupendous, and terrifying."

entire annual electrical production of the United States. Do we wonder that the explosion was of cosmic magnitude!

The official report of the test describes it in the following words: "At the appointed time there was a blinding flash lighting up the whole area brighter than the brightest daylight. A mountain range three miles from the observation point stood out in bold relief. Then came a tremendous sustained roar and a heavy pressure wave which knocked down two men outside the control center. Immediately thereafter, a huge multi-colored surging cloud boiled to an altitude of over 40,000 feet. Clouds in its path disappeared. Soon the shifting substratosphere winds dispersed the now grey mass. . . .

"The effects could well be called unprecedented, magnificient, beautiful, stupendous and terrifying. No man-made phenomenon of such tremendous power had ever occurred before. The lighting effects beggared description. The whole country was lighted by a searing light with the intensity many times that of the midday sun. It was golden, purple, violet, gray and blue. It lighted every peak, crevasse and ridge of the nearby mountain range with a clarity and beauty that cannot be described but must be seen to be imagined. It was that beauty the great poets dream about but describe most poorly and inadequately. Thirty seconds after, the explosion came first, the air blast pressing hard against the people and things, to be followed almost immediately by the strong, sustained, awesome roar which warned of doomsday and made us feel that we puny things were blasphemous to dare tamper with the forces hitherto reserved to the Almighty. Words were inadequate tools for the job of acquainting those not present with the physical, mental and psychological effects. It had to be witnessed to be realized."

# CHAPTER 13.

## SPECULATIONS ON COSMOLOGY.

"We had the sky up there all speckled with stars, and we used to lay on our backs and look up at them, and discuss about whether they was made or only just happened. Jim he allowed they was made, but I allowed they happened; I judged it would have took too long to make so many. Jim said the moon could 'a' laid them; well that looked kind of reasonable, so I didn't say nothing against it, because I've seen a frog lay most as many, so of course it could be done. We used to watch the stars that fell, too, and see them streak down. Jim allowed they'd got spoiled and was hove out of the nest."

Mark Twain in *Adventures of Huckleberry Finn.*

"Yet all the vast bodies that compose this mighty frame, how distant and remote soever, are by some secret mechanism, some Divine art and force, linked in a mutual dependence and intercourse with each other; even with this earth, which has almost split from my thoughts and lost in the crowd of worlds. Is not the whole system immense, beautiful, glorious beyond expression and beyond thought!"

George Berkeley in *Three Dialogues.*

### 1. *Some Problems of Cosmology.*

Man for his inspiration has always turned his face toward the stars; in them he has found the beginnings of his sciences; they have stirred his imagination and his reverence; his poetry and his religions would be sterile without their emotional appeal. There is something that moves us strangely in the following brief statement from a letter written by a young soldier just before he joined his last battle:

"I have been looking at the stars and thinking what an immense distance they are away. What an insignificant thing the loss of, say, 40 years of life is compared with them! It seems scarcely worth thinking about . . . This letter is going to be posted if . . . ".

Philosophical speculation began when man first raised his face out of the primordial slime and beheld the stars. One can scarcely controvert the observation that the scientific revolution emerged from the tables and the data of the ancient astronomers. Our survey of the problems which we have set would scarcely be complete if we did not turn to a splendid modern speculation which finds its origin in an ancient query of astronomy. What is the source of stellar energy?

But this question naturally poses others. What becomes of the energy of the stars? Does it radiate forever into empty space and finally dissipate itself beyond recovery in the very vastness of the universe? Or is there a mechanism by means of which radiant energy may be reconverted into matter and thus perpetuate a cycle of creation? Such questions though in changing forms have been proposed by the philosophers since the beginning of thought.

One is reminded at the outset of the difficulty of sifting testimony and arranging facts so that truth may be ascertained. The problem of stellar energy is a problem of fitting together the numerous parts of a scrambled puzzle. Great quantities of data about the luminosity and size of the stars, their densities, their pulsations, their stabilities, their distribution, and their spectra are elements of the complex problem to which the attempt to probe the mystery of the source of their energy leads. That conclusions obtained by experts should differ greatly, that the theory of one year should become the folly of the next, that the foundation of any particular speculation is built upon shifting sands, are observations which appear unnecessary to anyone who scans even casually the history of the subject.

The problem of stellar energy is two-fold as we have indicated above. We are interested not only in how the energy is generated and why it continues to pour out into space from the countless stars, but we are intrigued also by the speculation as to where its journey ends. One sees in this another aspect of the problem proposed in the first chapter. Why does energy never attain its fatal maximum? Why does the free energy of the universe continually waste itself and yet the supply appear to be inexhaustible? Is the macrocosm a great thermodynamic engine working perpetually upon a Carnot cycle which winds itself up as it runs down? Is the universe self-perpetuating? Is it meaningless to talk about its beginning and its end? Are these but terms which we may apply to the geometry of forms and not to their basic parts? Man's four-dimensional existence is bounded by the finite limits of the variables involved; the structure of all objects is wasted away by the "edacious tooth of duration," but the world-lines of the particles that compose the world-tube the existence of which we contemplate are boundless.

Let us look first at the historical struggle and see what speculation is suggested by the attempts which have been made to probe the secret of stellar energy. At the very outset we must admit that our real knowledge of the mysterious entity from which proceeds all the activity of the universe is still deficient. We do not know why a flying ball possesses that with which it flies; we say that the velocity, which is the manifestation of its energy, is only an instance of the application of the principle of least action; but this statement, in lieu of an explanation of deeper content, is mere description. In order to examine the source of energy it would be well to have more adequate knowledge of the thing itself, but this information so far has been denied to human knowledge. Hence, not knowing exactly what we seek, we embark upon the uncertain adventure of finding it. This is the spirit of true research.

### 2. *What Keeps the Sun Hot?*

We pass over the speculations of the ancients which have little to contribute to the modern investigation. Descartes in his *Principles* mentions the problem of stellar energy and points out the difference between the heat of the sun and the flame of the candle. The one, he observes, shines undiminished for many centuries without appreciable need of anything to sustain its activity; the other consumes the material from which it emerges.

Perhaps the first attempt to give a rational explanation of the source of the sun's heat was made by Julius Robert Mayer (1814-1878), who advanced the theory in 1848 that the heat of the sun was derived from a swarm of meteors continually falling upon its surface. This simple conjecture was easily shown to be untenable, however, because the meteoric swarm necessary to maintain heat for a century would need to be as massive as the earth and would double the mass of the sun in thirty million years. It is obvious that this source must be abandoned, and with it perish all attempts to find a cause exterior to the sun itself. Such hypotheses were doomed to failure in the beginning for as Eddington has observed: "*No source of energy is of any avail unless it liberates energy in the deep interior of the star.*[1]"

Another origin of the heat of the sun which was considered is that of combustion, or chemical transformation. A few figures, however, will make it readily apprehended that this suggestion is wholly inadequate. One must first remember the tremendous energy of the sun and the vast extent of time during which this supply has been pouring into empty space. We who dwell on a tiny object nearly a hundred million miles from the source, presenting with our disk only an infinitesimal target for the rays, may well marvel at the magnitude of the emanating radiation. The radio-active content of the rocks on the surface of our earth makes it highly probable that geological time must be regarded as an epoch not less than 1,500 million years in duration. The age of the sun would probably be greater than this.[2] At the present rate of the sun's radiation this estimate would lead to the conclusion that each gram mass of its huge bulk has radiated into space an amount of energy estimated to be not less than $9 \times 10^{16}$ ergs, a total from one hundred thousand to a million times greater than could be accounted for by the chemical combination of any two atoms.[3]

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[1] *The Internal Constitution of the Stars.* Cambridge, 1926, p. 291.

Herman L. V. von Helmholtz (1821-1894) in 1854 advanced his famous *contraction hypothesis.* In this Helmholtz sought to show that the gravitational potential energy lost as the surface of the sun shrunk inward toward the center would be of sufficient magnitude to account for the radiation loss. This was a fruitful idea which still possesses interest for us in spite of the modern discovery of other and greater sources of energy. To those uninitiated into the mysteries of energy transformations it is a novel idea that the contraction of the sun's surface could be of sufficient magnitude to make itself manifest in radiant energy. What is the mysterious bond between the intangible concept of potential energy and the more readily apprehended energy of radiation? In their eager search for a source of energy sufficiently large to account for the continuous heat of the sun the astronomers saw in the suggestion of Helmholtz a possible explanation. But that was in a day before the discovery of radio-active elements in the crust of the earth and at a time when the demands of the geologists were not so vast as they have recently become. When the modern estimate of the age of the earth is considered it is soon evident that the Helmholtz source is far too small to account for the energy demanded. Even though the sun's mass had been accumulated from an infinite dispersion, the total energy available would be about two per cent of the estimate which we have stated above.[4]

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[2] H. Jeffreys in *The Earth*, New York, sec. ed., 1929, p. 80 says: "Collecting our results, we have found the following: 1. From the eccentricity of the orbit of Mercury, we saw that the whole time since the rupture is probably between $10^9$ and $10^{10}$ years. 2. The ages of the oldest known minerals, found from the lead/uranium ratio, are about $1.3 \times 10^9$ years, and since the geological evidence indicates that some sedimentary rocks are still older, the age of the ocean must exceed $1.3 \times 10^9$ years. 3. The amounts of uranium, thorium, and lead in the crust as a whole indicate a time less than $3 \times 10^9$ years since solidification. The second and third results are in good agreement with the first. The age of the earth is therefore probably between $1.3 \times 10^9$ and $3 \times 10^9$ years."

[3] See J. H. Jeans: *Astronomy and Cosmogony*, Cambridge, 1928, p. 106.

But who knows the interior of a star? The picture given below of the mad dance of the atoms might be regarded as better poetry than science, and yet the quotation graces the artistic pages of Eddington's profound treatise on the internal structure of the stars. Who says that there is not work for poets to perform in translating our new knowledge into aesthetic pictures?

"The inside of a star is a hurly-burly of atoms, electrons, and ether waves. We have to call to aid the most recent discoveries of atomic physics to follow the intricacies of the dance. We start to explore the inside of a star; we soon find ourselves exploring the inside of an atom. Try to picture the tumult! Dishevelled atoms tear along at 50 miles a second with only a few tatters left of their elaborate cloaks of electrons torn from them in the scrimmage. The lost electrons are speeding a hundred times faster to find new resting-places. Look out! there is nearly a collision as an electron approaches an atomic nucleus; but putting on speed it sweeps around it in a sharp curve. A thousand narrow shaves happen to the electron in $10^{-10}$ of a second; sometimes there is a side-slip at the curve, but the electron still goes on with increased or decreased energy. Then comes a worse slip than usual; the electron is fairly caught and attached to the atom, and its career of freedom is at an end. But only for an instant. Barely has the atom arranged the new scalp on its girdle when a quantum of ether waves runs into it. With a great explosion the electron is off again for further adventures. Elsewhere two of the atoms are meeting in full tilt and rebounding, with further disaster to their scanty remains of vesture."[5]

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[4] If one gram of mass is brought from infinity to the sun it acquires a potential energy equal to $GM/r$, where $G$ is the gravitational constant $= 6.66 \times 10^{-8}$ c. g. s. units, $M$ is the mass of the sun $= 2 \times 10^{33}$ grams, and $r$ is the radius of the sun $= 7 \times 10^{10}$ centimeters. Substituting these figures in the expression for the potential energy, we get approximately $2 \times 10^{15}$ ergs for each gram mass which has fallen in from infinity. This is seen to be about two per cent of the required energy of $9 \times 10^{16}$ ergs per gram.

[5] *The Internal Constitution of the Stars*, Cambridge, 1926, p. 19.

We turn, therefore, from the older theories of the source of stellar energies to the new concepts of atomic physics which give us in some respects even more energy than we can use.

### 3. *Radio-Activity as a Source of Energy.*

With the discovery of radio-active substances new possibilities appeared. The recognition of the existence of uranium in the rocks of the earth did two things. These tiny atomic clocks extended the age of the earth by a factor of at least fifteen beyond the estimate of 100,000,000 years made by Lord Kelvin, based on the cooling of its surface. At the same time they furnished another and much more realistic source of energy to account for the greater age.

In a subject where so much is conjecture and where the extrapolation is so enormous compared with the extent of knowledge, one cannot be too careful in safeguarding and buttressing his statements. Observations which may appear to have been made as the elucidation of fact must be regarded as the most tentative estimates made for the sake of argument. Sciences progress in this way. Some daring spirit forms a hypothesis; others bring to bear upon his proposition their criticisms and their data. Out of the ensuing debate in which neither dogmatic assertion nor the weight of authority has the slightest force, a better approximation to the truth may be won. In a subject which presents as many variable factors as the origin of the energy of the stars there are vast possibilities for such scientific conjecture. Only the one fact of the enormity of the energy and its strange variability from that of the brilliant blue stars to that of the immense but relatively cool masses of the red giants forms our basic datum. Therefore we shall advance several tentative propositions and set forth the present status of the argument.

With the discovery of nuclear energy it became evident that the great internal temperature of the stars and their long interval of radiation might be accounted for readily by this powerful source. Three possibilities have presented themselves which we may enumerate as follows:

First, stellar energy might be accounted for by natural radioactivity, that is to say, by the transmutation of heavy, complex atoms into simpler elements.

Second, the mass of the stars may be actually in a process of annihilation. Protons, positrons, neutrons, and electrons may be combining with one another in the intense heat of the stellar interiors to create photons of radiation. The mass of the stars thus slowly melts away and the energy replaces the dissolving matter according to the Einstein law, $E = Mc^2$, which plays so conspicuous a role in all such modern transformations.

Third, the source of stellar energy may be found in a cosmic alchemy in which complex elements are created out of simpler ones.

These three theories, as will be readily apprehended, are not mutually exclusive ones. Thus the phenomenon of radio-activity enters into all transmutation processes which have yet been discovered. Moreover the generation of energy is always at the expense of matter and the masses of the stars would be dissolved by any theory which assumes an internal source. We thus see that the difference between the second theory and the other two is one rather of a definition of the mechanism by means of which the transformation from matter to energy takes place, than a radical departure on theoretical grounds. The differences in points of view expressed by the three propositions will appear in the arguments by means of which they are defended by their proponents.

There is quite enough energy in radio-active substances to account for any reasonable age of the sun. If its mass were composed wholly of radium, for example, there would be sufficient energy to enable it to pour out its radiation at the present rate for approximately 5,000 million years; if uranium were the active agent the life of the sun could be extended to 8,000 million years. Since these estimates are more than satisfactory from the point of view of the time-scale, the opponents of this theory have been forced to seek other objections to the hypothesis. One of these they have found in the half-lives of the radio-active substances. The half-life of radium, as we have indicated in another chapter, is approximately 1590 years. Hence a star composed of this element would pour forth a tremendous flood of energy for a brief period, as measured by the astronomers, but the rate of emission would decline far too rapidly to account for the great age required from other evidence. The star in a comparatively few centuries would finally lapse into coldness and death. Since human history itself extends back at least five thousand years, the evidence of such a heat-death for our own sun would be visible from the documents of the ancients. On the other hand the life of uranium is too great; its production of energy is slow and no method in the natural state of uranium is known which can be adduced to change this rate of emission.

That there is more than sufficient energy in the radio-active processes is readily seen from the amount released by uranium fission, which we have described earlier. Thus the fission of a single gram of uranium-235 gives an output of $9 \times 10^{17}$ ergs, which is ten times as great as that required by the present generous estimates of the time-scale. In other words, if the sun were originally composed of uranium-235 and there existed within its interior a mechanism for the fission of the material at a uniform rate, then the present radiation of energy could be maintained for fifteen billion years.

The two possibilities of atomic generation of energy by the radio-activity process appear to be either, first, that it is spontaneous, or second, that it is produced by one of the methods used in the stimulation of artificial radio-activity. Spontaneous generation is what we witness in the case of radium and uranium on the earth; it is a phenomenon associated only with time; there is neither acceleration nor retardation of its rate; the mechanisms that are at its roots are automatic; each second witnesses the liberation of a certain quantity of energy and never more nor less.

Arguments may be advanced, however, to show that a certain increase of the disintegration of uranium and radium atoms may take place under tremendous temperatures. But this process is not operative except at temperatures estimated at 120 billion degrees. Hence the search for the source of stellar energy in these disintegration phenomena, at least under present knowledge of radio-active substances, seems to reach a fatal impasse.

But one would be rash, indeed, to say at the present time that the entire theory must be rejected. One important argument may be adduced in the existence of heavy elements in many of the stars. Thus the spectrum of the sun reveals the presence of about sixty different elements, the heaviest of which is lead.

### 4. *The Annihilation of Matter.*

Sir James H. Jeans was perhaps the first to propose that the heat of the sun might be the direct consequence of the annihilation of matter. Considerable historical interest attaches to the observation that as early as 1904, one year before Einstein published his famous relationship between mass and energy, Jeans advanced the following theory:[6]

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[6] "A Suggested Explanation of Radio-activity," *Nature*, Vol. 70, 1904, p. 101.

"It seems probable that the rearrangement of the structure of the ether would consist of the combination and mutual annihilation of two ether streams of opposite kinds, i. e. in the coalescence of a positive and negative ion, and would therefore result in the disappearance of a certain amount of mass. There would, therefore, be conservation neither of mass nor of material energy; the process of radio-activity would consist in an increase of material energy at the expense of the destruction of a certain amount of matter."

Fourteen years later Jeans applied this idea to the problem of stellar energy and in 1928 elaborated his ideas into a definitive theory. His general argument is based upon the time-scale of the universe and the estimated rate of stellar evolution. Thus he says:[7]

"The older views of stellar radiation regarded a star's gravitational potential energy and the heat and chemical energy of its molecules as reservoirs from which a star's radiation was drawn. When we look at the matter in terms of a time-scale of millions of millions of years, we see that the capacity of all these reservoirs is quite negligible; the reservoir in which the star's future radiation is stored is the star's mass. The time-scale of millions of millions of years requires that the energy stored in each gram of a star's mass shall be of the order of magnitude of $9 \times 10^{20}$ ergs, and we know of no way in which such a great concentration of energy can be stored except in the actual matter of electrons and protons."

The weakness of the theory as here proposed is found in the lack of any mechanism by means of which the annihilation is accomplished. Perhaps the secret may be found ultimately when the source of cosmic radiation is explained. But until that time we can start with assurance only with the dance of the atoms. The depth of every star must be a place of inconceivable activity. Energy from its mysterious source is pouring out through the various layers of the star. The mad rush of the released photons is everywhere retarded by the envelope of electrons, photons, and neutrons. If Jeans' theory is correct then two of the primary particles comprising matter rush together. Perhaps positrons are created in the violent agitation of high-speed alpha particles within the mass and there combine with the free electrons which are everywhere abundant. Or perhaps some other mechanism is at work of which we now have no scientific knowledge. At any rate a quantity of energy is born and starts its journey toward the surface of the star. Perhaps its initial frequency is as great as that of cosmic radiation, which we shall describe later. But in the long journey to the boundary of its prison, the photosphere of the star, this primary photon must suffer innumerable collisions and reflections. According to the theory of the Compton effect the photon will continually reduce its initial energy as it proceeds, and when it emerges from the surface of the star its frequency will be that of one of the beams that fall upon our telescopes.

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[7] *Astronomy and Cosmogony*, p. 110.

There are various questions to be answered about this birth of energy. Is it accelerated by pressure within the depth of the star? Does temperature affect it? What influence does ionization, that is to say, the stripping of electrons from their constituent atoms, have upon the process?

If we assume the annihilation theory, as defined by Jeans, it is probable that neither density nor temperature will accelerate the process. About these factors Jeans says:[8] "We are thus led to the conclusion that the generation of energy must be a 'mono-atomic' process; the atom by which the energy is generated is alone involved. Since the concepts of density and temperature have no meaning with reference to single atoms, it follows that the process is independent both of density and temperature." Jeans estimates that the temperature under which the process of annihilation takes place would be of the order of 7,500 billion degrees. He arrives at this figure from certain theoretical considerations based upon the necessary energy of quanta which would conceivably influence the process. Recent discoveries of nuclear physics, however, might greatly change this estimate by supplying a very different mechanism for the annihilation process. Jeans appeals to the evidence of stellar observation to show that no correlation exists between the absolute luminosity of stars and their internal density.

The existence of white dwarf stars with their great densities has been presented as an argument in favor of the theory of annihilation. The observed densities are assumed to have been attained by an ionization of the constituent atoms. If this, indeed, is the case then a star from which most of the surface electrons have been stripped is not likely to dissolve rapidly into energy since there are few negative particles to use in destroying the protons or positrons. The white dwarf companion of Sirius, for example, which we have employed previously as an example substantiating one consequence of Einstein's law of gravitation, is conceivably a star the atoms of which are highly ionized. Its enormous density of the order of one ton per cubic inch may be derived from the fact that its constituent nuclei are closely packed together. The electrons have largely disappeared and hence the process of material annihilation is nearing its end. Jeans points out that the white dwarf stars in general emit an abnormally small amount of radiation, 0.007 ergs per second per gram as compared with 1.9 ergs per second per gram in the sun, which gives some substantial evidence in favor of the annihilation theory.

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[8] *Astronomy and Cosmogony*, p. 123.

Perhaps the strongest argument in favor of the annihilation hypothesis is found in the time-scale required by the currently accepted theory of stellar evolution, which we shall discuss later in this chapter. In this view of the development of the stars the vast red giants stand at one end of the time sequence and the insignificant red dwarfs at the other. The latter are but pale shadows of the great globes from which in the process of radiating energy they have finally descended. From the known rates of radiation it is readily ascertained that a very long time-scale is required for a red giant to evolve into a hot blue star, and thence dissolve into a red dwarf at the end of the sequence. Although many problems are presented by this picture, the order of magnitude of the time-scale can be approximated and this would seem to require something like the annihilation theory to make it plausible.

### 5. *The Theory of Celestial Alchemy.*

Another theory to account for the heat of the stars was made by J. Perrin in 1919[9] and elaborated by A. S. Eddington in 1920.[10] This involved the proposition that if a helium atom is formed from the combination of four atoms of hydrogen a certain quite considerable amount of energy is emitted in the process. To a lesser degree this is also true if certain other atoms at the lower end of the periodic table of the elements are created by uniting the constituents of atoms of smaller atomic weight. Thus in the case of helium we might regard its atomic weight of 4.00389 as derived from a combination of the material elements of four hydrogen atoms each of weight 1.00813. But the difference between the atomic weight of four hydrogen atoms and one helium atom is the appreciable amount 0.02863.[11] Hence if we multiply this residual atomic mass by 931, the factor for converting mass into megavolts of energy, we obtain the considerable energy release of 26.7 megavolts for each helium atom that is thus constructed. This is equivalent to $6.5 \times 10^{18}$ ergs per gram mass, a figure that is obviously large enough for cosmic purposes.

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[9] *Annales de Physique*, Vol. 2, 1919, p. 89; also *Revue du Mois*, Vol. 21, 1920, p. 113.

[10] *British Association*, Cardiff, 1920, Address to Section A.

Assuming, then, that the sun was originally a great ball of hydrogen, which transformed gradually into helium, this enormous energy per gram provides more than enough time to account for the estimated age of the sun. If we assume that the sun radiates into space each year some $6 \times 10^7$ ergs per gram mass, it is clear that the hydrogen-helium alchemy would provide a life-span in excess of $10^{11}$ years. This exceeds by a small fraction the age of the universe as it has been deduced from the arguments of the red-shift in the spectrum of the spiral nebulae, which we have set forth in an earlier chapter. It does not appear to be quite adequate, however, for the demands of stellar evolution. Only the annihilation theory provides us with sufficient energy, for it increases this vast epoch by more than a hundred fold.

A very suggestive theory has been proposed by H. A. Bethe to show how the hydrogen-helium alchemy could actually be achieved within the present knowledge of nuclear physics.[12] This is known as the *theory of the carbon cycle.* According to the mechanism thus described an atom of carbon captures a hydrogen nucleus and is transformed by this means into the unstable $N^{13}$ isotope of nitrogen. Since this element is only a transition state it immediately becomes the carbon isotope $C^{13}$ by the ejection of one positron, which combines with a free electron to produce two photons of energy. The carbon isotope then captures another hydrogen nucleus to become ordinary nitrogen, $N^{14}$. A third hydrogen nucleus transforms this into the radio-active isotope of oxygen, $0^{15}$, which transmutes into $N^{15}$ with the loss of another positron. The capture of the fourth nucleus of hydrogen then completes the cycle since $N^{15}$ breaks down into ordinary carbon, $C^{12}$, and helium. It is obvious that the transformations of the carbon cycle can occur only in the presence of high temperatures, but these have been estimated to be of the order of those prevailing within the interior of stars.[13]

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[11] In the original computations Aston's atomic weights of 1927 were used, namely 4.00126 for helium and 1.0078 for hydrogen.

[12] "Energy Production in Stars," *Physical Review,* Vol 55, 1939, pp. 434-456.

It is clear from the description that the original carbon and nitrogen in the star are not consumed in the process. As Bethe says: "*Carbon and nitrogen are true catalysts; what really takes place is the combination of four protons and two electrons into an alpha-particle.*" Hence on this theory in the evolution of the star the original quantity of hydrogen would slowly disappear to be replaced by an equivalent quantity of helium and radiant energy. There would be no sensible loss in either carbon or nitrogen in the transformation.

But appealing as the carbon cycle theory is, there are certain difficulties if one wants to apply it to explain all the complex problems of stellar energy. For one must account for the existence of giant and dwarf stars and the presumably vast time-scale which separates them in the process of evolution. One must also explain the existence of the intensely hot bluish white helium stars and the relatively cool red stars with prominent bands of titanium-oxide in their spectra. The carbon cycle also affords no explanation for the existence of heavy elements in the stars. Bethe must therefore assume "that the heavier elements were built up *before* the stars reached their present state of temperature and density."

### 6. *Celestial Alchemy and the Hydrogen Bomb.*

The picture of celestial alchemy which we have described above fired the imagination of science, for it suggested a strange and terrible idea. If it were indeed true that helium is born in the depths of the sun, created out of hydrogen in the crucible of the sun's core, generated spontaneously by the immense pressures and fantastic temperatures of that massive body, would it be possible to exhibit this transformation in the laboratories of the earth?

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[13] The technical transformations which we have described are represented symbolically as follows:

${}_6C^{12} \rightarrow {}_1H^1 \rightarrow {}_7N^{13}$, ${}_7N^{13} \rightarrow {}_6C^{13} + {}_1e^0$ (designation of a positron),

${}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14}$, ${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15}$,

${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0$ (second positron), ${}_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4$.

From the fission of uranium had come the terrible power of the first atomic bomb; but here, in the creation of a higher element from a lower, in the synthesis of helium from hydrogen, there existed a potential source of energy far greater than any which man had ever contemplated before.

From these musings came, indeed, the creation of the hydrogen bomb, an instrument of such terrifying potentialities, that the scientists themselves were shocked. Where a fission bomb could destroy a small city, the hydrogen bomb, if its full power were developed, would annihilate a vast industrial area.

But since the alchemy by means of which hydrogen is converted into helium must take place under conditions of enormous pressure and fantastic temperature, such, for example, as are found in the interior of the sun, where could such conditions be realized upon the surface of our planet? Obviously only one such possibility existed, namely in the interior of the fission bomb itself. In the presence of the vast energy generated at the core of such a bomb, it seemed to be possible to achieve the dreadful alchemy.

The nature of the experimental techniques which led to the construction of the hydrogen bomb has not been revealed, but it is known that the core of the mechanism is constructed out of hydrogen, deuterium, and tritium. The heavy hydrogens appear to be necessary to give speed to the transformation. Where the sun is able to generate its energy at a slow and regular rate, the bomb must explode in the fraction of a second. The heavy hydrogens appear to supply the necessary trigger and to hasten the conversion process.

This lethal combination of the three hydrogens is now embedded in a covering of fissionable material, which, at the moment of explosion, generates the temperature and pressure necessary to effect the transmutation.

It is of little interest to us to know the technical details of this terrifying instrument, but it is a matter worthy of a moment's reflection, to marvel at the ingenuity of the human mind. Contemplating the great sun, the object of man's veneration, the source of inspiration for his poets, the origin of all that man possesses, the scientist has asked the great question: "What keeps the sun hot? What is the source of its constant flow of light and heat?" And from the answer, by the combination of many factors, he finally constructs an instrument more terrifying in its power than anything ever dreamed of in his most imaginative musings. Out of the magic of his lamp, Aladdin was able to command the power of the genie, but out of "the lamp of day" man has commanded a force that transcends anything created by the Arabian romancer.

### 7. *The "Giant-Dwarf" Theory of the Evolution of the Stars.*

In the new astronomy there is a theory of the evolution of the stars which has an important bearing upon our problem. A brief

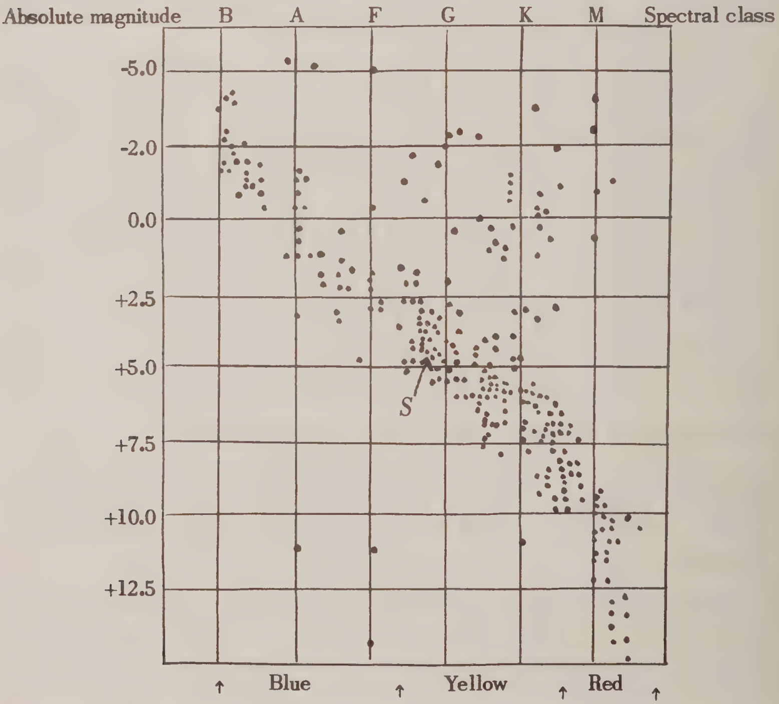

Figure 1. The absolute magnitude of stars, which measures their luminosity is correlated with their spectral class, which measures temperature. The line about which the stars seem to cluster is called the "main sequence." The approximate position of the sun is indicated by *S*.

sketch of it will indicate the numerous elements that must be considered by those who evolve a complete hypothesis to account for the source of stellar energy.

The stars "numerous as glittering gems of morning dew in the meadows of the sky" are not all of the same size nor have they the same intrinsic brightness. On one hand we find the vast giants, great red gaseous globes of world-stuff swinging their tremendous bulks in space; on the other hand we discover the tiny dwarfs, microscopic specks compared with the huge nebulous masses of their larger brethern. Both groups, however, excepting the numerically small class of the white dwarfs, have one significant thing in common, namely their comparatively low surface luminosity. This is shown by the fact that they are both red and hence belong to the lowest spectral class, which is known technically as class *M*. From these two groups, stretching like two arms across the diagram of their luminosity-density distribution, emerge the other stream.

To put the matter more precisely, let us arrange the stars, first according to their spectral class ranging from class *M*, namely those of lowest luminosity, to class *B*, namely those of highest luminosity; then, second, let us distribute them according to their absolute magnitudes. If one consults the resulting diagram (Figure 1) he will discover that the giants and the dwarfs are connected by a sequence of other stars.

There is, however, a very definite trend of stars along what Eddington calls the "main sequence" (Figure 1), that is to say, along the line which connects the tiny dwarfs with the brightest objects in the sky. A similar trend, though less clearly defined, connects the giants with the brighter stars of the main sequence. Between the great giants like Antares, Mira, and Betelgeuse with diameters equal respectively to 430, 282, and 216 million miles, to use recent approximations, and the insignificant dwarfs, there are no *M* type stars for more than six magnitudes. This unusual observation, first pointed out by E. Hertzsprung in 1905[14], was confirmed and amplified in 1914 by H. N. Russell.[15] It is the origin of what is generally called the "giant-dwarf" theory of stelllar evolution. Because of the existence of what are essentially two lines of stars across the luminosity-density diagram, the theory is sometimes referred to as the two-branch theory of star development.

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[14] *Zeitschrift für Wissenschaftliche Photog.*, Vol. 3, p. 442; also Vol. 5, p. 86 and *Astro. Nachrichten*, No. 4296.

[15] *Nature*, 1914. The conclusions of Hertzsprung and Russell were also confirmed by W. S. Adams and A. H. Joy in 1917, *Astrophysical Journal*, Vol. 46, 1917, p. 313.

Before the discovery of the relationship just described it had been agreed generally by astronomers that the younger stars are the hottest and that evolution in cosmology is a process of decay where the young and luminous bodies gradually radiate their energies into the absorbing vacuum of space. Red stars in this view are at the end of their career and will in the course of time lapse into dark, cold masses, derelicts of heaven whose only chance for renewed life would be a cataclysmic encounter with some other astronomical mass.

But another line of argument has been advanced which tends to discredit this concept of stellar evolution and to confirm the one suggested by the luminosity-density distribution described above. In 1870 J. H. Lane (1819-1880) published a paper entitled: "On the Theoretical Temperature of the Sun, under the Hypothesis of a Gaseous Mass Maintaining its Volume by its Internal Heat, and depending on the laws of Gases as known to Terrestrial Experiment."[16] This fruitful adventure in reflection was further advanced by Lord Kelvin and others and reached its climax in the exhaustive treatise on *Gas Spheres* published in 1907 by R. Emden.[17] The essential feature of these investigations was the information derived as to the thermal behavior of a spherical cloud of gas acting under the mutual attraction of its molecules and subject to the well known law of gases.

Lane reached the important conclusion, commonly known as *Lane's law*, that a ball of gas as it contracts and radiates its heat will nevertheless grow hotter. The energy gained at the expense of the gravitational potential is sufficient not only to replace the loss due to radiation, but will likewise add to the thermal content of the gas.

This striking theory has immediate application in the problem of the evolution of stars. Let us assume that a star is assembled out of the molecules of a tenuous nebula of gas. In the course of time these primary particles are drawn together and the temperature of the whole mass increases to incandescence. We are witnessing the birth of a giant star. As time goes on the great bulk with its small density contracts still further and the temperature continues to rise.

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[16] *American Journal of Science*, Vol. 50(2), 1870, pp. 57-74.

[17] *Gas Kugeln*, Leipzig, 1907.

The star advances from its low intensity of radiation as a star of type $M$ into the higher spectral classes of the main sequence. Finally, however, a limit is reached. Its surface luminosity becomes comparatively great; it enters the $B$ class, but its earlier bulk has been much reduced by this time. From its vast diameter of the order of that of Antares and Betelgeuse it has shrunk to a star of the diameter of Rigel, approximately 26 million miles, and Bellatrix, but at a gain in intense brilliance. The law of gases no longer prevails and further shrinkage must be made at the expense of luminosity. The star is sinking to its death; its energy slowly ebbs and its mass wastes away. Finally it has reduced into a dwarf, and the redness of its light indicates that the vast supply of energy which it had in its former glory as a giant has at last almost wholly disappeared.

### 8. *The Mystery of the "White Dwarf" Stars.*

As early as 1925 H. N. Russell offered the ingenious explanation that there is an interrelationship between the contraction hypothesis and the sub-atomic mechanism from which the stars received their main supply of energy.[18] He assumed a central temperature in the stars of the main sequence equal to thirty million degrees. When this temperature was attained the sub-atomic mechanism would operate, but if for any reason the temperature should fall below this figure then the generation of energy would automatically cease. The surface of the star, however, would continue to radiate energy and as the supply gradually failed contraction would set in. When this shrinkage had proceeded sufficiently far the temperature would rise according to Lane's law until the central temperature was again at the critical value. In other words thermal equilibrium was maintained by a kind of celestial thermostat, which shut off the energy when it was too great and released it again when the surface radiation fell below its normal value. This theory is a very intriguing one but has certain features difficult to reconcile with the hypothesis that subatomic radiation can neither be accelerated nor retarded by temperature or density factors. However, it is quite possible that modern nuclear studies may serve to overcome these difficulties.

The theory which Jeans substituted for the Russell picture has to do with the mechanism of ionization. In his belief the radiation of a star is retarded or accelerated by its degree of ionization. In the case of the white dwarfs, where their atoms are conceivably stripped of most of their outer shell of electrons, the possibility of the generation of energy by the coalescing of proton and electron, or in the more modern view, of positron and electron, and the annihilation of matter is seriously impaired. Jeans' theory of the process by which the stars shrink from the great giants to the tiny dwarfs is summarized descriptively in the following quotation:

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[18] "The Problem of Stellar Evolution," *Nature*, Vol. 116, 1925, p. 209.

"The contraction of the star is accompanied by an increase in its temperature and, by ionizing one ring after another of electrons, this causes the atoms to diminish in size. The star and its atoms contract together, but the star contracts steadily while the atoms, so to speak, contract by jerks. There will be times when the contraction of the star has rather outstripped that of the atoms, so that the atoms are jammed together. The ionization of a new ring of electrons may now relieve the congestion and set the whole structure free again."[19]

If this should prove to be the correct picture of the generation of energy within the stars, then we are faced with a discontinuous evolution. Transformations from one luminosity to another are not necessarily continuous ones but abrupt changes, quantum jumps of cosmological magnitude, so to speak, by which the star slides from level to level of its existence. "The evolution of the stars must no longer be compared to the steady march of an army through a perfect flat featureless plain, but rather to the movements of an army scrambling down, and possibly at times up, a succession of terraces," says Jeans.[20]

In this suggestive speculation we must give some thought to the mode of creation of the white dwarf stars the spectacular feature of which is their enormous densities, as we have mentioned earlier. How can they fit into the scheme of stellar evolution? It is evident that they do not conform to the theory outlined above because their abnormal density puts them below and to the left of the main sequence of stars upon the mass-luminosity diagram. They appear as detached specks upon the otherwise unoccupied segment of the figure below the line of the other stars. They are isolated phenomena, mysterious, inconceivable entities, whose existence seems easier to deny than to believe. How could such dense bodies ever come into being? What mechanism of the celestial cosmos could be brought into play to create matter fifty thousand times as dense as water? The densities of recently discovered dwarfs greatly exceed this figure. The star reported by G. P. Kuiper in 1935 is believed to have a density of the order of a thousand times that of the companion of Sirius.[21]

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[19] *Astronomy and Cosmogony*, Cambridge, 1928, p. 149.
[20] *Ibid.*, p. 175.

In the picturesque language of Jeans the situation seems to be the following:[22] ". . . the lowest terrace of all, the main sequence, does not lead to a further drop down. It is not bounded on its further side by a slippery unstable slope but by an impenetrable barrier formed by configurations in which the atoms lie as close together as they can be packed. In course of time most stars reach this barrier but cannot cross it, and sidle along it indefinitely. This explains the great concentration of stars along the left-hand edge of the main sequence, against which the stars seem to press like flies against a window-pane."

It is this last observation which leads to Jeans' theory of the creation of the white dwarfs. These stars on the extreme left of the main sequence are close to the edge of equilibrium. At any time factors which sustain their internal structure may give way and the star, falling through the window-pane, will slip down one of the terraces. This catastrophe in more scientific language is to be regarded as occasioned by a stripping of one of the layers of electrons from the constituent atoms and hence to a greater ionization of the material of the star. Of course the process by which this ionization takes place remains a mystery. Nothing yet discovered in nuclear physics gives any hint as to the mechanism by which it may be attained. However, the strange concentration of stars along the line representing the main sequence and the peculiar absence of stars in the region below it, excepting the mysterious white dwarfs with their abnormal concentrations, give considerable statistical support to the general theory.

In spite of the large amount of conjecture and the uncertainty of the scientific data in the present theory of stellar evolution, no one can deny the poetical nature of the speculations. The same element of melancholy beauty that we find in the lines of the *Rubaiyat* of Omar Khayyam (c. 1045-1123), another astronomer of an earlier time:

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[21] See Section 7 of Chapter 6.

[22] *Astronomy and Cosmogony*, p. 175.

> "One moment in Annihilation's Waste,
> One moment, of the Well of Life to taste —
> The stars are setting and the Caravan
> Starts for the Dawn of Nothing — Oh, make haste!"

is found again in the following suggestion of Jeans as to the possible fate of man:[23] "It is slightly disconcerting to find that our sun's position in the temperature-luminosity diagram suggests that it is pressing with perilous force against the dangerous edge of the main sequence, so that its collapse into a feebly luminous white dwarf may commence at any moment."

### 9. *The Cepheid Variables.*

In probing the source of stellar energy it is essential that we consider one of the most mysterious puzzles in the sky. This puzzle is presented by the variation in luminosity of a group of objects known as the Cepheid variables, stars which pass periodically through a regular variation in their radiation of light. The name is taken for this class of stars from Delta Cephei, an early known variable with a period of about 5.3 days, which was discovered as early as 1785 by John Goodricke. These stars, which are numerous in the globular clusters and in such objects as the Megallanic Clouds, appear like breathing spheres, pouring forth great volumes of radiation at one epoch of their cycle and lesser amounts at another, giant cosmic engines of variable production.

The most conspicuous and useful property possessed by the Cepheid variables is the relationship which exists between their intrinsic brightness and the length of time of their variation. We have already had occasion to use this law in Chapter 7 in discussing the distance of the spiral nebulae. Henrietta S. Leavitt, while studying about a hundred variable stars in the Smaller Megallanic Cloud observed in 1912 that the apparent brightness increased with the time required for the stars to pass through their points of maximum luminosity. In 1918 from a determination of the distance to several Cepheids, Harlow Shapley was able to construct a curve in which the absolute magnitude was represented as a function of their period in days.[24] This famous relationship, known as the period-luminosity curve of the Cepheid variables, is represented in Figure 2. As we have indicated earlier its great usefulness is found in the fact that the distance to objects like the spiral nebulae, which are too remote to be measured by the methods of parallax, can now be estimated accurately.

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[23] *Op. cit.,* p. 177.

Various theories have been proposed to account for the pulsation of these phenomenal stars, but the one most generally accepted was advanced by Shapley[25] and developed mathematically by Eddington.[26] This theory assumes that the variable stars, or at least a class of them, are visible examples of energy in a state of stable oscillation. This *pulsation theory*, as it is called, attributes the phenomenon to a contraction and expansion of the material of the stars under the mutual action of gravitational and thermal forces.

For example, suppose that a star like the sun were for some reason compressed. This change would augment internal forces because of gravitational attraction and elastic stresses. A considerable amount of energy would be stored in ensuing changes of potential. But this disturbance in equilibrium of the various elements of the star would then result in an increase in temperature. The gases would expand and the surface of the star, aided by the pressure of radiation, would be pushed out beyond its normal position of equilibrium. But this expansion could not be contained indefinitely. At some place it would slow down and stop. The gaseous sphere would again come under the control of its gravitational forces. Contraction would ensue and hence the great pendulum of energy would swing back and forth as the star oscillates between points of high and low density. For all of modern science there is yet a great deal of mystery in the vibration of a simple pendulum. How, for example, does the potential energy at the top of the swing transform into the kinetic energy at the bottom, and what is the mechanism which acts to bring about this transformation? How much greater then is the difficulty experienced in a contemplation of these stellar transformations where the exchange takes place between potential, thermal, and nuclear energies?

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[24] *Astrophysical Journal*, Vol. 48, 1918, p. 89; also *Mt. Wilson Contributions*, No. 151.

[25] "On the Nature and Cause of Cepheid Variation," *Astrophysical Journal*, Vol. 40, 1914, pp. 448-465. H. C. Plummer had as early as 1913 hinted that the cause of Cepheid variation was to be found in the radial movements of the atmosphere of the star. See *Monthly Notices*, Vol. 73, 1912-13, p. 665.

[26] "On the Pulsation of a Gaseous Star and the Problem of the Cepheid Variables," *Monthly Notices*, Vol. 79, 1918-19, pp. 2-22; 177-188. See also, *The Internal Constitution of the Stars*, Cambridge, 1926, Chap. 8.

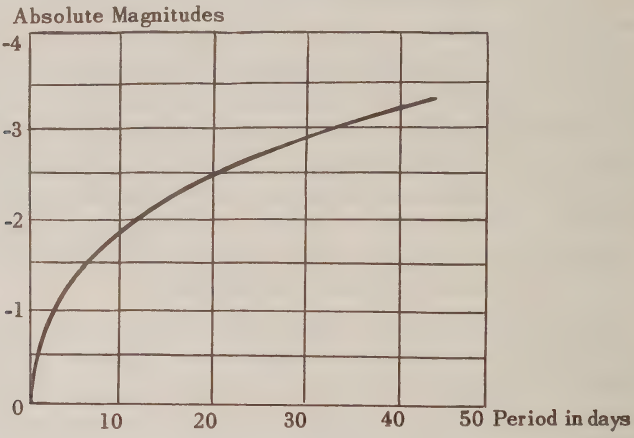

Figure 2. The period-luminosity curve of the Cepheids, which shows the absolute magnitude of these variable stars in terms of the period of their variable light-curve.

The impression must not be left, however, that the only alternative in explanation of the Cepheid variables is to be found in energy transformations, although this theory appears to give a convincing account of most of the characteristics of these curious stars. Jeans has advanced the hypothesis that the phenomenon is occasioned by a star in the process of dividing into two components, the so-called "fission theory of Cepheid variation."[27] The elongated ellipsoidal mass that would result in this process of evolution would cause both by its oscillation and by its rotation a very complex type of light curve the possibilities of which seem sufficiently numerous to account for a majority if not all of the observed facts. Several objections have been raised against this theory, perhaps the most cogent being the fact that it is necessary to assume the existence of a resisting medium surrounding the system. This medium is alternately heated or cooled by the irregular motion of the ellipsoidal star which thus generates the observed light curve. But if this medium is sufficiently dense to produce this considerable variation in energy, then it must exert large frictional forces which would alter the period in the course of time. Although in a problem as mysterious, and with as many complex elements as that of the Cepheid variables, it is not possible to reach a conclusion inevitably indicated by the data, the present opinion of the astronomers has accepted the pulsation theory as the most probable.

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[27] *Monthly Notices*, Vol. 85, 1925, p. 797. See also, *Astronomy and Cosmogony*, pp. 380-383.

### 10. *The Puzzling Novae, or New Stars.*

There are many mysteries in the sky that have to do with processes in which the nature of energy contains the kernel of the explanation. In none of these, however, is there more mystery than in the *novae*, or *new stars*, sometimes referred to more properly as *temporary stars*, which occasionally blaze up in the heavens, shine brilliantly for a short time, and then fade rapidly into oblivion. One of the most interesting and most brilliant ever recorded was the new star of Tycho Brahe (1546-1601), which appeared in 1572 in the constellation Cassiopeia. The account of its discovery follows from Tycho's own description:[28]

"On the preceding year on the eleventh day of the month of November, just after sun set, when according to my custom I was contemplating the stars in the serene heavens, I observed almost overhead that a new and unknown star shone forth, more conspicuously than the others about it. Since it was almost evident to me, who from boyhood have known all the stars of the sky perfectly (for there is no great difficulty in acquiring this knowledge), that none had ever existed in this part of the sky before, or at least no star of such conspicuous brightness, I was overcome with astonishment at this thing and was not ashamed to doubt the evidence of my eyes. But when the place had been pointed out to others and I observed that they too saw it, then doubt no longer assailed me."

This magnificent spectacle is not an infrequent one in the annals of astronomy, although none in modern times has rivaled the brilliance of Tycho's star which was equal to Venus at its brightest and hence could be observed in the daytime. In 1604 Johannes Kepler (1571-1630) reported a nova in the constellation Ophiuchus the light of which rivaled that from Jupiter.

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[28] *See Tychonis Brahe Opera Omnia*, edited by J. L. E. Dreyer, 1913, Vol. 1, p. 16. "De nova et nullius aevi memoria, a mundi exordio prius conspecta stella, quae in fine anni superioris omnium primo apparuit."

During the first part of the present century five brilliant novae have been recorded and a number of lesser ones. The most conspicuous of these was Nova Aquilae which appeared in 1918 with an intensity of light (apparent magnitude of -1.1) just short of that of Sirius. Since its distance from the earth was estimated to be of the order of 1200 light years, this remarkable nova had an absolute magnitude of -8.8, which gives an intrinsic brilliance equal to three hundred thousand suns. Even this fantastic figure was exceeded by one nova in a distant nebula which, attaining an absolute magnitude of -16.6, shone with the intensity of 380 million suns. In the brief moment of its existence it actually poured forth more radiation than all the other stars of the nebula combined.

In our search for the source of stellar energy the truth that lies within these cosmic cataclysms must not be overlooked. Apparently there exists beneath the surface of the stars a tremendous quantity of energy that can be released with spectacular speed. The expansion of Nova Aquilae, for example, proceeded at the rate of approximately one astronomical unit (93 million miles) per day, and the star grew from one of eleventh magnitude to one only a half magnitude less brilliant than Sirius within the period of a week. The velocity of expansion of its shell of gas attained at maximum approximately 1400 miles per second.

What strange mechanism could create so suddenly this tremendous burst of energy? As in everything connected with the ultimate about energy the answer is still in doubt. The phenomenon might be caused by the collision between two stars, but this possibility seems to be ruled out by the fact that more novae occur than would be accounted for by statistical probability derived from the relative distances between the stars.

The *detonator theory* advanced by W. H. Pickering (1858-1938)[29] assumes that a small object of size not greater than the earth might collide with a star and, sinking below the surface, vastly accelerate its hidden source of energy.

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[29] "A Suggested Explanation of the Phenomena Presented by a Nova," *Popular Astronomy*, Vol. 26, 1918, pp. 599-607.

But whatever the cause of the explosive character of the nova it is clear today that the only possible source for such a vast outpouring of radiant energy must be found somewhere in the mysteries of nuclear transformations. But why should this be released in such a lavish and explosive manner? Moreover, why should the class of novae appear to be divided rather sharply into two kinds, the first or common novae being of a different order of luminosity from the second class, now generally referred to as *super-novae?* Thus the average luminosity of all new stars has been estimated by Lundmark to be about 25,000 as great as that of the sun, while the brilliance of super-nova exceeds this by a factor which varies from 10 to more than 10,000. It is quite probable that the stars of Tycho Brahe and Kepler were super-novae.

The answer to these cosmic questions must await further knowledge both of the phenomena itself and of the mysteries of nuclear energy. It is suggestive to reflect, however, that the novae need not originate in massive stars in so far as the energy-matter relationship is concerned. For if an object no greater than the earth should, through some mechanism, be dissolved suddenly into radiant energy it would produce a cosmic spectacle whose energy-emission would be equivalent to about fifty million times the annual emission from the sun. Such an explosion would certainly be of the order of those observed in the most brilliant examples of the super-nova.

This raises the question whether our own sun might not some day become a nova. In such an event the results so far as the earth is concerned are graphically pictured by the imagination of the astronomer: "The earth could hardly survive such an explosion. So rapidly would the catastrophe proceed that the earth and all of its inhabitants would experience the most precipitous form of sudden death. The entire sunward side of the earth would be burned to a crisp in half an hour, and clouds of live steam from boiled-away oceans would devastate the night hemisphere even before the rotation of the earth exposed it to the sun. Within a few days, great clouds of metallic vapors would be ejected from the sun, to envelop and vaporize the earth. The only remaining trace of our earth would be a tiny condensation in a rapidly expanding shell of gas."[30] But such an event is highly improbable, although an observed variability in the solar constant by as much as five per cent shows that the sun itself is a variable star. Present inference from careful studies of the spectra of novae indicate that the stars of nova class probably belong to types resembling the white dwarfs. When the explosion has died away and the enveloping cloud of energy has been largely dissipated, the star that remains appears to be one of very high density and low intrinsic luminosity. Can we assume from this that the nova originated in a star of similar character?[31]

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[30] L. Goldberg and L. H. Aller: *Atoms, Stars, and Nebulae,* Philadelphia, 1943, 323 p.; in particular, p. 153.

### 11. *What Happens to Radiant Energy?*

In summarising the results of preceding paragraphs we must be impressed by the evidence of mystery that still envelops the nature of energy. In other chapters we have found that its relationship with the ether, or if this concept now seems obsolete, with space itself is strangely perplexing. In the present discussion we have found that the source of energy in the stars is similarly difficult, although the progress of modern nuclear physics has provided important guides toward the solution of the problem. Our present knowledge indicates that the energy is obtained at the expense of matter itself. This creation of radiant energy may take place in several ways, the most plausible at the present moment being the creation of helium out of hydrogen through the mechanism of the carbon cycle. In some stages of the evolution of the stars it is possible that the transmutation of the elements of higher atomic weights into those of lower atomic weights may be the essential factor, although no mechanism has yet been suggested for this process. Or there may be at work an annihilation of matter through the coalescing of positrons and electrons in a manner as yet not understood. Each of these theories, however, rests upon the assumption that matter, whether it disappears in the process of celestial alchemy or in the mechanism of annihilation, is transformed into energy according to Einstein's law.

We turn next to a consideration of the second problem set by the present chapter, which may throw some light upon the mystery of solar radiation. The second has to do with what we might call the density of radiation in space. Where do these vast stores of energy go which pour so steadily from the surfaces of the stars? If the universe is finite, as the theories of Einstein and de Sitter assume, then the energy of radiation cannot be dissipated in the infinite reaches of space. As the matter of the stars dissolves the density of radiant energy in space must increase correspondingly. Or is it building up a new universe at some remote distance from the boundary of the stars? Does it transform again into matter by a process as mysterious as the mechanism by which it is generated in the sun? Are the vast regions of interstellar space, which with their temperature in the neighborhood of the absolute zero seem to be the very antithesis of the interior of stars, the laboratory within which radiant energy is recondensed into the atoms from the annihilation that created it?

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[31] For a discussion of the various theories of the cause of novae see A. C. Gifford: "The Physical and Chemical Principles that underlie the Interpretation of Novae," *Scientia*, Vol. 49, 1931, pp. 169-182. For more recent developments consult L. Goldberg and L. H. Aller: *Atoms, Stars, and Nebulae*, (loc. cit.), Chap. 8.

Curiously enough, some insight into this problem is gained from a very simple question which at first appearance seems to be inconceivably remote from cosmic speculation. What is the thief that steals the charge from an electroscope? An *electroscope* is an elementary instrument consisting of a metal rod supported on a cork or other piece of insulating material and carrying two leaves of gold foil suspended from its lower end, the leaves being enclosed in a glass vessel. If a charge of electricity is placed upon the foil, the leaves will immediatey diverge, due to the repulsive action of the charge.

Now it has been observed for a long time that if a charged electroscope was sealed and set aside the divergent leaves would gradually draw together again. Some unknown thief was stealing the charge. The long search to find the thief is one of the romances of modern physical science. It was finally discovered in what is now called cosmic radiation. The history of the discovery of this phenomenon, and an account of the results of recent investigation will be discussed in the next chapter.

# CHAPTER 14.

# THE ROMANCE OF COSMIC RADIATION.

"There may be many shapes of mystery. . . And the end men looked for cometh not, and a path is there where no man thought."

Euripides in *The Bacchae.*

"The distinctive character of our own times lies in the vast and constantly increasing part which is played by natural knowledge. Not only is our daily life shaped by it, not only does the prosperity of millions of men depend upon it, but our whole theory of life has long been influenced, consciously or unconsciously, by the general conceptions of the universe, which have been forced upon us by physical science."

T. H. Huxley in *Science and Culture.*

## 1. *Cosmic Radiation.*

We return again to a further speculation about the nature of energy and its relationship to the universe as a whole. In the domain of what we might call micro-physics we have seen that the boundary between a material particle and a radiant particle becomes increasingly difficult to define. Both possess energy. More than that, the evidence converges upon the proposition that if these particles were to lose their energy entirely, then they would also lose identity of any kind. In a word the mystery of energy is also in a real sense the mystery of matter, and the identification of one involves the identification of the other. Hence the problem of micro-physics expands into the problem of what we might call macro-physics, that is to say the study of matter and energy as a whole, and we turn our speculation to the universe itself where the transfer of matter into energy is found on a cosmic scale.

The most spectacular adventure of science in this field of the relationship of matter and energy in extra-terrestrial phenomena is found in the discovery and exploration of what is now called *cosmic radiation.* The adventure began in the early years of the present century and has become one of the major interests of physical science.

The problem of cosmic radiation challenges the imagination because of the fantastic size of the energies which it has exhibited. So great are these in the cosmic particles which have been studied that unusual difficulty has attended the identification of the particles. It will be recalled that the energy of photons is proportional to their frequency, while the energy of electrons, and of other material particles, is a function of their velocities. In the case of photons the energy depends upon the place of the radiant particle in the spectrum. At one end we find the long-wave radio waves whose propagation is little impeded by material objects, but which have infinitesimal capacity to penetrate matter. At the other end of the spectrum are the gamma rays with wave lengths of the order of $5 \times 10^{-11}$ centimeters. These rays will penetrate a block of lead several inches thick. But the hardest of these rays consist of photons with energies less than three megavolts. In similar way the electrons, when accelerated by the most powerful devices of modern science, can assume energies of an order not greater than 30 megavolts. The identification of electrons of this energy is readily accomplished by bending their paths into circles with powerful electro-magnets.

But in the case of cosmic radiation particles have been found which penetrate as much as 1,100 meters of water and which have energies of the order of 30,000 megavolts. An electron which possesses so vast an energy would have a velocity which differed from the velocity of light by about four centimeters. It is clear that even the most powerful electro-magnet could make only a very small change in the curvature of the path of such a particle. Nothing even remotely resembling such incredible energies can be generated by any mechanism yet conceived by the daring imagination of the nuclear physicist. We are thus faced with the spectacular problem of the origin and character of these remarkable particles. We can do no better than begin with the history of the adventure and trace the progress of discovery from its origin in what we might call the puzzle of the electroscope.

## 2. *The Discovery of Cosmic Radiation.*

Study of the problem of cosmic radiation began in 1903, when it was observed by several British physicists[1] that the leakage from an electroscope was reduced by as much as thirty per cent by enclosing the instrument within an air-tight metal box several centimeters in thickness. This was interpreted to mean that the loss of charge experienced by the metal foil of the instrument was due to the ionizing of the air by some unknown radiation which was sufficiently *hard*, that is to say, penetrating, so that it could pass through the metal covering surrounding the electroscope. The question then was asked: What is the nature of this mysterious radiation and whence does it come?

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[1] J. C. McLennan and E. F. Burton: "Some Experiments on the Electrical Conductivity of Atmospheric Air," *Physical Review*, Vol. 16, 1903, pp. 184-192. E. Rutherford and H. L. Cooke: "A Penetrating Radiation from the Earth's Surface," *Ibid.*, Vol. 16, 1903, p. 183.

The first speculation turned to the possibility that the rays were emanating from radio-active elements in the earth and were, perhaps, identical with the gamma rays of radium. In order to test this hypothesis A. Gockel, a Swiss physicist, in 1910 took an enclosed electroscope with him in a balloon to a height of 4,500 meters (15,000 feet) and found that the rate of discharge at this elevation was not significantly different from that found at the surface of the earth.[2] The source of the mysterious radiation thus appeared to be located in the upper atmosphere or beyond it.

The intriguing possibilities awakened by this report struck fire in the imagination of the physicist, and from 1911 to 1914 the balloon measurements of Gockel were repeated by V. F. Hess,[3] a Swiss physicist, and W. Kolhörster,[4] a German. The latter attained a height of 9,000 meters (5.6 miles) and reported that the radiation affecting the electroscope first decreased slightly and then increased in a marked manner, attaining at the flight's highest point a value eight times that at the surface of the earth. As a result of these experiments Hess reached the conclusion as early as 1912 that the radiation had an extraterrestrial origin, hence its name of cosmic radiation, and entered our atmosphere almost uniformly from all directions.[5]

The first world war then put an end to the experimental work on this problem, but the investigation was resumed by R. A. Millikan and I. S. Bowen in the spring of 1922.[6] These men sent up sounding balloons equipped with specially constructed electroscopes to altitudes as great as ten miles. To their surprise they found that the rate of discharge, while it increased with altitude, as had been reported by the European investigators, was very much less than that expected. Although at the great altitudes attained by their apparatus only twelve per cent of the atmosphere was above the electroscopes to serve as a screen for the bombarding radiation, the acceleration of discharge of the instruments was only slightly greater than at lower elevations. More recent soundings, specially in three stratosphere flights which attained heights extending to 60,000 feet, have amply confirmed this altitude effect reported by the earlier investigations.[7] The conclusion derived from these studies was that the rays under investigation were of tremendous penetrating power, much greater than that of any hitherto known radiation.

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[2] "Observation of Atmospheric Electricity from a Balloon," *Phys. Zeitschrift*, Vol. 11, 1910, pp. 280-282.

[3] "Measurement of the Earth's Penetrating Radiation in a Balloon," *Akad. Wiss.*, Wien, *Berichte*, Vol. 120, 2a, pp. 1575-1585; also *Phys. Zeitschrift*, Vol. 12, 1911, p. 998; Vol. 13, 1912, p. 1084.

[4] "Penetrating Atmospheric Radiation," *Phys. Zeitschrift*, Vol. 14, 1913, pp. 1066-1069; 1153-1155.

[5] See also for an account complete to 1915, A. Gockel: "The Penetrating Radiation Present in the Atmosphere," *Phys. Zeitschrift*, Vol. 16, 1915, pp. 345-352.

[6] "High Frequency Rays of Cosmic Origin, I. Sounding Balloon Observations at Extreme Altitudes," *Phys. Review*, Vol. 27 (2), 1926, pp. 353-361.

In the meantime Kolhörster[8] had continued his researches carrying them on this time in holes and crevasses in Alpine glaciers. From the results thus obtained he had reduced very materially his estimate of the alteration of the rate of discharge with altitude. This coincided with further discoveries made by Millikan and R. Otis,[9] who experimented with the phenomenon at the top of Pikes Peak. The evidence converged to the conclusion that the cosmic rays must be of a different character from any previously known to science.

The next move in the adventure was to find some critical method by means of which the nature of the rays and, in fact, their existence could be unequivocably determined. Millikan, in collaboration with Harvey Cameron, decided to seek this proof in the waters of some deep snow-fed lake whose waters would be free from any appreciable radio-active constituent which might interfere with the experiment.[10] Such a body of water was found in beautiful Muir Lake (11,800 feet high) just under the brow of Mount Whitney in the Sierra Nevada range of Southern California.

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[7] For a summary of these explorations see I. S. Bowen, R. A. Millikan, and H. V. Neher: *Physical Review*, Vol. 34, 1933, p. 246; Vol. 46, 1934, p. 641.

[8] "Intensity and Direction Measurements of Penetrating Radiation," *Preuss. Akad. Wiss.*, Berlin *Sitzungsberichte*, Vol. 34, 1923, pp. 366-377.

[9] "High Frequency Rays of Cosmic Origin, II. Mountain Peak and Airplane Observation," *Phys. Review*, Vol. 27, 1926, pp. 645-658.

[10] "High Frequency Rays of Cosmic Origin, III. Measurements in Snowfed Lakes at high Altitudes," *Phys. Review*, Vol. 28, 1926, pp. 851-868.

Here in the heart of the mountains the incontrovertible proof was found for the existence of a radiation of immense penetrating power. During the last ten days of August, 1925, the two experimenters lowered their electroscopes into the waters of the lake to varying depths down to 67 feet. Is there not poetry in the adventure? "I have stolen the golden vases of the Egyptians," said Kepler in the exultation of his discovery of the harmonic law of planetary motion. Is there less thrill for us in the simple language of the American experimenters in their more recent probing of cosmic mysteries?

*"Our experiments brought to light altogether unambiguously a radiation of such extraordinary penetrating power that the electroscope-readings kept decreasing down to a depth of* 50 *feet below the surface."*[11]

The results of the experiment showed that a radiation of sufficient penetrating power to pass through the depths of our atmosphere and through fifty feet of water besides, is bombarding the surface of the earth from some source within the mysterious region known as interstellar space. This penetration was at least as great as 73 feet of water, the mass of the atmosphere above the lake being estimated as equivalent to 23 feet of water. This statement translated into other terms means that these rays would exhibit their activity through six feet of lead. When one reflects that the most penetrating X-rays used therapeutically will not penetrate a half-inch of lead the remarkable character of the new radiation is strikingly revealed.

In order to check their conclusions the experimenters next moved to Lake Arrowhead, an expanse of water in the San Bernadino Mountains, 300 miles south of Muir Lake and 6,700 feet lower in altitude. The experiments were again repeated and identical results obtained when the corrections were made for the absorbing power of the blanket of air represented by the difference between the two altitudes. The report says:

. . .*"Within the limits of observational error, every reading in Arrowhead Lake corresponded to a reading* 6 *feet farther down in Muir Lake, thus showing that the rays do come in definitely from above, and that their origin is entirely outside the layer of atmosphere between the levels of the two lakes."*[12]

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[11] *Loc. cit.*, p. 855; see also *Proc. of National Academy of Sciences*, Vol. 12, 1926, p. 51.

[12] *Loc. cit.*, p. 856; see also, *Proc. of the National Academy of Sciences*, Vol. 12, 1926, p. 52.

Since these original experiments were made, the conclusion that there exists a radiation of extraordinary energy has been amply confirmed. In 1934 J. Clay[13] reported that the rays penetrated several hundred meters of water. V. C. Wilson[14] measured the penetrating power of the radiation through a quarter of a mile of rock, equivalent to 1,100 meters of water, and found that the intensity was one ten-thousandth that of the radiation incident upon the upper surface of the rock. An even greater penetration was reported in 1941.[15] The amazing energy of the rays may be ascertained from the fact that approximately 99 per cent of the most penetrating component of the radiation will pass through material equivalent to a meter thickness of water.

Many questions were inevitably invoked by the discovery of these remarkable rays. From what part of the sky do they come in the greatest abundance? Is it from the concentration of stars in the Milky Way? What is their nature? Are they photons of a wave-length far smaller than that of the most penetrating gamma radiation? Or are they particles of a velocity hitherto unknown in the laboratories of the earth? Are the rays manifestations of energy transformations on the surface or within the depths of stars, or have they, perhaps a more mysterious origin in the cold recesses of space itself?

### 3. *Geographical Distribution of Cosmic Rays.*

The important questions which we have proposed in the preceding section were next explored.[16] The scientists whose adventures we are describing decided to continue their investigations in the lakes of the high Andes in South America. Readings similar to those taken at Muir and Arrowhead Lakes in the northern hemisphere were made in Lake Miguilla, near Caracoles, Bolivia, and in Lake Titicaca. The former is a snow-fed lake 125 feet deep at an altitude of 15,000 feet and is entirely surrounded by mountains. This was regarded as a particularly favorable situation for the investigation of the source of cosmic rays because the mountains formed a perfect screen against radiations which might be generated by the lightning in thunderstorms, possible cause of the rays.

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[13] *Physica*, Vol. 1, 1934, p. 363.

[14] *Physical Review*, Vol. 53, 1938, pp. 204 and 337.

[15] Y. Nishina, Y. Sekido, Y. Miyazaki, and T. Masuda: *Physical Review*, Vol. 59, 1941, p. 401.

[16] "High Altitude Tests on the Geographical, Directional, and Spectral Distribution of Cosmic Rays," *Physical Review*, Vol. 31, 1928, pp. 163-173.

The results in South America agreed substantially with those found in California and were interpreted by the investigators as pointing unquestionably to the cosmic origin of the radiation. The rays, according to the report, enter the atmosphere of the earth with undiminished intensity both night and day from all directions.

Speculations as to the place of their origin naturally directed attention to the question whether or not the position of the Milky Way had any influence upon the discharge of the electroscope. This question had been investigated previously by Kolhörster,[17] who reported that at an altitude of 11,650 feet the presence of the Milky Way overhead caused an increase of approximately fifteen per cent in the intensity of the radiation, although he failed to detect such variation at sea level. The experiment was repeated by Millikan and Cameron in a pocket in the mountains at a height of 15,400 feet where the Milky Way was practically out of sight for four hours and a half. These experimenters reached the conclusion that the influence either of acceleration or diminution of the intensity to be traced to the Milky Way is at most very slight. A similar conclusion holds for the influence of the sun in the creation of cosmic radiation.

With the establishment of the existence of a radiation of great intensity, attention was next directed to its character. Was it essentially a stream of photons of very short wave length, or did it consist of high-speed particles? Millikan and his collaborators inclined to the first hypothesis and as the result of experiments conducted at Churchill (730 miles due south of the North pole on the west side of Hudson bay) they reached the conclusion: "First, *that the cosmic rays enter the earth uniformly from all portions of the sky;* Second, *that they consist as they enter the earth's atmosphere of ether waves, not of electrons.*"[18]

This conclusion, the photon character of the rays, had been criticized through arguments advanced by W. Bothe and Kolhörster[19] to show that the phenomena attributed to cosmic radiation might be due to high-speed electrons. The answer to this controversy would depend upon the discovery of a latitude effect, for if the rays were essentially charged particles this would be revealed in the influence exerted upon them by the magnetic field of the earth. For a while confusion prevailed in the matter. J. Clay, a Dutch physicist, sailed on a series of voyages between Amsterdam (53° N) and Batavia (6° S) and took readings on the intensity of the radiation.[20] He reported a large latitude effect, the variation between latitudes in the temperate zone and those in the equatorial region being as great as fifteen to twenty per cent. On the other hand Bothe and Kolhőrster, travelling northward from latitude 53° to latitude 81°, found no change in the intensity of the radiation.

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[17] *Sitzungsberichte der Preus. Akad.*, Vol. 34, 1923, p. 366.

[18] "On the Question of the Constancy of the Cosmic Radiation, and the Relation of these Rays to Meteorology," *Physical Review*, Vol. 36, 1930. pp. 1595-1603.

[19] *Zeitschrift für Physik*, Vol. 56. 1929, p. 751.

But the mystery was soon elucidated with the publication by P. S. Epstein[21] of a calculation showing the effect of the earth's magnetic field upon electrons entering the earth's atmosphere. His conclusion was that even very high-speed electrons, with energies of the order of a thousand megavolts, could not penetrate the earth's magnetic field at magnetic latitudes lower than 58° north, while above this latitude they would enter the atmosphere with uniform intensity. A series of observations made over the earth soon confirmed these conclusions.

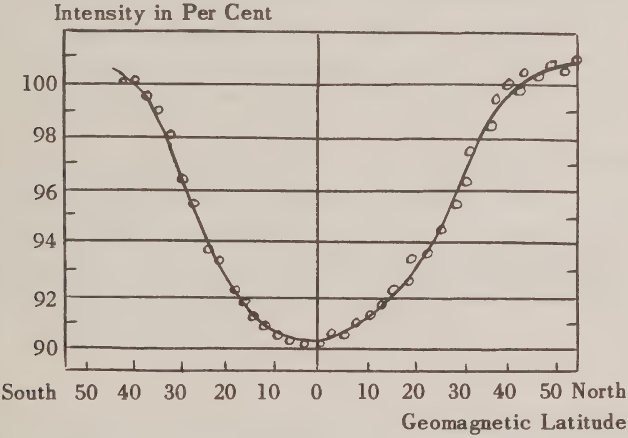

Figure 1. The geomagnetic effect of cosmic radiation.

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[20] *Proc. Acad. Wetensch.*, Amsterdam, Vol. 30, 1927, p. 1115; Vol. 31, 1928, p. 1091; Vol. 33, 1930, p. 711.

[21] *Proc. of the National Academy of Sciences*, Vol. 16, 1930, p. 658.

Expeditions by Millikan and H. V. Neher,[22] by A. H. Compton and R. N. Turner,[23] and by an extensive cooperating group of physicists, who made measurements at more than a hundred widely separated points on the earth's surface,[24] all showed this latitude effect. A careful mapping along geo-magnetic lines was thus achieved, the results of which are shown in Figure 1.[25]

The conclusion of these studies led to the immediate interpretation that cosmic radiation must consist in part at least of charged particles, since photons would be unaffected by the magnetic field of the earth. A second inference derived from these investigations was that the energy of such particles must be of an order greater than 2,500 megavolts, since energy of such magnitude would be required for them both to penetrate the atmosphere and to show the proper latitude effect. T. H. Johnson[26] has estimated that the average energy of cosmic ray particles coming in at the equator must be of the order of 30,000 megavolts. When one reflects that the highest laboratory energies are of the order of thirty megavolts[27] and the energies of atomic fission are of the order of 218 megavolts, the incomprehensible nature of the new particles is readily apprehended.

With the establishment of the latitude effect of the new radiation, the question arose as to the possibility of a longitude effect as well. This was a matter of considerable significance since, if the cosmic rays consisted of charged particles as the latitude effect appeared to indicate, an asymmetry with respect to longitude would disclose the nature of the preponderating charge on the particle.

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[24] *Physical Review,* Vol. 41, 1932, pp. 111, 381; Vol. 43, 1933, p. 387.

[23] *Physical Review,* Vol. 52, 1937, p. 799.

[22] *Physical Review,* Vol. 50, 1936, p. 15.

[25] For a comprehensive account of these explorations and for the state of progress in the study of cosmic radiation the reader is referred to the following works: (1) R. A. Millikan: *Electrons* (+ and —), *Protons, Photons, Neutrons, and Cosmic Rays,* Chicago, 1935, x + 492 p. (2) Millikan: *Cosmic Rays*, New York, 1939, viii + 134 p. (3) P. Auger: *What are Cosmic Rays?* (translation by M. M. Shapiro), Chicago, 1945, vi + 128 p. + 22 plates. (4) J. D. Stranathan: *The "Particles" of Modern Physics,* Philadelphia, 1942, xvi + 571 p., in particular, chapters 12 and 13. (5) D. J. X. Montgomery: *Cosmic Ray Physics,* Princeton, 1949. viii + 370 p. (6) J. G. Wilson: (Editor): *Progress in Cosmic Ray Physics,* Amsterdam and New York, 1952, xvi + 557 p.

[26] *Physical Review,* Vol. 53, 1938, p. 499.

[27] A recently constructed cosmotron has been reported to have generated energies of the order of 2300 megavolts.

If these were positive, either positrons or protons, then the direction of the horizontal component of the earth's magnetic field would give to them a characteristic westerly direction. On the other hand, if the particles carried a negative charge, then the magnetic field would alter their course in an easterly direction. If the particles were equally distributed between positive and negative no difference would exist.

If we represent by $W$ and $E$ the respective intensities of the radiation from the west and east, then the per cent of asymmetry can be defined by the following formula:

$$\text{Per cent asymmetry} = 200\,\frac{W-E}{W+E}.$$

Using this definition, experimenters reported that when the intensity of the radiation coming in at an angle 45° from the zenith was measured near the magnetic equator, a positive asymmetry of about 14 per cent was found. That is to say, there was definitely an excess of cosmic rays from the westerly direction. This asymmetry declined to 2 or 3 per cent at latitudes of 40° and almost disappeared at latitudes of 50° and over.[28]

Hence we reach the tentative conclusion that there is a predominance in the rays of positively charged particles. But a mystery still remains! For an abundance of such particles should make itself felt in a large increase in the potential of the earth itself. Since no such effect is observed there must exist some compensating neutralization by negative particles.

### 4. *The Altitude Effect of Cosmic Radiation.*

While the extensive studies which we have described were being made to show the geographical effects of cosmic radiation, the intensity of the radiation with respect to altitude was also the subject of vigorous investigation. Continuing the pioneer work of the discoverers of cosmic radiation, scientists on numerous expeditions sounded the depths of the atmosphere with balloons, airplanes, and flights into the stratosphere.

These flights have extended to within less than two per cent of the top of the atmosphere, that is to say, to a height where more than 98 per cent of the air by weight was below the instruments.

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[28] T. H. Johnson: *Physical Review*, Vol. 45, 1934, p. 569; Vol. 48, 1935, p. 287.

The results obtained by these explorations revealed an astonishing and unexpected effect, namely, that at a certain altitude, depending upon the geomagnetic latitude of the flight, the intensity of the cosmic radiation reached a maximum and at higher altitudes declined.

In soundings made at Madras, India, magnetic latitude 3° North, the number of pairs of ions formed per cubic centimeter per second in an ionization chamber with pressure at one atmosphere was approximately 95, while the same maximum at Fort Sam Houston, Texas, magnetic latitude 38.5° North, was more than twice this number, namely 240. On the other hand the stratosphere flights of Settle and Fordney in 1933 and of Kepner, Stevens, and Anderson in 1934 at the magnetic latitude of 53° N did not reveal the maximum, which, if it existed, exceeded a count of 500 ion pairs.

The maximum at Madras, India occurred at a point 90 per cent of the distance to the top of the atmosphere and that observed at Fort

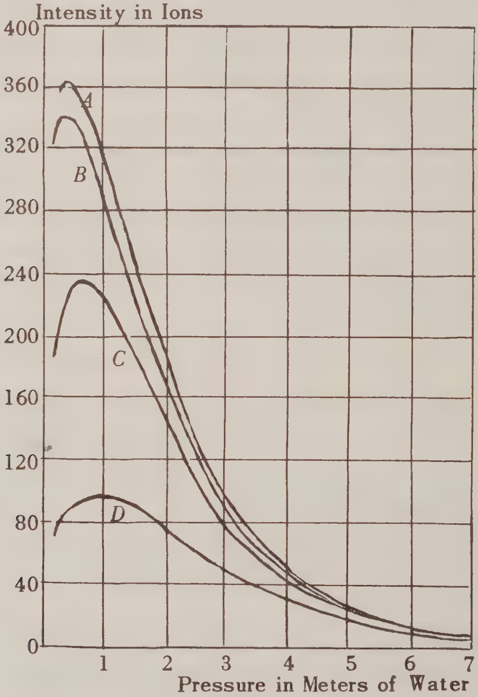

Figure 2. Latitude-altitude effect of cosmic radiation. Ionization (measured in the no. of ion pairs per cubic centimeter at a pressure of one atmosphere) observed at altitudes (measured in terms of meters of water where 10 meters = 1 atmosphere) and geomagnetic latitudes as follows: *A*: Saskatoon, Canada, 60° N; *B*: Omaha, Nebraska, 51° N; *C*: San Antonio, Texas, 39° N; *D*: Madras, India, 3° N.

Sam Houston at a point 94 per cent of the distance. Since the stratosphere flights did not penetrate more than 93 per cent of the air, and since the point of maximum intensity increased with increasing latitude, it is clear that the flights did not attain an elevation great enough to reveal the maximum. Later balloon explorations at latitudes of 51° (Omaha, Nebraska) and 60° (Saskatoon, Canada), which penetrated 98.8 per cent of the atmosphere, did reveal the existence of maxima exactly as in the case of the flights at lower geomagnetic latitudes. In the first instance the maximum of 337 ion pairs appeared when the pressure was equivalent to 0.3 of a meter of water (97 per cent penetration) and in the second the maximum of 361 ion pairs was found at a pressure of 0.4 of a meter of water. The results of these soundings are shown graphically in Figure 2.

Even higher altitude data have been obtained, which confirm the conclusions already recorded. At Lima, Peru, on the magnetic equator, soundings were made to an altitude of 21,300 meters (70,000 feet) and at Washington, D. C. (magnetic latitude 50°) to 35,300 meters (116,000 feet). The pressure at this altitude was approximately 4 millimeters of mercury (5 centimeters of water), which meant that only one half of one per cent of the atmosphere remained above the highest point attained by the instruments. The significant fact thus obtained by these flights was that the ionization reduced rapidly after maximum point had been attained.[29]

These experiments threw considerable light upon the nature of the radiation which had been so dramatically discovered. The magnetic effect, as we have said before, appeared to answer unmistakably the question: Was the fundamental constituent of the radiation a group of photons of enormously high frequency, or was it a swarm of charged particles with velocities approaching that of light? The new experiments tended to confirm the previous answer to the question, namely, that the primary radiation consisted of charged particles. They also indicated that the effects measured near the bottom of the atmosphere were largely those derived from the secondary effects caused by the collision of the primary radiation with the constituents of the atoms that made up the atmosphere itself.

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[29] The results which we have just given are found in the following papers: (1) I. S. Bowen, R. A. Millikan, and N. W. Neher: *Physical Review*, Vol. 44, 1933, pp. 246-252; Vol. 46, 1934, pp. 641-652; Vol. 52, 1937, pp. 80-88; Vol. 53, 1938, pp. 855-861; (2) R. A. Millikan, H. V. Neher, and S. K. Haynes: *Physical Review*, Vol. 50, 1936, pp. 992-998; (3) S. A. Korff, L. F. Curtiss, and A. V. Astin: *Physical Review*, Vol. 53, 1938, pp. 14-22; (4) L. F. Curtiss, A. V. Astin, L. L. Stockman, and B. W. Brown: *Physical Review*, Vol. 53, 1938, pp. 23-29.

To summarize the situation as it now appeared, the following conclusions were derived from the information obtained from the depth-soundings of the atmosphere:

1. The latitude effect is more pronounced in the upper atmosphere than at sea-level, which indicates that part of the radiation must consist of charged particles, the directions of which are influenced by the magnetic field of the earth.

2. Most of the measurable effects of cosmic radiation are generated in the atmosphere of the earth.

3. The radiation consists of at least two parts. The first of these is a primary ray of unbelievable intensity which is isotropic in distribution, that is to say, enters the earth's atmosphere from all directions. It is capable of penetrating the atmosphere itself (equal to 10 meters of water) and is found at depths of water and rock equivalent to an additional 1,000 feet or more of water. The second component, which is the origin of the measurable phenomena, is a secondary effect that is generated in our own atmosphere as the result of the bombardment of atomic nuclei by the primary radiation.

### 5. *The Nature of Cosmic Radiation.*

From the description of the phenomena attributed to cosmic radiation it is at once clear that the nature of the radiation is complex. Its curious geomagnetic and altitude distributions, combined with measurements of its vast penetrating power, indicate that the radiation must have more than a single component.

The most logical interpretation of results at the present time is that the primary rays approach the earth from all directions, that is to say, they are distributed isotropically in space. But when the atmosphere is bombarded by the primary radiation secondary effects result which are what we observe in the various devices used to measure them. Among these effects one of the most dramatic is that of high momentary ionizations to which have been given the name of *bursts.* First observed in 1927 by G. Hoffman, these bursts have been the subject of much study since.

Closely related to the bursts and probably essentially the same phenomenon, are cosmic ray showers. A *shower* is observed when several counters, non-linearly arranged, are triggered simultaneously. Since the counters are not in line with one another, it is clear that the disturbances are not caused by the same particles; but it is equally obvious that the probability is very small that the counters should be simultaneously affected by unrelated particles. Hence, it must be assumed that there is a common origin for the showers and the most likely explanation appears to be that the showers are created by the ions released from the bursts.

The simplest mechanism that will account for the burst and the simultaneous shower is that of nuclear collision. A primary ray strikes the nucleus of some atom and the phenomena observed follow as secondary effects. The *nucleons,* a generic name for the protons and neutrons contained in the nucleus, are scattered in all directions. The constituent particles also doubtless contain electrons of both signs and mesons which were described earlier in Chapter 11. The number of particles in a single shower can be as great as 300 and the total energy can reach a magnitude of fifteen thousand megavolts.

From the mass of observation which has been studied relating to these phenomena, current theory has assumed the following propositions:

The primary ray, originating in interstellar space, enters the upper atmosphere and there makes a nuclear encounter. The energies involved in this collision are of almost incredible magnitude, since many of them are in excess of $7.5 \times 10^3$ megavolts and some have been reported of the order of $10^9$ megavolts. The nucleons are scattered in all directions with the high velocities derived from the encounter and out of the destruction of the nucleus is born the ephemeral pi-meson. These particles together constitute the *hard* component of cosmic radiation.

The *soft* component of cosmic radiation is derived from the collision of the particles of the primary encounter with other nuclei in the atmosphere of the earth. The principle constituent of this component is the mu-meson, which in its rapid decay contributes energies to the nucleons of an order of $2.5 \times 10^3$ megavolts.

Since these conclusions are, in a sense, conjectures, that is, deductions from the available data, it was a matter of considerable interest when the announcement was made in 1948 that mesons had actually been created in the Berkeley cyclotron. E. Gardner and C.

Lattès, using a beam of high-speed alpha particles, bombarded targets made of carbon, beryllium, copper, or uranium and observed that pi-mesons of both signs resulted from the encounter.

### 6. *Theories on the Origin of Cosmic Radiation.*

The facts have now been presented to the reader as they have emerged from the work of a legion of investigators studying the tracks and stars in their ionization chambers and their photographic emulsions. Although still subject to modification and re-evaluation, the general theory has assumed a form that has correlated in a consistent manner the diverse observations. But the great question has not yet been answered: What is the origin of the primary ray? Whence come the vast energies that pour so abundantly from the depths of interstellar space?

In an appraisal of the philosophical aspects of the new physics it is necessary at least to raise the question. The astronomers, surveying the great problem of the evolution of the stars, have made demands for new matter with which the cosmic cycle can be maintained. The recent theories of F. Hoyle and R. A. Lyttleton and of H. Bondi and T. Gold[30] have required the creation of enough matter to replace the galaxies which disappear over the horizon of space.

We have shown in earlier chapters how man has finally been able to unlock the door to one of the great problems of the universe, the conversion of matter into luminous energy. We have also had a brief glimpse, but nothing more, of the reverse mechanism by means of which photons can be transformed into matter. But the matter thus created had only a momentary existence in the form of a positron which disappeared again almost at the moment of its creation. How, then, can the astronomer obtain a continuous creation of the hydrogen atoms which, accumulating in the dark regions of space, gradually cohere under gravitation and slowly build up into the luminaries of the sky?

Various theories have been propounded in a very cautious and tentative manner to account for the existence of cosmic radiation. The first of these was proposed in 1928 by R. A. Millikan and G. H.

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[30] For an exposition of these theories see H. Bondi: *Cosmology*, Cambridge, 1952.

Cameron[31], who sought to find the necessary energy in celestial alchemy by means of which one atom was created from another. Their most interesting assumption is found in the proposition that cosmic energy originates in interstellar space and not in the stars themselves. "The essentially new element that we have introduced," they said, "is the experimental observation that the creative, or atom-building, processes do not appear to take place at all in the stars, or in those parts of the universe where matter is found in appreciable densities and temperatures, but only in the interstellar spaces where densities and temperatures are essentially zero."

But it was soon evident that atom-building could not be the answer to the problem, for the energy of the transformations was far below the vast requirements of cosmic radiation. Many years later Millikan again returned to the problem and, in collaboration with H. V. Neher and W. H. Pickering,[32] sought to find the origin in the annihilation of matter itself. But here again, even in the complete extinction of the heaviest atoms, there was not sufficient energy to account for that carried by some of the hardest rays. Uranium itself would not yield energies exceeding 120 thousand megavolts, and some of the observed radiations exceeded this limit by a factor of ten.

Since the evidence has seemed to point inevitably to the conclusion that the origin of cosmic radiation must be in the dark reaches of space itself and cannot be found in the energy transformations of the stars, even in the novae and the super-novae, attention has been directed to the possibility of finding accelerating mechanisms in interstellar space. An ingenious theory was proposed by E. Fermi[33] in 1949, who sought for the source of the high energies of the cosmic particles in the galaxy itself. The data which he considered in his speculation were the particles of cosmic dust assumed to fill interstellar space.

Recognizing the uncertainty in such estimates, Fermi assumed that interstellar space is filled with matter at low density, let us say, of the order of one hydrogen atom per cubic centimeter. Condensations with densities ten or a hundred times as great occur in many places, as one may infer from the dark regions observed in many places in the sky. A further hypothesis was introduced which gave to these great clouds of cosmic dust a rotational velocity of the order of 30 kilometers per second. From this rotation there is generated a magnetic field. Into this magnetic field there comes a proton, which already possesses a high energy of its own. Then, according to the theory, this particle will spiral around the lines of force in a circular path with a radius of the order of $10^{12}$ centimeters, and will derive from the field an accelerated velocity which ultimately reaches the energy levels observed in the primary rays of cosmic radiation.

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[31] "The Origin of Cosmic Rays," *Physical Review*, Vol. 32, 1928, pp. 533-557.

[32] *Physical Review*, Vol. 61, 1942, pp. 397-407.

[33] "On the Origin of the Cosmic Radiation," *Physical Review*, Vol. 75, 1949, pp. 1169-1174.

The nature of the forces which contribute to the original motion of the matter in interstellar space is not discussed, but the author assumes that there must exist a regenerating factor to maintain the energy. "The motions of the interstellar matter are not quite conservative," says Fermi, "in spite of the reduced friction, caused by the magnetic fields. One should assume, therefore, that some source is present which steadily delivers kinetic energy into the streaming motions of interstellar matter. Probably such a source of energy ultimately involves conversion of energy from the large supplies in the interior of stars."

The theory of Fermi has recently been modified by R. D. Richtmeyer and E.Teller[34] and by H. Alfven,[35] who prefer to find the origin of the great energies of the particles in the space around the sun. They assume the presence of extended electromagnetic fields in interplanetary space, which are the accelerating mechanisms for the primary particles of cosmic radiation. High velocity protons are swept up in these moving fields and circulate in the neighborhood of the planets for thousands of years until the radiation becomes isotropic.

Other theories have been proposed but, as in those which we have surveyed, they are based upon hypotheses difficult to establish by experiment. But from these grand extrapolations there emerges one common hope, that the universe of stars as we know it now is a regenerating machine. The energy which is poured so lavishly into space at the expense of matter, must in some distant region of the sky be reconverted into other material objects to replace those which have been destroyed in the interior of the stars.

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[34] "On the Origin of Cosmic Rays," *Physical Review*, Vol. 75, 1949, pp. 1729-1731.

[35] "On the Solar Origin of Cosmic Radiation," *Physical Review*, Vol. 75, 1949, pp. 1732-1735; Vol. 77, 1950, pp. 375-379.

It is an intriguing idea to assume that the stars and interstellar space form the two parts of a great cosmic engine, working on a perfect Carnot cycle. Within the stars we have the highest temperatures that can be attained by matter in the universe; within the dark reaches of interstellar space we have conditions that approximate absolute zero. An engine working between these great extremes would have perfect efficiency, the exact conditions for perpetual regeneration. From the surfaces of the stars the photons pour, energized at the expense of the material which has been consumed. Reaching finally the regions of absolute zero, these electro-magnetic waves are spontaneously reconverted into stable matter. This cosmic dust, under the mysterious force of gravitation, in the course of time coheres and forms a nebulous cloud. As this contracts, heat is generated according to Lane's law described earlier and the dark nebula is converted into a red giant. The evolutionary process continues until the height of the cycle is reached in the blazing magnificence of the blue giants.

But in this evolutionary process one element is missing. If cosmic radiation is an evidence of the creation of cosmic dust from the photons poured into space by the stars, how do the particles attain their enormous energies? No evidence exists that photons of the proper energy escape from the surface of the stars. Their average energies, as determined from the observed temperatures of the hottest stars, are far below that of even the least energetic of the cosmic particles. But is it possible that there is a link between the inertial framework of the galaxy and the photons, or between the gravitational field of the universe and the photons, which increases their frequencies as they approach the empty regions of space? Since no answer to such a speculation is even hinted at by cosmoligists or physicists, we can only wait until more knowledge is obtained from the busy laboratories of those who are studying these strange cosmic particles that bombard our atmosphere from all directions.

# CHAPTER 15.

# MAN AND HIS RELATIONSHIP TO THE NEW KNOWLEDGE.

"I know of no way of judging the future, but by the past."
Patrick Henry in a *Speech*, March, 1775.

"In ten or twenty thousand years, we may imagine, the face of the earth has changed but little. The sky, the monotonous wash of the sea, the hills, the plains, the yellow wastes of the desert, must have looked very much to the cave-dwellers as they do to us. The human animal, too, has not varied greatly—his dress, his speech, his social relations, a little; the main activities of his daily life, his pleasures, his moral and economic problems, the struggle for existence, the ceaseless round of birth and death, remain much the same. What has so wonderfully changed in these ten or twenty thousand years is the human mind and its outlook on the world."
Carl Snyder in *The World Machine* (1907).

## 1. *Man's Relationship to the Universe.*

In the preceding chapters of this book we have traced the origin of man's speculations where they turned to the world of matter, space, time, and the activity known as energy. We have explored the methods that man has used to probe the mysteries of nature. We have reviewed the things that he has discovered, the instruments that he has used, the speculations that have arisen in his attempts to synthesize his new knowledge. We have come finally to his discovery of how to utilize the unbelievable energy of the atom and the problems which it has suddenly thrust upon an awe-struck world.

We must turn, therefore, to that phase of the problem which is after all the most important. What is man's relationship to the universe of matter, energy, and their geometrical framework of four dimensions? What part has the human mechanism in the cosmic scheme? What is man to believe about the mysteries into which he has probed so deeply? And how is he to use the immense forces which modern science has finally exhibited?

Man's experience is greatly limited. Unassisted he peers into the unknown reality about him through tiny key-holes. We have exhibited the size of one of these key-holes in comparison with the great gate in the case of radiation. There the infinitesimal slit, the band of visible light, is less than the span of fifty-millionths of a centimeter. What an insignificant space is this in comparison with the total range of the spectrum, which, beginning with the inconceivably small wave lengths of a cosmic ray photon, extends to electromagnetic waves hundreds of miles in length? How is man to attain knowledge of the real world through such a tiny crack in the wall of nature? It perpetually amazes us to think that he has been able to see so far and to probe so deeply.

In one sense the question: What is the nature of reality? is but the phrasing of another. What is the nature of man who asks the question? Into his fragile hands has now been given a control of natural forces of such magnitude that his character, rather than the mysterious mechanism of nature, promises to be the dominating factor in the world. For the first time his activities take precedence over the vagaries of the natural world. It will be instructive, therefore, to examine man himself in the concluding chapter of our history.

Man has been examined from various points of view by the philosophers, the poets, and the scientists of the past. A few of these appraisals may furnish a proper setting for a more scientific examination. "Man is a two legged animal without feathers,"[1] said Plato, a statement which so aroused the disgust of Blaise Pascal (1623-1662), mathematician and philosopher, that he himself essayed a definition:[2] "What a chimera, then, is man! what a novelty, what a monster, what a chaos, what a subject of contradiction, what a prodigy! A judge of all things, feeble worm of the earth, depository of the truth, cloaca of uncertainty and error, the glory and shame of the universe." Does this satisfy any more than the words of Plato? Turn then to the anthropologists. In the words of A. R. Wallace (1823-1913), English naturalist and propounder of the theory of natural selection, they place man apart, "as not only the head and culminating point of the grand series of organic nature, but as in some degree a new and distinct order of being."

Can the psychologists and philosophers tell us more? They are evasive; they will not give a definition; they will only cite characteristic attributes and behaviours of the assemblage of atoms that constitute his being. In the words of William James (1842-1910) we find:[3] "*In its widest possible sense, however, a man's Self is the sum total of all that he can call his,* not only his body and his psychic powers, but his clothes, and his house, his wife and children, his ancestors and friends, his reputation and works, his lands and horses, and yacht and bank account." The behaviourist school would have us think in the summary of J. B. Watson: "Our personality is thus the result of what we start with and what we have lived through. It is the 'reaction mass' as a whole."[4]

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[1] "Man is the plumeless genus of bipeds, birds are the plumed." *Politicus,* 266. See also Diogenes Laertius: *Life of Diogenes,* vi.

[2] *Thoughts,* Chap. 10.

[3] *Principles of Psychology,* Vol. 1, 1890, p. 291.

Are these generalizations satisfactory or do we require more? Let us turn to the poets for the literary appraisal. "What a piece of work is man!" mused Hamlet, "how noble in reason! how infinite in faculty! in form and moving how express and admirable! in action how like an angel! in apprehension how like a god! the beauty of the world! the paragon of animals! And yet, to me, what is this quintessence of dust?"[5]

Perhaps we cannot go beyond these estimates and perhaps it is impossible for man to explain his nature to himself. Avoiding esoteric thoughts, however, we prefer to regard man as a mechanism, mysterious though the mind-stuff which moves it may be, that peers into the unknown through five tiny cracks in the wall dividing sensuous perception from the thing perceived. That which is designated under the psychologist's term of behaviour means to us only the power to assemble world-points into world-lines and world-lines into the world-tubes along which our knowledge moves. This is the mechanism that wants to explore the meaning of reality. This is the thing that would look into the unknown and the unknowable,—an entity which receives stimuli from five insignificant sources and which has the mysterious power of classification and arrangement. Man is characterized by his ability to distinguish between cause and effect, to recognize curious recurrences in inanimate things and to arrange them into the system of science. Is it any wonder that a being so circumscribed by ignorance, with vision so narrow and with entrances to truth so restricted, should be puzzled at what he finds? The wonder would be many times multiplied were it found that all stimuli which came through the narrow crevices of perception could be joined into world-tubes that would everywhere accord with rational expectation.

In the final summary we must review the great speculative material through which we have passed. We must see what it means in terms understandable to this mysterious creature who has been able to feel so marvelously along the great spectrum of radiation. We are like the three blind men who, according to the tale, fell to examining an elephant that they had met on their travels. The first, who felt of its leg exclaimed: "Why the thing is like a tree"; the second, whose examination began with the trunk, cried out: "No, you are wrong; the thing resembles a serpent"; the third who had grasped the elephant's tail, said: "Both of you err; this thing is like a rope." Any problem of epistemology approached from different points of view must lead to similar divergent opinions.

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[4] *Psychology from the Standpoint of a Behaviorist,* Philadelphia, 1919, p. 420.
[5] Act 2, Scene 2, l. 316.

### 2. *Man as a Subjective Philosopher.*

In our proposed summary let us first turn to man as the subjective philosopher. This was his earliest role and one in which he has perhaps been the most successful.

The struggle in subjective thought has been waged around the question: Is nature continuous or is nature atomic? We find among the geometers a curious blending of the two concepts, and this attempt to mingle two fundamentally different modes of thinking has reached fruition in the postulates of the mathematical theory of point-sets.

The material ether of the nineteenth century was very far from furnishing a continuous medium. As soon as the concepts of elasticity or of density were invoked an atomistic philosophy of nature was necesssarily postulated. However, the wraith of the ether that remains in the twentieth century, if it may be thought of in terms of reality at all, is fully as continuous as the psi-function of the wave mechanics. And less metaphysically than this, a medium, material or otherwise, which is capable of supporting undulations as small as those forced upon us by the discovery of cosmic radiation has aspects of continuity.

The modern theory of the quanta of energy and of the electrons, protons, and neutrons of matter again modifies the answer. The full atomistic philosophy emerges. Even time itself may share the concept, and the psychologist's picture of "the flow of duration" is a statistical illusion occasioned merely by insufficient refinement of sensuous experience.

But the behaviour of light in space, the sharp diffraction patterns of the laboratory, are arguments which the atomist must yield to his opponents.

The atomist deals in integers; his theories must ultimately be reduced to whole numbers; he must be able to count the points on a line and enumerate the atoms in a piece of matter. His universe is bounded in fundamental ways; it is broken up into units; his spatial measure must have a lower limit whether it is the electron or whether it is something smaller; his time must be ticked off in multiples of the vibrations of atomic clocks, in terms of the rate of disintegration of heavy atoms such as uranium and radium. Quantities can be sorted and arranged. There is a first thing and a last thing in every sequence. The paradoxes of Zeno have no place in the philosophy of the atomist. Achilles passes over a finite number of electrons and his rate of passing must be measured in terms of the atomic units of time. There are no points without breadth, thickness, and width. These are but creations of the imagination, from whose activities beautiful subjective theories may be evolved. No deduction which depends wholly upon the assumption of "absence of dimension" will have meaning in his world of physics.

On the contrary, the continuist affirms that the picture of reality is a flow, that the substratum of the universe cannot be divided into unitary particles, that there is no categorical imperative to define the first and last of any sequence. The points in a straight line form a non-denumerable class; the integers are insufficient to count the quantities in the universe. The continuists may well have admitted a shaken faith when the discovery of cathode rays pointed unmistakably to the atomic nature of matter and of electricity; but the more recent argument is on their side. The mystery of radiation, once it has emerged from matter as a quantum, is again defined by a continuous function. The medium in which the undulation takes place has lost precise meaning, but the continuous function and the evidence of diffraction remain as powerful arguments as before.

It is instructive to debate these matters with others. The arguments seem to pile up on both sides. One almost believes that both points of view may be true. Energy is an undulation in a continuous ether and yet possesses atomic properties. Matter is a group of indivisible electrons and protons and yet behaves as a train of waves. Nature appears to be in a great conspiracy to prevent us from giving a categorical answer to the question.

### 3. *Man as a Geometer.*

The second rationalistic concept with which the Greeks struggled and which they passed on to their successors is the concept of space and time. That is to say, the Greeks studied the subject of geometry.

We commonly regard geometry as a painful discipline of the school room, with a disagreeable maze of intersecting lines and bothersome angles from which unimportant truths are to be discovered. Nothing is further from the truth. Geometry is a way of surveying the substratum of nature. "Geometry, no doubt, is the knowledge of what externally exists," said Plato. "Then, my noble friend, geometry will draw the soul towards truth, and create the mind of philosophy, and raise up that which is now unhappily allowed to fall down."[6]

In this noble subject one finds the rationalistic powers of man at their highest. In it he has learned to postulate. From a contemplation of its history he has learned the one great truth: that there is probably no absolute truth. All knowledge is deduction from a set of beliefs. You and I belong to the same intellectual faith because there is accord between our systems of postulates.

"Geometry in every proposition speaks a language which experience never dares to utter; and indeed of which she but half comprehends the meaning," said William Whewell. "Experience sees that the assertions are true, but she sees not how profound and absolute is their truth."[7] The subjective model which geometry constructs is the best example that man has found to exhibit the nature of postulates. The mind, unaided by laboratory experiment, may probe a long way into the nature of itself, but finally there is a limit. A time comes when a choice must be made and this choice cannot be made fortuitously. This is exhibited in man's three geometries, the discovery of which was certainly among the most profound contributions passed on by the nineteenth century to the twentieth.

The intellectual excitement of the present century has centered around another discovery which has a curious relationship with these ideals. "Plato said that God geometrizes continually" affirmed Plutarch.[8] This idea has reached fruition in the modern period. One way of paraphrasing Einstein's theories would be to say that: "Nature is the great geometer." We can look at the world of reality in terms of this subjective edifice which the Greeks founded and which modern minds have explored. We need merely add the fourth dimension of time. With this addition the metric of geometry, the distance between two points, is sufficient to tell us the story of the activity of nature. Subjective intuition was not enough to give us a choice in the basic postulates; but a casual survey of some experimental facts unlocks the mystery. We can make an objective choice if we are willing to throw the difficulties back, not upon the mystery of infinity, but upon physical assumption. Light, for example, is the ultimate velocity.

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[6] *The Republic,* book 7.

[7] *The Philosophy of the Inductive Sciences,* London 1858, pt. 1, bk. 1, chap. 6, sec. 1.

[8] *Convivialium dispectationum,* book 8, 1. 2.

The law of gravitation is in one sense merely an agent to aid us in geometrizing. It helps us to decide which of the three logical edifices to occupy. The law of gravitation is an aspect of space-time; its reality is in the subjective creation; it resides in a set of equations.

So what shall we believe? Is the activity of nature but an illusion and is the reality, after all, the subjective geometry which we have discovered in ourselves? Which is the shadow and which the substance? Or is this statement but a manner of speech? Are we after all indulging in arguments which would put to shame the metaphysics of the schoolmen?

### 4. *The Problem of Determinism.*

We turn to another problem which has long been a battle-ground for the philosopher. What is the nature of cause and effect? Is there a determinism in the physical universe? If we were infinitely wise could we tell the events of the future? Is the destiny of the human race already written in the stars?

The mathematician boasts: Tell me the initial conditions of a planet, the coordinates of its position, its velocity at a given instant, the forces under which it is to move, and I shall tell you for all time to come its history. With what feeling of satisfaction Leverrier, the French astronomer, must have looked at the planet Jupiter and reflected: "Here in my tables, embodied in these rows of figures, is the future of your course. For a thousand years my speculative eye is upon your path. Your movements are known to me. Nothing that happens to your course will surprise me. The problem is solved. No younger astronomer will come along and do more than alter figures a few places removed from my decimal points." Is it any wonder that the assembly rose when a mortal so god-like entered to issue his decrees?

Science has been pursuing its course on the assumption that nature follows laws; that one event predetermines another; that a determinism is at work in the natural world. There are intimations among the philosophers that determinism is not limited to inanimate nature, but that man, as a complex of atoms, must also share this universal law. Mark Twain in the cynicism which marked his last philosophy has explained the matter through the words of *The Mysterious Stranger*.[9]

"Among you boys you have a game: you stand a row of bricks on end a few inches apart; you push a brick, it knocks its neighbor over, the neighbor knocks over the next brick—and so on till all the row is prostrate. That is human life. A child's first act knocks over the initial brick, and the rest will follow inexorably. If you could see into the future, as I can, you would see everything that was going to happen to that creature; for nothing can change the order of its life after the first event has determined it. That is, nothing will change it, because each act unfailingly begets an act, that act begets another, and so on to the end, and the seer can look forward down the line and see just when each act is to have birth, from cradle to grave."

In the complicated acts of life we are not able to trace the effects of all the events. Many things are in unstable equilibrium, and the smallest cause may produce a conflagration. A mutilated telegram may create a war devastating a nation. A careless match may destroy a city. The merest breath from a random breeze will upset a top balanced upon its peg.

But the complexities of many situations are solved for us in the doctrines of the theory of chance. We cannot trace the adventures which befall every life, but we can tell with high accuracy the future of a group. The path of Jupiter is not more certainly known than the span of life of the average mortal. No business is more solvent than that of life insurance. Is it due, perhaps, to the statistical model that we are determinists? Is it from our observation of the infallibility of the law of averages that we believe all things can be determined? Perhaps we have lost sight of the adventures of the individual in the behaviour of the mob.

It is of immense speculative interest, therefore, that we see the first failure of statistical law. The equipartition of energy lapses when we try to apply a normal scheme to the mysterious radiations of the ether. We are greatly perplexed at this unkind revelation that nature is not all she seems to be.

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[9] New York, 1916, p. 87.

The Compton effect is another mystery in kind and has led to the formalization by Heisenberg of a "principle of indeterminacy." This startling principle found in the pages of the new physics, that subject devoted to the exploration of the realities, the foe to mysticism and excoriator of magic, is nothing but a denial of the philosophy of determinism.

According to the new principle it is impossible, as was explained earlier in the book, to define the position and velocity of an electron at the same time. The consequences of this revolutionary postulate are of the highest philosophical interest. If we define the position of a thing, but by so doing have destroyed our ability to predict its velocity, then our most cherished belief in the determinacy of natural phenomena falls to the ground. Practically this belief may have no effect upon the ordinary affairs of life, since the principle of indeterminacy is confined to the activities of nature's smallest unit, but its implications are important in the foundations of our philosophy.

P. W. Bridgman, reflecting upon the consequences of this principle takes the following melancholy position:

"The physicist thus finds himself in a world from which the bottom has dropped clean out; as he penetrates deeper and deeper it eludes him and fades away by the highly unsportsmanlike device of just becoming meaningless. No refinement of measurement will avail to carry him beyond the portals of this shadowy domain which he cannot even mention without logical inconsistency. A bound is thus forever set to the curiosity of the physicist. What is more, the mere existence of this bound means that he must give up his most cherished convictions and faith. The world is not a world of reason, understandable by the intellect of man, but as we penetrate ever deeper, the very law of cause and effect, which we had thought to be a formula to which we could force God Himself to subscribe, ceases to have meaning. The world is not intrinsically reasonable or understandable; it acquires these properties in ever-increasing degree as we ascend from the realm of the very little to the realm of everyday things; here we may eventually hope for an understanding sufficiently good for all practical purposes, but no more."[10]

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[10] "The New Vision of Science," *Harper's Magazine*, Vol. 158,1929, pp. 443-451;

Is this denial of determinism in the microcosmic universe the end of exploration? Does it impose a fundamental bound to physical insight? It may be. We may, perhaps, have proceeded as far as we can into the mysteries of the infinitely little. That realm far beyond the direct reach of our senses may be an undiscoverable country to which the intelligence of man has been denied a passport. We have reached an impasse in the case of the luminiferous ether. This new situation is as disturbing as the problem of the medium filling space. As we have advanced to meet it it has forever retreated from our grasp. It has had all the aspects of a mirage. The electron may be a similar illusion and its activities may be forever beyond the comprehension and the reach of man.

However, the human spirit always hopes. These gloomy speculations on the ultimate futility of knowledge may be without foundation.

> "So, when dark thoughts my boding spirit shroud,
> Sweet Hope! celestial influence round me shed
> Waving thy silver pinions o'er my head."[11]

It seems strange when one thinks about it that the discovery of a lack of determinism in nature, a free will of individual electrons, so to speak, should evoke such gloomy speculation. Those who have adhered to the freedom of the human will as one of the individual glories of man, will now decry a similar aspect in primary nature. On the one hand we demand our own freedom, but on the other this very freedom demands an absolute determinism in nature which, while it gives us the power of absolute prediction, sets no bound upon the accuracy with which we shall know nature.

The intrusion of indeterminacy into the problem of knowledge, even though it is confined to the microcosm, raises interesting questions about the foundations of logic itself. With this deep problem, however, we have not been primarily concerned, since we have found no blocking of the progress of science by the failure of the logical processes. When our subjective intuitions have proved inadequate, the logic of mathematics has always been able to surmount the difficulties and knowledge has been advanced.

The logical methods of induction and deduction have been continually employed. By means of induction we have been able to arrive at general law from a contemplation of particular examples; by means of deduction we have been able to derive the natural consequences of these laws. It has been the laws themselves which have surprised us on some occasions, not the methods by means of which they have been discovered. Indeterminacy has entered the logical pattern only when the questions at issue belong to the transfinite.

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[11] Keats: "To Hope."

An able and extensive presentation of the problems of the logical process has recently been published by J. R. Kantor.[12] The reader is referred to this work for a broad appraisal of logic and its relationship to science. At the end of his treatise the author reaches the following suggestive conclusions:

"For logic cosmos is system; it is the organization and ordering of the systemizer's field of interest. But systemizing means creative operations. Systems imply selection, the application of criteria of use or of completeness. Under no circumstances must we lose sight of the operational field. There is always a plenum—a set of events, things and entities—which can never be exhausted by the structuring operations."

### 5. *Man and Power.*

Within the last few years, since the first edition of this work was published, another element has entered into man's quest for the answer to the nature of energy and matter. The story which has been told in the preceding pages has been one of great adventure, but the adventure has been largely one of the mind. The connection between the laboratory of the scientist and the world of politics has been indiscernible. One deals with problems of the microcosm, the other with the use of massive power. The scientist does his best work in solitude, but the leader of government must work in full view of the great public.[13]

Within the recent past, however, the scientist's quest has assumed an aspect of high drama. The spectacle of the terrifying clouds generated by the atomic bomb has shocked all mankind. For here at last was something from the laboratories of the scientists which could be understood by everyone. The man of science had provided a vision of a source of power that transcended anything ever witnessed before by the people of the world.

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[12] *Psychology and Logic*. Bloomington, Indiana; Vol. 1, 1945; Vol. 2, 1950.

[13] Much of this section is taken from an article by the author: "The Growth of Power and its Economic Significance," *School Science and Mathematics*, Vol. 47, 1947, pp. 124-138.

But the most remarkable aspect of this drama is not the pillar of clouds of the atomic bomb; it is found, rather in the chain of ideas which led step by step within the compass of half a century to the remarkable spectacle. Even the wildest imagination never could have foreseen the consequences of an attempt to measure the velocity of the earth with respect to the ocean of ether through which it was assumed to move. How could the most daring dreamer have conjectured that man's exploration of the mysterious puzzle of the increasing mass of the electron with increasing velocity would, in less than half a century, bring to an abrupt conclusion one of the most savage wars in the history of the world? For the first time the lay public and the leaders of governments turned to inspect the scientist who had created this dramatic spectacle. No longer could he be left in his former isolation to continue his impractical musings.

The scientist has seldom been a popular hero in the public mind. The history which we have traced, from the period of Eudoxus, Democritus, and their colleagues through twenty-five centuries, has shown his struggle to probe the mysteries of nature. But the work of these men has never intrigued the public fancy in a measure comparable with that accorded the social and political events of any age and the people associated with them. In Thomas Carlyle's *Hero and Hero Worship* (1840) we find the hero as priest, the hero as a god, the hero as a king, the hero as a man of letters, but there is no evidence that the man of science is regarded as a hero. There has been at times even fanatical hostility to his work as one finds in the long struggle between the proponents of the geocentric theory of the solar system and those who advocated the heliocentric theory. One still remembers the savage words with which the French tribunal condemned Lavoisier to the guillotine: "The Republic has no need for scientists."

There are many reasons for this attitude toward the scientist and his work. He is an austere character; his discoveries often seem to be the work of black magic; he frequently produces theories which challenge general popular belief; he is sometimes an unpleasant fellow for he will get the better of an argument with his tables of data and his pages of formulas; he appears to work more for the glory of discovery than for the goal of gain; and he is a great fool since he publishes his discoveries free that the world may read and benefit by them. In a word he does not appear to care for the accumulation of power, which is the general goal of political leaders. His code of life we have already given earlier in the words of Poincaré: "The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. . ."

However, during the last few decades, imperceptibly if one observes the change from day to day, but in a truly astounding manner, if a longer view of the situation is taken, the scientist has created the greatest social revolution in the history of mankind. Probing the mysteries of the tiny electron, he invented the vacuum tube which, through the endless variety of its uses, has done more, perhaps, to ameliorate the living of the world than the work of all the social agencies from the beginning of time. The publication of the first table of mortality in 1692 by Edmund Halley, combined with the development of the laws of probability and the theory of interest, brought into the world the great edifice of life insurance, which as a source of social improvement can scarcely be estimated. These instances could be multiplied without end to show the vast changes which have been wrought in our civilization by the steady accumulation of exact knowledge and its interpretation by the scientists.

Among the many practical achievements of science, one stands out more prominently than most others. This is the development of power. Since power, precisely defined, means the capacity to do work or to use energy, the development of power is essentially a consequence of the discovery of the nature and sources of energy. This has been the principle theme of the present work—the story of energy. Without the basic experiments of Faraday which brought magnetic and electric forces into a single discipline, there would have been no electrical industry as we know it today. Without the practical researches of James Watt (1736-1819) and the formulation of the principles of thermodynamics by Carnot, Clausius and their successors there could never have been created the gigantic turbines which have contributed so fundamentally to our industrial growth.

Up to the present time the sources of power which we have used have been derived from relatively inefficient sources when compared with the atomic power now within our sight, if not actually within our grasp. The chemist, in his remarkably varied transformations and in the exchanges of energy which he effects, has made use of the shells of electrons that surround the nuclear core of the atom. The burning of a ton of coal is what might be described as a chain reaction in which the valence electrons of the carbon combine with the valence electrons of oxygen to form the oxides of carbon. The energy extracted from the burning of a ton of coal is of the order of 7,600 kilowatt hours. But the energy extracted from the fission of a ton of uranium is of the order of twenty billion kilowatt hours, while that from the complete annihilation of a ton of matter (not yet accomplished) is of the order of twenty trillion kilowatt hours. We thus see again, as we have stated earlier, the incomprehensibly greater energy that resides in the depth of the atom as compared with that in the peripheral shell of electrons with which the chemist has constructed his science.

Let us turn from this dream of power to another and more important matter, the impact of this dream upon the human state. In order to have a broad view of the situation the reader is invited to consider a few figures relating to the growth of population. It has been possible from the work of a number of scholars to construct the curve of population growth for Europe from about 300 B. C. to the present time. Over many centuries this curve remained essentially stable with a fluctuation ranging between 20 and 60 million people. Crude though much of the evidence may be, the curve exhibits with some accuracy the general pattern of the varying fortunes of human society for more than twenty-two centuries. The great plagues that struck the ancient world, the pestilence which raged during the reign of Justinian in the sixth century, and the Black Death of the fourteenth century, are represented in substantial dips in the wavering line of the chart.

But suddenly a change came. Near the beginning of the sixteenth century, and without apparent reason, the population of Europe began to increase. By 1700 the rate of growth had become a noteworthy matter and in the subsequent years developed into a phenomenon such as the world had never seen before. From a population estimated at around 87 millions in 1600 the nations of Europe had increased to the astounding total of 535 million in 1940, a six-fold growth in the space of 340 years. No history can portray more vividly the causes of the incessant tumults of Europe than this simple fact. For the people of each nation press with ever increasing force against the boundaries of their neighbors and seek to escape the fate described by the theory of T. R. Malthus (1766-1834) that populations will ultimately outgrow their food supply.

With this astonishing growth of population before us, we turn to another and brighter aspect of the picture. For the increase in population has been attended by a concomitant increase in standards of living. A few figures will show how great this has actually been, although we, who survey the past from the great height of well-being that has prevailed in our own times, can scarcely credit the testimony of ancient documents. The budget of Cato's slave has come down to us from about 200 B. C. and this provides a clear picture of the standards of living that prevailed in earlier times since it is a measure of the marginal income necessary to sustain life. Measured in terms of wheat, instead of the fluctuating coinage of that date, the annual cost of the food and clothing of this slave was equivalent to 31 bushels of wheat. In the famous edict of Diocletian (301 A. D.), which placed a ceiling upon all the prices in the Roman empire and decreed death for those who operated black markets anywhere, the price of common labor was set at 28 bushels of wheat per year plus maintenance. If we estimate 300 working days in the year and maintenance at the level of that of Cato's slave, these figures show that common labor was purchased for about 12 pounds of wheat per day. The more skilled occupations brought not more than twice this amount. The school teacher who was fortunate enough to have a class of 30 pupils received the equivalent of 45 bushels of wheat, or 9 pounds per working day, while even the highly paid professor of literature and geometry received only four times this amount. The evidence shows that the level of wages never varied greatly from these figures throughout classical times, nor did it differ significantly from the scale paid in England at the beginning of the seventeenth century.

One might be inclined to doubt the evidence, however, except that the same standard of living has been observed in many places in the modern world. A detailed budget study, published in 1928, showed that the scale of wages which prevailed in modern Shanghai was still that of ancient Rome and of England in the sixteenth century. The mill-workers in this thriving city received a per capita annual wage equal to 32 bushels of wheat, almost exactly that given by Cato to his slave, and the mill foremen received only slightly more than twice this amount.

But in those countries which have made practical use of the new knowledge, which have employed kilowatt hours to turn the wheels of their machines, the standard of living has increased in a spectacular manner in spite of the rapid growth of population. In 1600 the daily wages of the laboring man in England would purchase 16 pounds of wheat per day, a figure little better than that which prevailed throughout the Roman world. During the first quarter of the nineteenth century this purchasing power of labor had doubled. By 1882 it had doubled again, and by 1914, at the beginning of the first world war, the price of a day's work had reached the level of 100 pounds of wheat. During the quarter of a century which spans the period between the two world wars there was a vigorous increase in the creation of machines and in the development of power. Electrical production in the United States, which was less than 40 billion kilowatt hours in 1918, was 218 billion in 1942. Wages in England increased during this period at least 150 per cent, and stood in 1940 at an equivalence of 250 pounds of wheat per day. In the United States the prevailing wage-level for common labor has reached the incredible figure of 350 pounds of wheat per day, an increase of three thousand per cent over the wage level observed in the ancient world.

Today the world stands upon the threshold of new discoveries. Within the core of the atom there resides a store of energy, and a source of power, that beggars the imagination. If this store can finally be reached, the standards of living of mankind can be raised much higher than they are today. But whether this great supply of energy will be used for good or evil depends upon the basic nature of man himself. His mind has been able to probe the depths of the atom and will doubtless find a way to unlock the energy which he has discovered. But will his emotional nature in its approach to social and political problems ultimately make the new knowledge the engine for his destruction? Only the future knows the answer to this question.

### 6. *The Wisdom of Prospero.*

Has there been mysticism in this picture which we have traced in the earlier chapters of the book? Have we shown that the universe is irrational? Does it appear that man, peering through his tiny keyhole, will ultimately be unable to explain to himself all that he has seen?

We have traced the concept of the ether from the substantial postulates with which it started down to this modern period in which only the most tenuous wraith remains. We have explained gravitation in terms of a geometry difficult to comprehend. The mystery of least action we have shown is as perplexing as it was when first formulated by Maupertius. The only modern clue is in the fact that we find it occupying a prominent role in the wave theory of the electron.

The most hopeful concept emerging from these mysterious adventures of modern science is that which negates the gloomy law of Clausius. The universe does not appear to be running down. There will never be a time when all the cosmic energy will have attained a universal level and all activity will have ceased. By some mysterious methods, which we do not comprehend but which are implied in the discovery of cosmic radiation, there is a mutual conversion of energy into matter and matter into energy, which assures the eternity of cosmic activity.

Much, however, remains mysterious in nature. We have set many problems and have resolved but few. Wherever we probe we find that new difficulties beset our pathway. But all this is gain. The puzzle that has been solved always lacks savor. The brief satisfaction which is felt when one finds that the answer to his problem is correct is not to be compared with the zest of the conquest. The object of the present book will have been attained if it has set forth the problems of modern science as living things; if it has shown that the nature of man is intimately bound up with his test tubes, his formulas, his galvanometers, and his telescopes. It will be a sorry day for man when he has once definitely answered the questions: Is space-time limited? Does determinism rule the activities of nature? The quest of modern science is not a material one. Its motto could well be the dictum of Keats:[14]

> " 'Beauty is truth, truth beauty,'—that is all
> Ye know on earth, and all ye need to know."

Have we appeared too mystical in all of this? Do the struggles of man with the mysterious ether, the elusive energy, the subjective concepts of space and time, have the appearance of battles waged in the mists of unreality? Or is there a place in the nature of man for these unscientific musings? Is it this sense of the strangeness of the physical facts, of the data of the laboratory itself, that accounts for man's yearning for the beauty best expressed in the language of poetry and music? Can we conclude these philosophical musings better than with the summary of Prospero, whose reflections are typical of the mystery of man and man's relationship to nature?[15]

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[14] "Ode on a Grecian Urn."

"These our actors,
As I foretold you, were all spirits, and
Are melted into air, into thin air:
And, like the baseless fabric of this vision,
The cloud-capp'd towers, the gorgeous palaces,
The solemn temples, the great globe itself,
Yea, all which it inherit, shall dissolve,
And, like this unsubstantial pageant faded,
Leave not a rack behind. We are such stuff
As dreams are made on; and our little life
Is rounded with a sleep."

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[15] *The Tempest,* Act 4, sc. I, 1. 148.

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